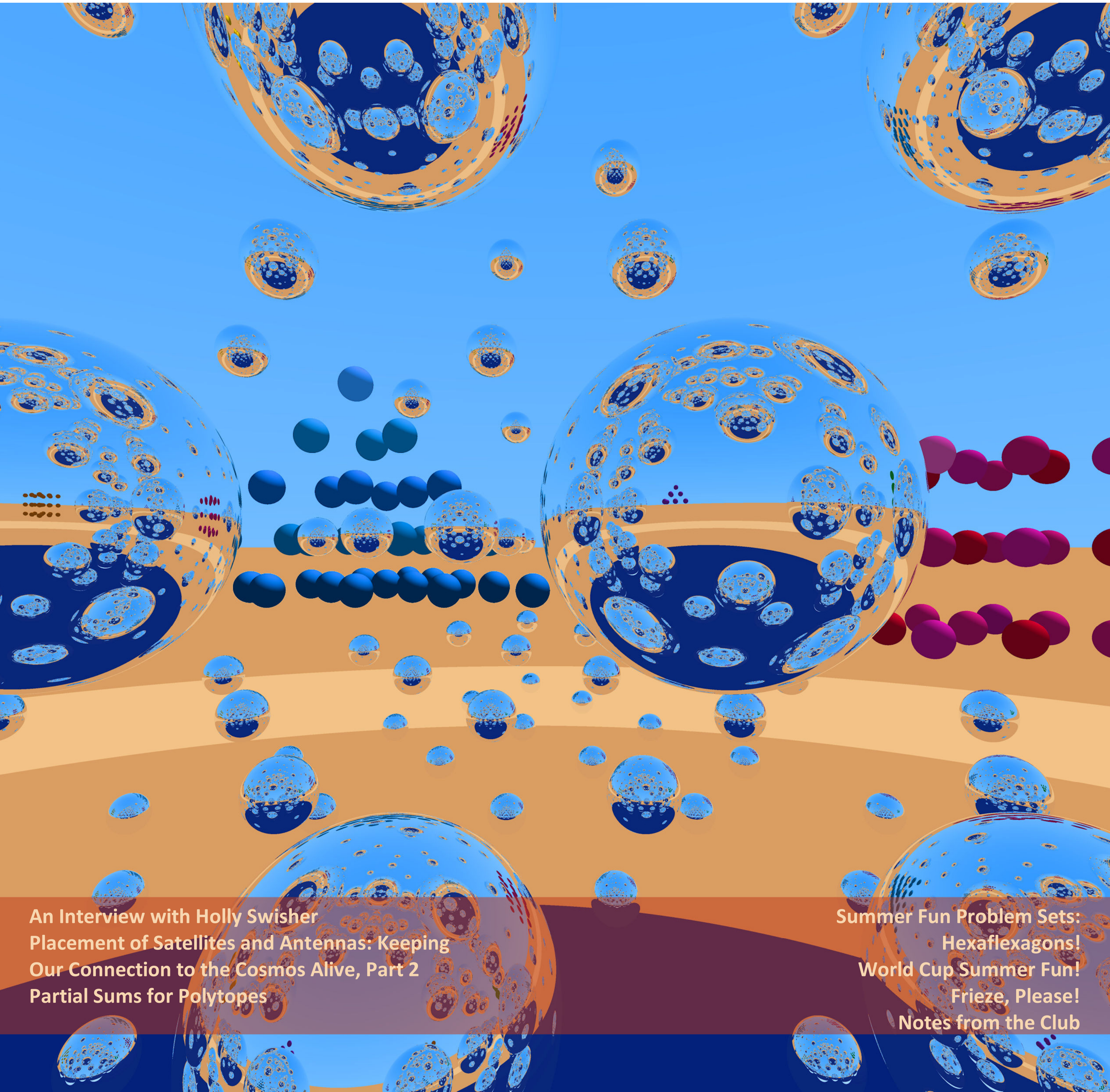
Girls' Angle Bulletin

June/July 2026 • Volume 19 • Number 5

To Foster and Nurture Girls' Interest in Mathematics



From the Founder

Robert Donley was a great friend to Girls' Angle. He single-handedly wrote over 5% of the Bulletin content. He passed away suddenly and unexpectedly on June 24, 2026. This issue is dedicated to him. He will be dearly missed. -Ken Fan, President and Founder

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Editors: Amanda Galtman
Jennifer Sidney
Executive Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: *For Bob Donley* by C. Kenneth Fan. Generated via procedural ray-tracing algorithms written in collaboration with AI. See page 11.

An Interview with Holly Swisher

Starting small and building off existing work that interests me is nearly always how I start approaching a problem.

Holly Swisher is Professor of Mathematics at Oregon State University. She graduated with a Bachelor's degree in mathematics, with honors, from the University of Oregon in 2000. She went on to receive both a Master's degree and a PhD in Mathematics from the University of Wisconsin-Madison in 2005 under the supervision of Ken Ono. She is an expert in number theory.

Ken: What were the key moments that led to you becoming a mathematician?

Holly: My dad was taking a trigonometry class for his plumbing apprenticeship (if I recall correctly) when I was a kid and he really enjoyed talking about math with me. He would challenge me to think a little more creatively about what I was learning at school as well. (For example, he would ask me to subtract a larger number from a smaller number and I would staunchly defend my teacher and say that you weren't allowed to do that! Haha. But then it would get me thinking...). Another key moment was when I found one of my dad's elementary algebra textbooks that had problems in the margins and answers in tiny script at the bottom of the pages so you could check your work. The first section was on elementary set theory and it was like nothing I had ever seen before! It was so delightfully abstract! When I saw that I was indeed learning the definitions and solving the problems correctly I was absolutely tickled.

Ken: Could you please share with us a mathematical idea that you found exciting when you were growing up?

Holly: I really enjoyed prime numbers and also π. I also loved elementary logic once I first saw it, but I didn't see that until early in college.

Ken: Did you experience rough moments in the process of becoming a mathematician? How did you overcome them?

Holly: When I was in college and first moved into advanced undergraduate/early graduate level math from the junior level courses I struggled a lot at first. I had been used to being able to follow at the pace of lectures and didn't have to study so much just to understand the definitions and theorems before even starting the homework problems. The pace was too fast for me to follow that way and I had to retake my 400-level real analysis course again the next year. I also had to work on my study skills and spend more time understanding the definitions. Retaking that course the next year I was able to do much better so it taught me to keep trying and sometimes things just take a while. Another moment that was hard was when I failed my first qualifying exam in graduate school. It was tricky for me to figure out the best way for me to study, but once I figured out that I needed a lot of time on my own to study I was able to pass the second time, and I was able to pass my second exam on the first try. So, I learned a lot about myself in the process.

Ken: One of my favorite quotes from your website is: "One of the beautiful things about number theory is that seemingly simple questions, when deeply investigated, can blossom into rich and intricate discoveries." Could you please describe an example of this?

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Thank you and best wishes,
Ken Fan
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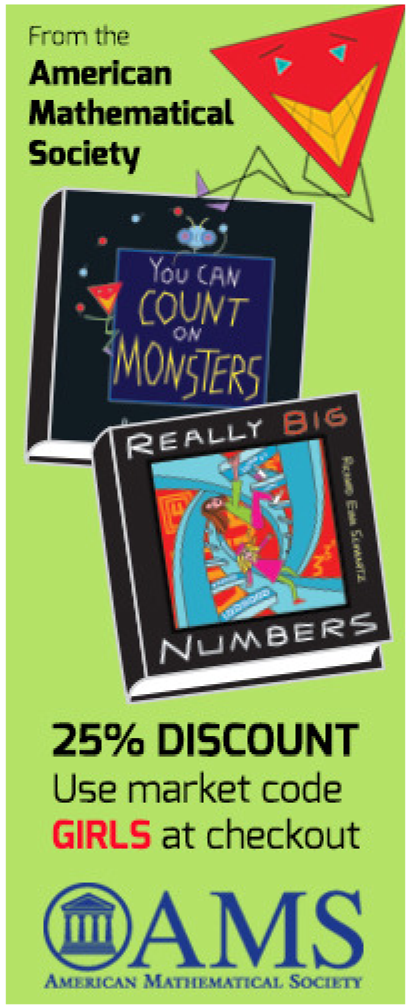

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Placement of Satellites and Antennas: Keeping Our Connection to the Cosmos Alive, Part 2

by Lilly Carrillo, Jillian Cervantes, and Dr. Pamela E. Harris | edited by Jennifer Sidney

Let us explore a toy model of satellite placement. Consider the 3-by-12 grid graph in Figure 10, representing possible locations at which satellites can be placed in the airspace around the Earth. We set the satellite strength t to 3 and the reception strength r to 1. Our task is to determine the optimal positions for these (3, 1) satellites so that they effectively cover the entire 3-by-12 grid graph.[1]

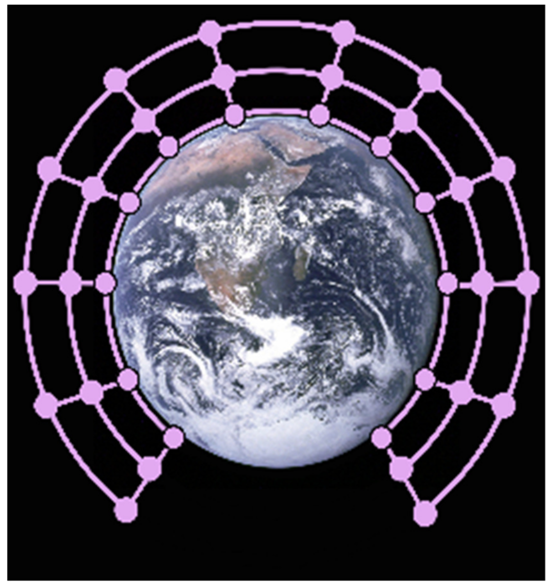


Figure 10. An example of a 3-by-12 grid graph surrounding Earth.
explorer1.jpl.nasa.gov/galleries/earth-from-space/#gallery-10

Example 2. Find a (3, 1) broadcast dominating set for the 3-by-12 grid graph and try to minimize the number of (3, 1) satellites used.

Hint: If you place a (3, 1) satellite at a vertex that has more edges connected to it (i.e., use vertices with the highest **degree**), then you might be able to use fewer satellites!

Let's walk through one possible construction of a (3, 1) broadcast dominating set. We try to be as efficient as possible in our placement; to identify where we place satellites, we use the following notation:

Let c_1 through c_{12} represent the columns of this graph and r_1 through r_3 represent the rows (see Figure 11). The vertex at column i and row j is described by the pair (c_i, r_j).

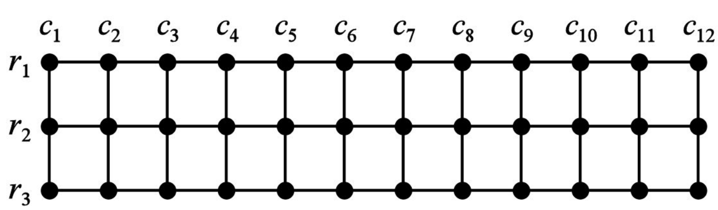


Figure 11. A 3-by-12 grid graph with rows and columns labeled.

We place the first (3, 1) satellite at vertex (c_2, r_2), as in Figure 12. We place checkmarks on the vertices that receive a reception of 1 or greater. You might notice corners in grid graphs are "high maintenance"; by this we mean that it is hard for these vertices to receive reception because they have fewer edges.

[1] See the previous installment in Volume 19, Number 4 for the definition of a (t, r) satellite.

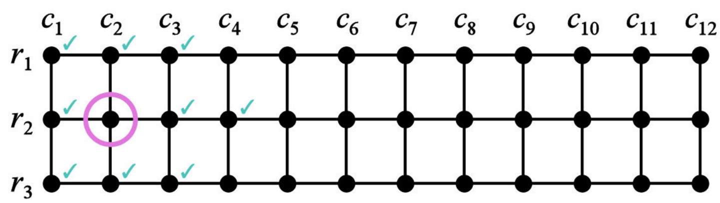


Figure 12. A 3-by-12 grid graph with one (3, 1) broadcast dominating vertex.

In our attempt to be as efficient as possible, we don't want to place the next (3, 1) satellite too close to the (3, 1) satellite at (c_2, r_2). To ensure that the vertices (c_4, r_1) and (c_4, r_3) receive enough reception, the farthest we can place a (3, 1) satellite is on (c_5, r_2) (see Figure 13).

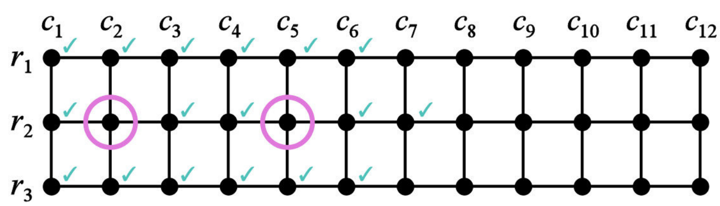


Figure 13. A 3-by-12 grid graph with two (3, 1) satellites.

By following the same pattern of our placement of the last (3, 1) satellites, we add another (3, 1) satellite on (c_8, r_2), as in Figure 14.

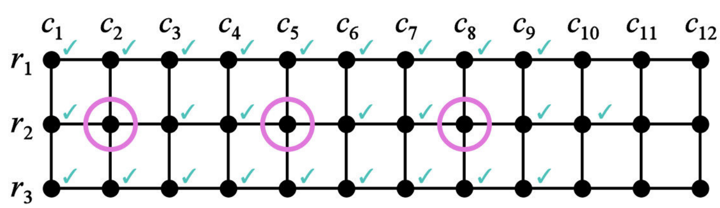


Figure 14. A 3-by-12 grid graph with three (3, 1) satellites.

Finally, we place our last (3, 1) satellite on (c_{11}, r_2), as in Figure 15. We do this to ensure that the corner vertices in c_{12} get reception, too!

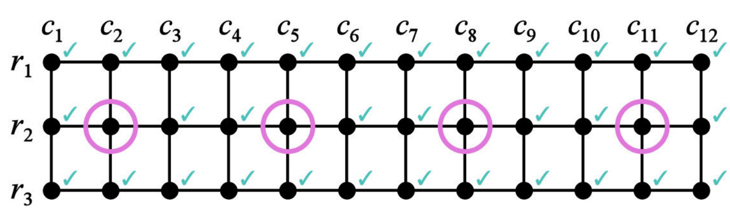


Figure 15. A 3-by-12 grid graph with a (3, 1) broadcast dominating set.

We placed vertices by using our intuition, trying to use as few (3, 1) satellites as possible. The number of (3, 1) satellites used in this construction of a (3, 1) broadcast dominating set is what we call an **upper bound** of the (3, 1) broadcast domination number. It may or may not be the actual (3, 1) broadcast domination number, but at least we know it works with four, and anything greater than four is not the (3, 1) broadcast domination number.

We now set out to find a **lower bound** for the (3, 1) broadcast domination number of the 3-by-12 grid graph. To prove the actual (3, 1) broadcast domination number of the 3-by-12 grid graph, we would need to establish a lower bound that is equal to the upper bound. In general, to show that some number L is a lower bound for the (t, r) broadcast domination number of a graph, you would have to show that if you use fewer than L (t, r) satellites, then there exist vertices that receive reception less than r. With the aid of symmetry, coding, and considering various cases, it is possible to prove a lower bound, but this type of proof does get significantly harder on larger grid graphs!

We introduce some helpful notation. Let us count the number of (3, 1) satellites in each column and represent it as a **pattern** a_1-a_2-…-a_{12}, where a_i is the number of (3, 1) satellites in column c_i. In Figure 15, we have zero (3, 1) satellites in c_1, one in c_2, zero in c_3, and so on. If we continue counting all the way to c_1, we get the pattern 0-1-0-0-1-0-0-1-0-0-1-0, which we illustrate in Figure 16. It is a string of binary numbers – how cool! Since this pattern is a (3, 1) broadcast dominating set for the 3-by-12 grid graph, we will refer to this as a **dominating pattern**, and to any string of numbers within a dominating pattern as a **dominating subpattern**. With that in mind, we leave you to explore the next challenge problem, and we encourage you to use the given hint in your solution.

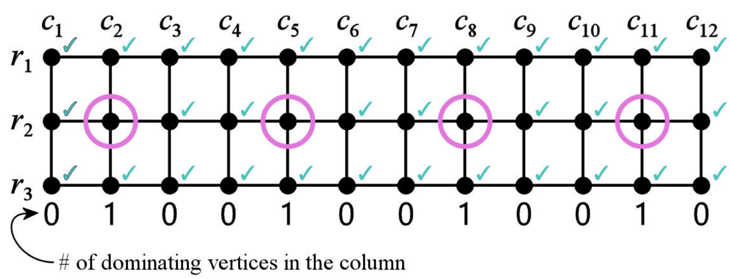


Figure 16. A 3-by-12 grid graph with a (3, 1) broadcast dominating set and labels showing the number of satellites in every column.

Challenge Problem. Show that four is a lower bound of the (3, 1) broadcast domination number for the 3-by-12 grid graph in Example 2.

Hint: Show that a dominating pattern for the 3-by-12 grid graph cannot contain the subpattern 0-0-0 (this just means that there cannot be three columns in a row without a satellite). Conclude that this is equivalent to showing that the 3-by-12 grid graph cannot have a (3, 1) broadcast dominating set of size three.[2]

What if I told you that for any 3-by-n grid graph, with n being any positive integer indicating the number of columns, we can find the (3, 1) broadcast domination number with a formula? Blessing et al[2] used their algorithm (which utilizes subpatterns of grid graphs like we described in the previous problem) to assist in this proof. The formula to find the (3, 1) broadcast domination number for any 3-by-n grid graph G is:

$$\gamma_{3,1}(G) = \left\lceil \frac{n}{3} \right\rceil.$$

[2] In "On (t, r) Broadcast Domination Numbers of Grids," which appeared in *Discrete Applied Mathematics*, Volume 187, Pages 19-40, 2015, Blessing, Johnson, Mauretour, and Insko give a similar algorithm to find lower bounds for the (t, r) broadcast domination numbers of grid graphs. If you are interested in algorithms, check it out!

Test the formula on Example 2!

Challenge Problem. Find an upper bound for the (3, 2) broadcast domination number on a 3-by-12 grid graph as in Figure 17.

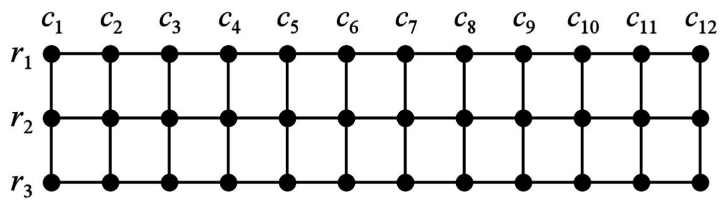


Figure 17. A 3-by-12 grid graph.

To determine the (3, 2) broadcast domination number for this grid graph, you would have to establish a lower bound for the (3, 2) broadcast domination number that is equal to an upper bound found by a construction. Can you also determine the (3, 2) broadcast domination numbers for the 3-by-4, 3-by-5, …, 3-by-12 grid graphs? Using those numbers, can you spot a pattern and make a conjecture (an educated guess) for a formula that gives the (3, 2) broadcast domination number for any 3-by-n grid graph?

The technologies of Voyager 1 have allowed it to send broadcasts that are still active to this day! Some of the best-known discoveries were unveiling a thin ring around Jupiter and two new Jovian moons, Thebe and Metis. If not for Voyager 1's ability to broadcast information through its network, there would be many things about our universe that we wouldn't have discovered. Information passing through a network, or in our case, graphs, keeps connections alive.

To recap our journey, we learned a type of graph domination called (t, r) broadcast domination. We also explored (t, r) broadcast dominating sets on grid graphs, (t, r) broadcast domination numbers on grid graphs, and a formula for (3, 1) broadcast domination of a 3-by-n grid graph.

Since (t, r) broadcast domination is fairly new to graph theory, there is much left to be explored! For example, if you know the (t, r) broadcast domination number, in how many ways can you place those (t, r) satellites so that you still have a (t, r) broadcast dominating set? Coding to the rescue? You can get a PhD in this; come join us![3]

References

[1] J. Cervantes and P. E. Harris, (t,r) Broadcast Domination Numbers and Densities of the Truncated Square Tiling Graph. Under peer review. https://arxiv.org/abs/2408.13331 (2024).

[2] J. Cervantes and P. E. Harris, Optimal Resource Placement: From Disneyland to Dominating Sets, Part 1, *Girls' Angle Bulletin*, Volume 17, No. 3, pp. 14-17 (2024).

[3] J. Cervantes and P. E. Harris, Optimal Resource Placement: From Disneyland to Dominating Sets, Part 2, *Girls' Angle Bulletin*, Volume 17, No. 4, pp. 8-12 (2024).

[3] As Tony Yu Chang did! If you're interested in his thesis, see "Domination Numbers of Grid Graphs," ProQuest LLC, Ann Arbor, MI, 1992. Thesis (Ph.D.)–University of South Florida.

Partial Sums for Polytopes[1]

by Robert Donley[2]
edited by Amanda Galtman

In this installment, we consider the geometry underlying some of the counting formulas of previous installments, in particular, Pascal's identity and the hockey stick rule. The new objects of interest are **convex polytopes,** with familiar examples given by regular polygons in two dimensions and the Platonic solids in three dimensions. Rather than work in full generality with formal definitions, we confine ourselves to the special cases of simplices, cubes, and cross-polytopes, which we approach through their defining equations.

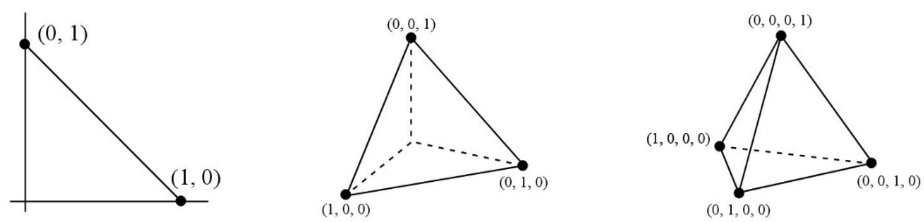
For polytopes in three dimensions, we use the common notions of vertex, edge, and face. We also consider faces in higher dimensions when applicable. We begin with simplices.

Definition: The ***m*-simplex** Δ_m is the set of real-valued solutions $(x_0, x_1, \ldots, x_m)$ to the equation

$$x_0 + x_1 + \ldots + x_m = 1$$

with each $x_i \geq 0$.

For example, Δ_1 is a segment of the line $y = -x + 1$ in the xy-plane, Δ_2 is a triangle in three-space, and, while defined in four-space, Δ_3 can be identified with the usual tetrahedron in three-space.



The **vertex** v_i of Δ_m is the distinguished point of Δ_m with $x_i = 1$ and all other entries equal to 0.

Exercise: Prove that Δ_m has $m + 1$ vertices. If every pair of vertices of Δ_m has an edge joining them, how many edges are there? If every set of $n + 1$ vertices in Δ_m span an n-dimensional face, how many n-dimensional faces are there?

Definition: Fix a positive integer k. The **dilation** of Δ_m by a factor of k, denoted $k\Delta_m$, is the set of all solutions to

$$x_0 + x_1 + \ldots + x_m = k$$

with each $x_i \geq 0$.

Dilation produces a similar shape, but the scaling can increase the number of integer solutions. The integer points in $k\Delta_m$ are the weak compositions of k with $m + 1$ parts, and, as seen in the

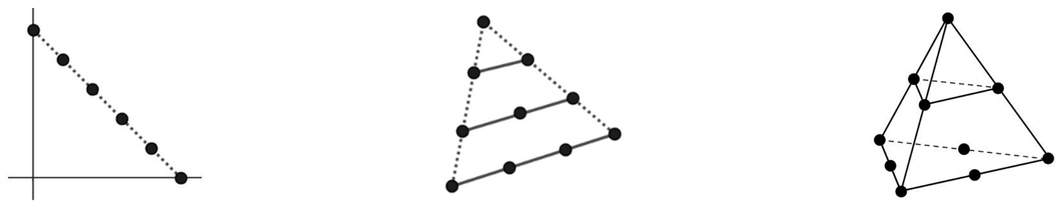
[1] This installment is 26th in a series that began in Volume 15, Number 3 and the fourth of a new subseries.
[2] This content is supported in part by a grant from MathWorks.

previous installment, their number is $\binom{m+k}{m}$. We partition these solutions by fixing $x_m = n$ for each $0 \le n \le k$. That is, given such n, we now solve

$$x_0 + x_1 + \ldots + x_{m-1} = k - n,$$

which in turn has $\binom{m+k-n-1}{m-1}$ solutions. This approach effectively slices $k\Delta_m$ into dilated simplices of one less dimension, as seen in the following exercise.

Exercise: Label the integer points in the following figures, which represent $5\Delta_1$, $3\Delta_2$, and $2\Delta_3$, respectively. Be sure to respect the partitioning by x_m. Express the total number of integer points as a sum of the number of integer points in the smaller simplices.



As seen in the previous exercise, the counting formula now refines the hockey stick rule

$$\binom{m+k}{m} = \binom{m+k-1}{m-1} + \binom{m+k-2}{m-1} + \binom{m+k-3}{m-1} + \ldots + \binom{m-1}{m-1}$$

from the preceding installments. Furthermore, the partial sum operation is expressed geometrically as a partitioning of a dilated m-simplex into $(m-1)$-simplices of progressively smaller scaling.

Exercise: In $3\Delta_3$, find the pattern of the binomial coefficients in Pascal's triangle corresponding to this way of partitioning

Exercise: Describe Pascal's identity as a partition of the integer points of a dilated m-simplex $k\Delta_m$. For the figures above, give the corresponding formula and circle the corresponding subsets of integer points for $m = 1, 2, 3$.

In particular, this interpretation of Pascal's identity splits the integer points of $k\Delta_m$ into those of $k\Delta_{m-1}$ (with $x_m = 0$) and $(k-1)\Delta_m$ (with $x_m > 0$).

Exercise: Use the hockey stick rule to draw $2\Delta_4$ as a stack of tetrahedra. Label the integer points. How many tetrahedra are needed to draw $k\Delta_4$?

An equivalent way to realize $k\Delta_m$ follows from converting the integer points to integer partitions. That is, we consider the coordinates of an integer point in $k\Delta_m$ as the multiplicities of parts taken from $\{0, 1, \ldots, m\}$. See the series that begins with Volume 16, Number 5 of the Bulletin for notation and more theory on integer partitions.

For $0 \le n \le m$, let x_n denote the multiplicity of n in the corresponding partition. This definition allows parts of size 0, so the number of positive parts may vary. Since our convention is to list parts in non-increasing order, the integer points (2, 2, 0, 0), (1, 1, 1, 1) and (0, 0, 2, 2) in $4\Delta_3$ correspond to the partitions 1100, 3210, and 3322.

Exercise: For the simplices $5\Delta_1$, $3\Delta_2$, and $2\Delta_3$ above, relabel each integer point with its corresponding integer partition. Characterize the full set of partitions obtained from $k\Delta_m$.

Consider the family of m-cubes C_m; we studied the m-cube graphs Q_m in Volume 18, Number 1 of the Bulletin. The vertices of C_m are the m-tuples $(x_1, \ldots, x_m)$ with each $x_i = 0$ or 1. For $0 \le d \le m$, the faces of C_m of dimension d are given by setting $m - d$ of the variables x_i to 0 or 1 and letting the remaining variables take all possible values between 0 and 1.

Exercise: Draw C_2 and C_3, and identify each face in terms of fixed x_i. Then identify each face of Q_4 in this manner. How many faces of each dimension are there in each case?

Exercise: Prove that, if we dilate C_m by a factor of k and define $\rho_m(k)$ as the number of integer points in kC_m, then $\rho_m(k)$ equals $(k + 1)^m$.

Exercise: With the notion of reciprocity from the previous installment, describe the reciprocity rule for $\rho_m(k)$ and the rule geometrically.

A version of Pascal's identity for kC_m is given simply by

$$(k + 1)^m = ((k + 1)^m - k^m) + k^m$$

with corresponding hockey stick rule

$$(k + 1)^m = ((k + 1)^m - k^m) + (k^m - (k - 1)^m) + \ldots + (k - 1) + 1.$$

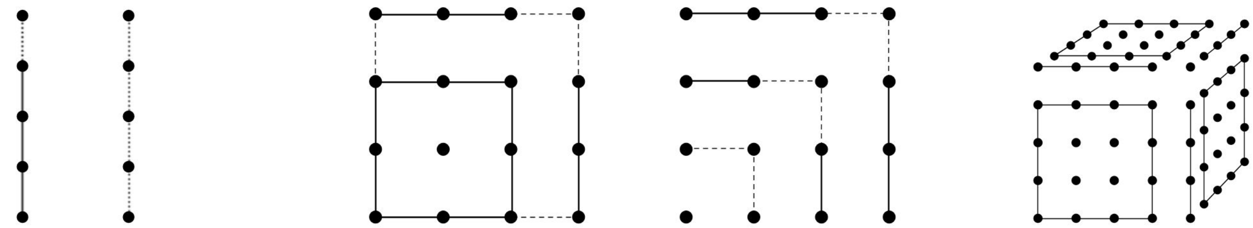
In the first equation, k^m represents the number of integer points of the $(k - 1)C_m$ based at the origin, and the difference counts all integer points in faces (of all possible dimensions) containing $(k, \ldots, k)$, located at the upper right corner in the figures at the top of the next page. For the expression $(k + 1)^m - k^m$ in the first equation, the binomial theorem applies directly to give

$$(k + 1)^m - k^m = \binom{m}{m-1}k^{m-1} + \ldots + \binom{m}{2}k^2 + \binom{m}{1}k + \binom{m}{0}.$$

The following exercise gives an interpretation of this sum in terms of faces.

Exercise: Prove that there are m edges of C_m adjacent to $(1, \ldots, 1)$. Then prove that there are $\binom{m}{d}$ faces of C_m of dimension d adjacent to $(1, \ldots, 1)$ and that the interior of a corresponding face of kC_m contains k^d integer points.

As with the usual hockey stick rule, iteration of the Pascal's identity for C_m gives a full decomposition of the cube into nested collections of face sets. On the top of the next page , the first and second figures (both vertical segmented sticks, side by side) depict Pascal's identity and the hockey stick rule for $4C_1$. The third and fourth figures depict Pascal's identity and the hockey stick rule for $3C_2$. The final figure depict $4C_3$ with $3C_3$ removed.



Of course, in the case of kC_2, $(k + 1)^2 - k^2 = 2k + 1$. In this case, the hockey stick rule gives the familiar identity

$$1 + 3 + 5 + \ldots + (2k + 1) = (k + 1)^2,$$

with a well-known visual interpretation as seen in the second square above.

Exercise: Draw $3C_3$ with its integer points. Verify Pascal's identity by counting faces adjacent to (3, 3, 3), and draw the partition corresponding to the hockey stick rule as in the above figures.

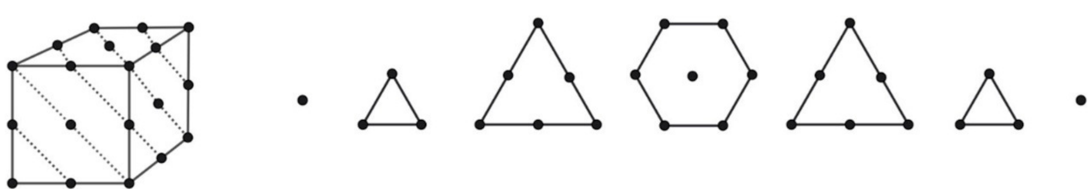
We can find the generating function $F_m(x)$ for $\rho_m(k)$. It follows from $\rho_1(k) = k + 1$ that

$$F_1(x) = 1 + 2x + 3x^2 + 4x^3 + \ldots = \frac{1}{(1-x)^2}.$$

The general case is given in the following exercise.

Exercise (calculus required): In calculus, the power rule says that $(x^{k+1})' = (k + 1)x^k$. Noting that $\rho_{m+1}(k) = (k + 1)\rho_m(k)$, use the power rule to prove that $F_{m+1}(x) = F_m(x) + xF_m'(x)$. Calculate $F_m(x)$ for $m = 2, 3$.

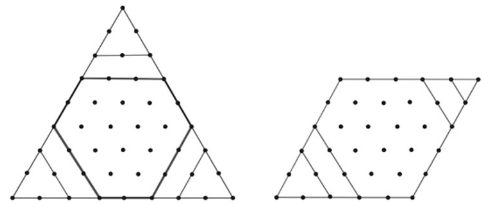
For an alternative partition of the integer points in kC_3, we can intersect kC_3 with each equilateral triangle $x_1 + x_2 + x_3 = n$ for $0 \le n \le 3k$. Now, each x_i satisfies $0 \le x_i \le k$. Also, for each n, the sections undergo a progression from triangles to hexagons back to triangles in a specific way that allows for counting. The figures below show the cases $k = 1$ and 2.



These cases yield the respective formulas $1 + 3 + 3 + 1 = 8$ and $1 + 3 + 6 + 7 + 6 + 3 + 1 = 27$.

For the general formula, we have two parts to consider. For $0 \le n \le k$, the nth triangle has $\binom{n+2}{2}$ integer points; the situation is similar when $2k \le n \le 3k$. For $k < n < 2k$, the section is a hexagon with at most two side lengths.

To count the integer points, first note that the hexagon is obtained by removing the three triangles outside the hexagon from the larger triangle. On the other hand, if we keep one of these triangles in place, we can complete the figure to a parallelogram by adding another triangle, possibly of different size. The figure to the right shows the section for $k = 5$ and $m = 8$, which gives a hexagon with sides lengths 2 and 3.



Exercise: Label the integer points of the large triangle above. In particular, verify that the points in the hexagon satisfy both defining conditions.

Exercise: For general k and $k < n \leq 1.5k$, prove that the number of integer points in the corresponding hexagon is given by the formula

$$(k+1)^2 - \binom{n-k+1}{2} - \binom{2k-n+1}{2}.$$

Exercise: Write $(k + 1)^3$ as a sum according to these triangular cross sections for $k = 3$, 4, and 5. Draw the hexagons for each k, and label each set of integer points. You can use the triangle above as a template for spacing points.

Exercise: Can you show that the terms of the sum can be rearranged into $k + 1$ sums, each equal to $(k + 1)^2$? Hint: each hexagon completes to a parallelogram using two triangles, possibly of different sizes.

Exercise: Rearrange the sums in this manner for $k = 2, 3, 4, 5$.

Exercise: Formulate the corresponding problem for kC_4. Note that, in this case, each dilated simplex is a tetrahedron.

For our final class of polytopes, we have the cross-polytopes, which generalize the octahedron.

Definition: The **cross-polytope** CP_m of dimension m is the set of all solutions $(x_1, \ldots, x_m)$ to the inequality

$$|x_1| + \ldots + |x_m| \leq 1.$$

Since each $|x_i| \leq 1$, it follows that $-1 \leq x_i \leq 1$, and CP_m is a subset of an m-cube with side length 2. The first three CP_m are a line segment, a square, and an octahedron.

To analyze CP_m in general, we first consider the points in the sector with all $x_i \geq 0$. Denote this set by CP_m^+. All other solutions are obtained by sign changes to a solution in CP_m^+.

Exercise: Prove that CP_m partitions into 2^m copies of CP_m^+ with overlaps on the boundaries. Draw the partitions for $m = 1, 2, 3$.

As with simplices and cubes, we are interested in counting integer points in the dilated cross-polytope kCP_m. The latter space is the set of solutions to

$$|x_1| + \ldots + |x_m| \leq k.$$

If we fix some $0 \le n \le k$, the solutions to $x_1 + \ldots + x_m = n$ in kCP_m^+ form a dilated $(m - 1)$-simplex, and the integer points of CP_m^+ partition accordingly. That is, the number of integer points in CP_m^+ is

$$\binom{m-1}{m-1} + \binom{m}{m-1} + \ldots + \binom{m+k-1}{m-1} = \binom{m+k}{m}.$$

Exercise: Calculate this count directly by appending an extra variable x_{m+1}.

To calculate the number of integer points in kCP_m using the counting formula from kCP_m^+, we would need complicated bookkeeping for where the boundaries overlap. Instead, we derive the generating function of the counting function $\rho_m(k)$ for the integer points in kCP_m. For each m, we define $\rho_m(0) = 1$.

Exercise: Prove that $\rho_1(k) = 2k + 1$ and $\rho_2(k) = 2k^2 + 2k + 1$.

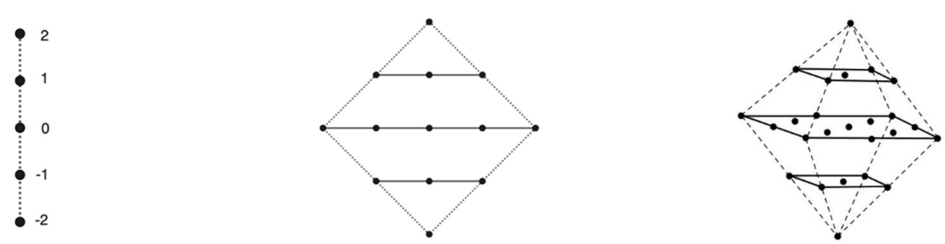
For the general case, we see recursively that

$$\rho_m(k) = \rho_{m-1}(k) + 2\rho_{m-1}(k-1) + 2\rho_{m-1}(k-2) + \ldots + 2\rho_{m-1}(0).$$

When $x_m = 0$, we obtain a single copy of kCP_{m-1}; otherwise, for $|x_m| = n$ with $0 < n \le k$, we obtain two copies of $(k - n)CP_{m-1}$. See the figures for $2CP_m$ below for $m = 1, 2, 3$. Note that each $(k - n)CP_{m-1}$ occurs as a horizontal slice of the figure.

We now proceed exactly as in the installment "A Generating Function for Conjoined Compositions" (Volume 19, Number 2). Let $F_m(x)$ denote the generating function for $\rho_m(k)$. Recursively, we have $F_1(x) = \frac{1+x}{(1-x)^2}$, $F_{m+1}(x) = \frac{1+x}{1-x} F_m(x)$, so that

$$F_m(x) = \frac{(1+x)^m}{(1-x)^{m+1}}.$$



Exercise: Construct the table of values for $\rho_m(k)$ similar to the table at the end of the noted installment, using either the corresponding hockey stick rule or four-term Pascal's identity. Give a general formula for $\rho_m(k)$ in terms of binomial coefficients, describe the zeros of $\rho_m(k)$, and find the corresponding reciprocity formula.

For a proper treatment of general polytopes in any dimension, we use either convex hulls of sets of points, which amounts to filling in all lines between points, or intersections of half-spaces. In the next few installments, we consider one more familiar example, the Birkhoff polytope B_3, for which the integer points are the semi-magic squares of size three.

Summer Fun!

The best way to learn math is to do math. Here are the 2026 Summer Fun problem sets.

We invite all members and subscribers to send any questions and solutions to us at girlsangle@gmail.com. We'll give you feedback and might put your solutions in the Bulletin!

The goal may be the lake, but who knows what wonders you'll discover along the way?

In the August/September issue, we will provide some solutions. You could wait to see the answers, but you will learn a lot more if you try to solve these problems on your own first.

Some problems are very challenging and could take several weeks to solve, so please don't approach these problems with the idea that you must solve them all. Our main goal is to give you some interesting things to think about.

If you get stuck, try to formulate a related question that you can see a way to actively explore to get your mind moving and your pencil writing. If you don't understand a question, email us.

If you're used to solving problems fast, it can be frustrating to work on problems that take weeks to solve. Try to enjoy the journey and don't be afraid to follow detours. It's like hiking up a mountain. Getting to the top rewards one with a spectacular view, but during the journey, there's a lot to see and experience. So here's a meta-problem for those of you who feel frustrated when doing these problems: see if you can dissolve that frustration and replace it with a relaxed, optimistic sense of adventure!

This is Summer Fun, not Summer Torture!

Summer Fun!

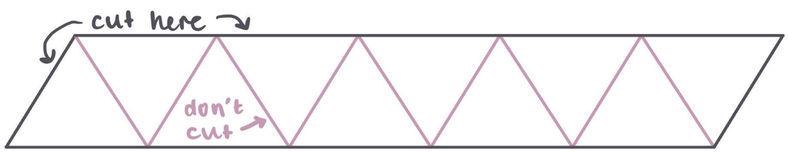
Hexaflexagons!

By AnaMaria Perez

Ready to build a hexaflexagon? This magical hexagon has 3 distinct faces! Build as many as you like—they're perfect for drawing on, cutting up, or even sending secret messages.

Download and print the template at https://tinyurl.com/2x2e7n94, or use the QR code. Don't have a printer? Make your own template! (See box.)

If you don't have access to a printer, you can make your own template. The goal is to make a strip of 10 congruent equilateral triangles in the following configuration:



Can you figure out how to do that as accurately as you can? Please give it a go. (If you want to use tools, like a compass and straightedge, feel free to do so!) If you're stuck, here's a hint: To draw a really accurate equilateral triangle, you can use tools such as a ruler and compass or a ruler and a protractor. To do it with a ruler and compass, check out the Learn By Doing in Volume 8, Number 2 of this Bulletin.

After you've made one equilateral triangle, make 9 more in the configuration above. Alternatively, you could cut the paper into a strip the height of the triangle by cutting along parallel lines, one through a side of the triangle and the other through the vertex opposite that side, then fold creases to mark out the other triangles in the strip.

Now everyone is in the same place whether you printed and cut or drew it yourself!

Making the Hexaflexagon

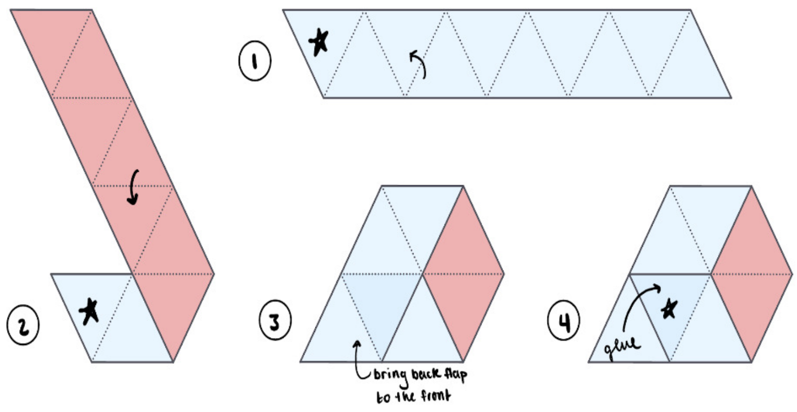
Lay the strip with the leftmost triangle pointing down. Mark this triangle with a star; it will remain fixed throughout. Follow the following folding steps:

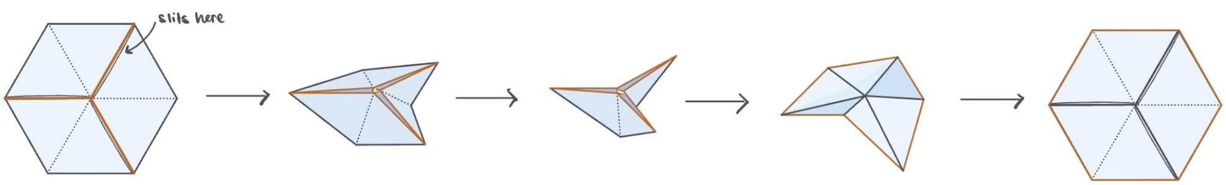
Step 1. Fold the paper up along the edge separating the 3rd and 4th triangles.
Step 2. Fold the paper down along the line separating the 6th and 7th triangles. You should not need to lift the paper up to do this, and the result will cover the original triangle.
Step 3. Bring the starred triangle in front of the strip covering it.
Step 4. Glue the last triangle to the first triangle.

Congrats! You have made a hexaflexagon!

Summer Fun!

It may look like any hexagon, but unlike a drawing on a flat piece of paper, this is a physical object you can transform. The hexaflexagon has hidden layers, and you can cleverly fold and unfold it to reveal a completely different face from the ones showing, comprised of 6 completely different triangles!





1. Can you find all 3 different faces? Try folding and unfolding to reveal the third one.

2. Draw a circle in the center of one of the faces. Now, flex the hexaflexagon to reveal the hidden third face. What happens to the circle you drew? Is it still visible on the surface, or has it disappeared into the structure?

3. What happens when you cut the hexaflexagon? Try cutting it in different ways to see if anything cool happens.

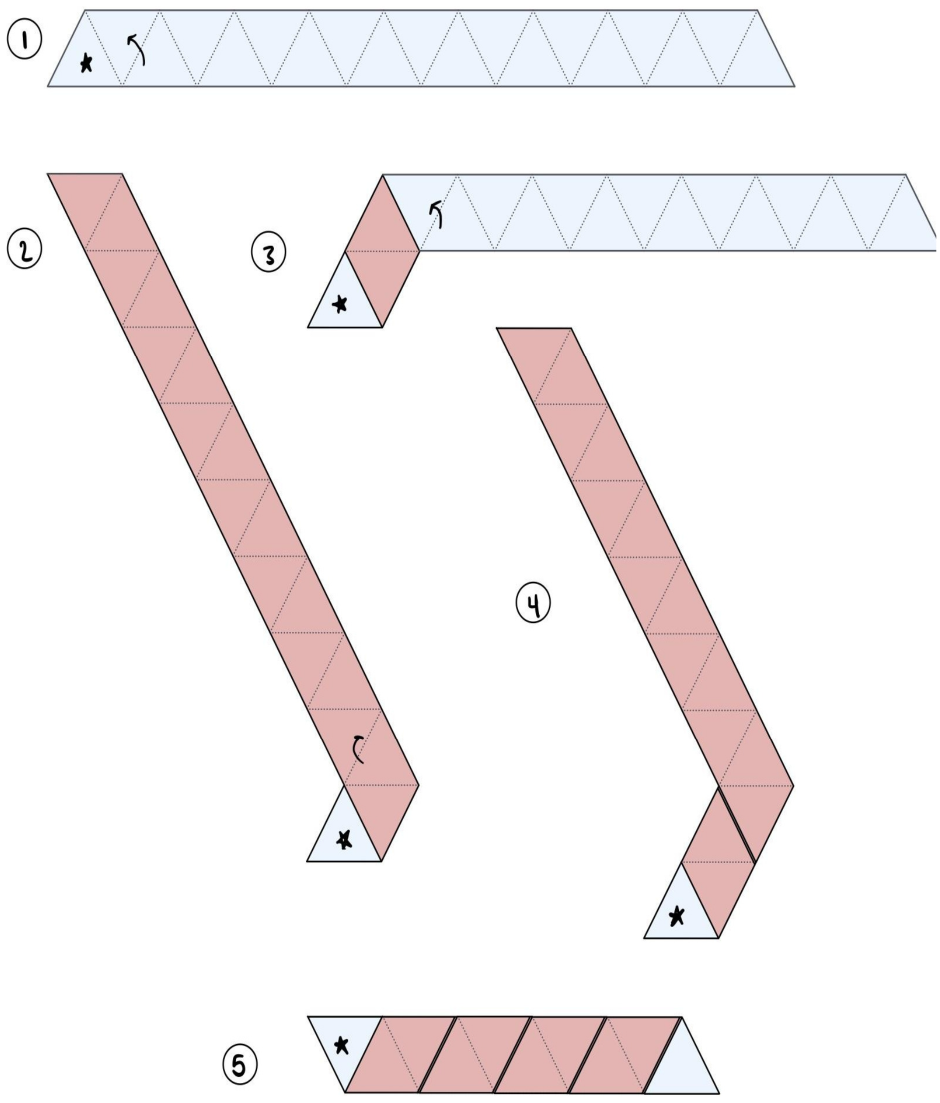
Can you build a hexaflexagon with more different “faces”? See if you can follow these steps to make a hexaflexagon with 6 faces. A diagram is provided on the next page. Start with a strip of 19 equilateral triangles, with the base of the first triangle at the bottom.

Step 1. Fold the strip up between the 2nd and 3rd triangles.
Step 2. Fold the strip behind between the 4th and 5th hexagons. You should be starting to wrap around.
Step 3. Fold up again between the 6th and 7th hexagons.
Step 4. Continue until it is entirely folded in this fashion. You should end with a strip that looks like the one for the 3-sided hexaflexagon!
Step 5. Use this strip to follow the 3-sided hexaflexagon instructions.

4. How many faces does this version have? How does this compare to your initial expectation based on the strip's length? Use numbers or colors to track the faces as you explore!

5. Try and come up with a system to quickly navigate from face to face. There are many correct answers to this, and one possible solution will be given in the next Bulletin!

6. Can you build a hexaflexagon with even MORE faces? How could you do this? How many triangles would you need? If paper were infinitely thin so you could fold it as many times as you want, what process could you use to generate hexaflexagons with more and more faces?



Summer Fun!

World Cup Summer Fun!

by Ella Wilson

This summer, the 2026 Men's FIFA World Cup—the premier international soccer tournament—is taking place across North America. Held quadrennially, this year's event features 48 men's national teams competing for the title of World Champion. One of the locations where matches are being held is Gillette Stadium in Foxborough, MA, just south of Boston!

The tournament spans 16 major host cities. In order for teams and fans to travel between these cities as quickly as possible, they will have to fly. Because Earth is a sphere, the shortest flight paths aren't straight lines on a flat map but segments of "great circles" (circles that split the Earth into two equal hemispheres, much like the equator).

1. **Try this out yourself!** Take a spherical ball (such as a soccer ball) and a long piece of string. Choose any two points on the surface of the ball. Hold the string down with one finger at the first point you chose, then stretch it to the second point. Now, holding the string tight at both points, continue to wrap the string around the ball until you get back to the first point (you may need to ask a friend to help hold everything down tightly). You should find that the string wraps to form a great circle! Try this with several different pairs of points.

2. Use the intuition you built with the string to explain why, no matter what pair of points you choose, the shortest path between them will always lie along a great circle. *Hint: It doesn't matter what point you choose to be your starting point, so you can always assume you are beginning at the North Pole. Make sure you understand why this assumption is valid!*

On the surface of a sphere, we can construct polygons out of segments of great circles, just like we do in standard Euclidean (flat) geometry using straight line segments. However, there are quite a few differences between spherical polygons and those in Euclidean space. The first major difference is that while a Euclidean polygon must have at least three sides, spherical geometry allows for 2-sided polygons, which are called lunes.

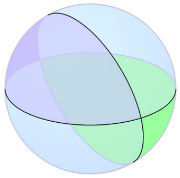


Figure 1. Four 2-sided spherical polygons called lunes

In both spherical and Euclidean geometry, we call a polygon "regular" if all of its side lengths are equal to one another and all of its interior angles are equal to one another.

3. Show that all lunes are regular polygons.

4. In Euclidean geometry, every regular polygon of a specific type has the same interior angle measure regardless of its size; for example, an equilateral triangle always has angles measuring 60° (or $\pi/3$ radians). Is the same true for spherical polygons? Why or why not?

Summer Fun!

5. Use what you learned above and the fact that the total surface area of a sphere of radius R is $4\pi R^2$ to derive a formula for the area of a lune.

A **spherical triangle** is a figure composed of three segments belonging to different great circles such that:

- Every interior angle is less than π radians (180°).
- Every side length is less than half the circumference of the sphere.

6. In Euclidean geometry, the angles of every triangle add up to exactly 180° (π radians). Try to construct some examples of spherical triangles whose angle measures do not add up to π. Is there a maximum possible angle sum for a spherical triangle? Is there a minimum?

7. Extend all three sides of a spherical triangle so that they form three complete great circles around the sphere. How many distinct spherical triangles are created on the surface? How many unique lunes is each triangle contained within?

One of the most remarkable properties of spherical geometry is that we can determine the exact area of a triangle simply by looking at the sum of its interior angles.

Girard's Theorem. Let $\triangle ABC$ be a triangle on a sphere of radius R with interior angles α, β, and γ measured in radians. Then the area of $\triangle ABC$ is $R^2(\alpha + \beta + \gamma - \pi)$.

8. Use your findings from questions 5 and 7 to write a formal mathematical proof of Girard's Theorem.

Note: If you want to learn how to calculate the area of a spherical quadrilateral, see the "Ice Hockey Shot Angles" article in the February edition of the Bulletin!

Just like the classical Euclidean "Law of Cosines" allows us to calculate the angles of a flat triangle given its three side lengths, there is a **Spherical Law of Cosines**. If we have a spherical triangle on a sphere of radius R with side lengths (arc lengths) a, b, and c, and interior angles A, B, and C opposite those respective sides, then:

$$\cos\frac{c}{R} = \cos\frac{a}{R}\cos\frac{b}{R} + \sin\frac{a}{R}\sin\frac{b}{R}\cos C\,.$$

9. Below is a matrix showing the great-circle distances (in kilometers) between the primary airports of the 16 World Cup host cities. Using these values, an estimated Earth radius of $R = 6378$ kilometers, and the Spherical Law of Cosines, identify the three cities that form a triangle with the largest sum of interior angles, and the three cities that form a triangle with the smallest sum of interior angles.

Summer Fun!

City	Atlanta	Boston	Dallas	New York/NJ	Guadalajara	Houston	Los Angeles	Kansas City	Mexico City	Miami	Monterrey	Philadelphia	Seattle	San Francisco	Vancouver	Toronto
Atlanta	0	1521	1175	1198	2364	1107	3126	1113	2144	959	1745	1071	3505	3435	3612	1191
Boston	1521	0	2509	323	3870	2567	4193	2016	3665	2028	3227	450	4005	4341	4034	715
Dallas	1175	2509	0	2204	1510	362	1983	741	1511	1803	846	2093	2668	2352	2819	1929
New York/NJ	1198	323	2204	0	3549	2249	3941	1753	3342	1750	2909	128	3855	4118	3899	558
Guadalajara	2364	3870	1510	3549	0	1322	2106	2243	459	2422	670	3425	3448	2644	3643	3382
Houston	1107	2567	362	2249	1322	0	2215	1036	1232	1549	663	2129	3013	2628	3169	2060
Los Angeles	3126	4193	1983	3941	2106	2215	0	2190	2500	3763	1980	3856	1537	544	1741	3495
Kansas City	1113	2016	741	1753	2243	1036	2190	0	2248	2015	1585	1666	2392	2407	2501	1345
Mexico City	2144	3665	1511	3342	459	1232	2500	2248	0	2051	713	3215	3757	3029	3945	3247
Miami	959	2028	1803	1750	2422	1549	3763	2015	2051	0	1983	1634	4380	4154	4503	1989
Monterrey	1745	3227	846	2909	670	663	1980	1585	713	1983	0	2787	3097	2475	3278	2717
Philadelphia	1071	450	2093	128	3425	2129	3856	1666	3215	1634	2787	0	3817	4048	3869	557
Seattle	3505	4005	2668	3855	3448	3013	1537	2392	3757	4380	3097	3817	0	1093	204	3307
San Francisco	3435	4341	2352	4118	2644	2628	544	2407	3029	4154	2475	4048	1093	0	1288	3628
Vancouver	3612	4034	2819	3899	3643	3169	1741	2501	3945	4503	3278	3869	204	1288	0	3346
Toronto	1191	715	1929	558	3382	2060	3495	1345	3247	1989	2717	557	3307	3628	3346	0

Table 1: Distances Between World Cup Host Cities (in Kilometers)

The geometry of a soccer ball itself poses some very interesting questions! There are many different ways to tile a sphere using polygons. A traditional soccer ball is tiled with spherical hexagons and pentagons forming a truncated icosahedron pattern. It is made of exactly 32 panels (20 white hexagons, 12 black pentagons). This tiling pattern follows a strict structural rule: every single pentagon is completely surrounded by five hexagons, and every hexagon borders exactly three pentagons.

10. Is this specific arrangement the unique way to tile a sphere using regular pentagons and hexagons, or do other valid configurations exist?

Official World Cup match balls stand out because they challenge traditional manufacturing patterns and are custom-designed every four years. This year, the tournament match ball engineered by Adidas features the fewest panels ever seen on a World Cup ball: just 4 panels! The boundaries of these panels are smooth, sweeping curves inspired by the moving geometry of a stadium "Mexican Wave." Now, it is your turn to start designing soccer balls!

11. A **monohedral tiling** is a pattern where every single tile is perfectly congruent to one another (they share the exact same size and shape), though tiles can be rotated or reflected to fit together seamlessly. What are all the possible monohedral tilings that can be formed using regular spherical polygons?

12. Design your own custom match ball! Create a unique tiling scheme across the surface of a sphere utilizing the principles of spherical geometry you have mastered throughout this problem set.

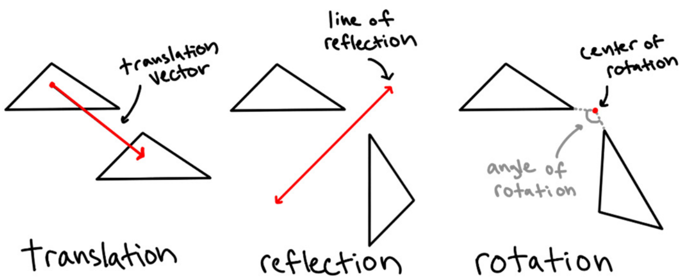
Summer Fun!

Frieze, Please!

By Hanna Mularczyk

If you've visited ancient ruins in Greece or India, owned an ornate rug, or are sitting in an old Cambridge building with a fancy crown molding, you've probably seen frieze patterns. Put simply, a frieze pattern is a two-dimensional design that repeats itself in one direction. These patterns are particularly beautiful because of the mathematical symmetries they contain.

A **transformation** of a frieze pattern is a way to move the frieze pattern in the 2-dimensional plane. For our purposes, the transformations we will consider here are **translations** (shifting the pattern by some amount in some direction), **reflections**, **rotations**, and combinations of these.



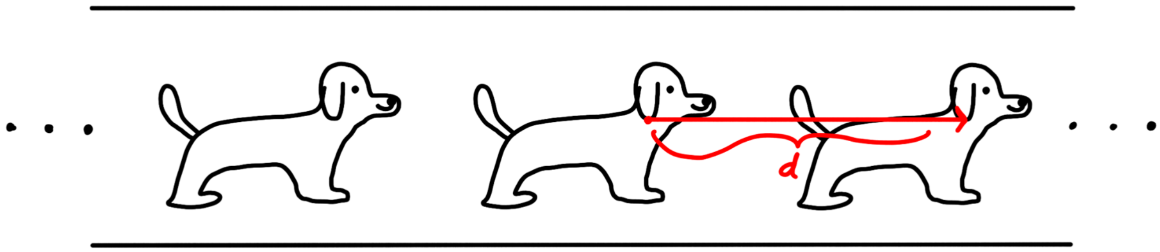
A **symmetry** of a frieze pattern is a transformation of the frieze pattern that does not change the appearance of the frieze pattern. We say that such a transformation "maps the pattern to itself." This might seem counterintuitive; why would we care about doing nothing? But this is also how we classify symmetries of other objects! We know that a circle has radial symmetry (that is, it is rotationally symmetric), and this is exactly because rotating a circle by any amount around its center leaves you with the same circle.

The **symmetry group** of a frieze pattern is the group of all of its symmetries. We call it a group, and not just a set, because the symmetries have more structure than a set. For example:

1. Show that if I have two symmetries in a symmetry group, if I **compose** them (that is, if I do one of the transformations and then the other), this transformation is also a symmetry.

Recall from above that a frieze pattern repeats itself in one direction. In our new terminology, this means that its symmetry group contains a translation to the right (or equivalently, left). To create such a frieze pattern, all I need to do is to draw a picture (say, of a dog), draw an identical version of the dog some distance *d* to the right (perhaps using the bottom of the dog's ear as a reference point), and also *d* to the left, and keep repeating this. We define the frieze pattern to extend infinitely in both directions (signified by "…"), since otherwise no translation would map the pattern to itself.

Summer Fun!



By construction, translation to the right by d is in the symmetry group, because moving the entire pattern to the right by this distance leaves you with the same pattern. Note that the same can be said of translations by $2d$, $3d$, $4d$, and so on, both to the left and to the right. We often choose the smallest distance d, because translations by kd for some integer k are just translation by d, done k times (and if k is negative, done to the left). In this case, translation by d **generates** all other translations in our symmetry group.

We require all frieze groups to contain some translation in them. What about the other symmetries? In general, a frieze pattern lives on an infinitely long blank strip, so regardless of the pattern itself, a symmetry must send the strip to itself. This is why we only allow translations left or right, and not translations that move the strip up or down, because these translations would physically change the location of the strip, which is not allowed.

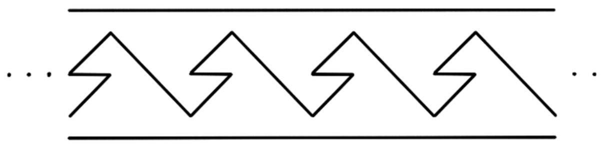
2. Which reflections send a blank strip to itself? Specify the line of reflection, that is, the line the reflection is done over. (These are the possible contenders for reflections in any symmetry group of a frieze pattern).

3. Which rotations send a blank strip to itself? Specify the degree of rotation and the center of rotation. (These are the possible contenders for rotations in any symmetry group of a frieze pattern.) We do not include rotations by 360 degrees, since this always does nothing.

4. Verify that the symmetry group of the dog frieze above contains no reflection or rotation.

Things get more interesting once we consider frieze patterns with more symmetries than just translations. Luckily I don't have to look very far! Below is a picture of the border of the rug in my bedroom. It contains three different frieze patterns. Although my rug is only a few feet long, we can imagine that this pattern goes on forever both to the left and to the right.

Let’s start by looking at the upper-most one.

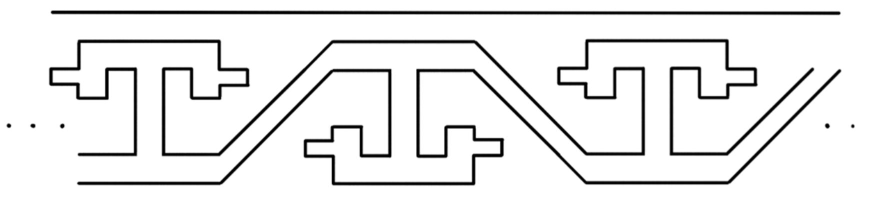


5. a. Draw a line marking the smallest distance of a translation in the symmetry group.

b. Which reflections are in this symmetry group, if any? Specify the line of reflection for each.

c. Which rotations are in this symmetry group, if any? Specify the degree of rotation and the center of rotation for each.

Let’s move on to the next frieze.



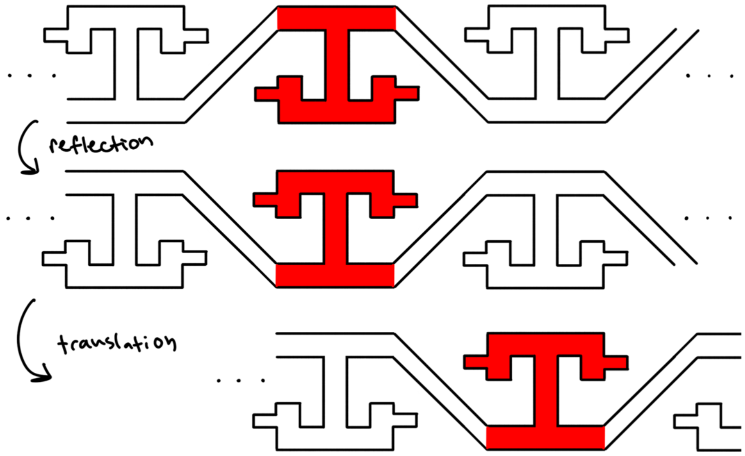
6. a. Draw a line marking the smallest distance of a translation in the symmetry group.

b. Which reflections are in this symmetry group, if any? Specify the line of reflection for each.

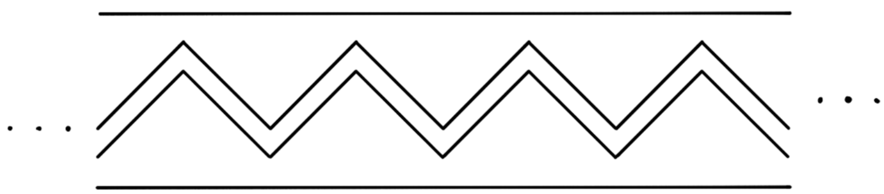
c. Which rotations are in this symmetry group, if any? Specify the degree of rotation and the center of rotation for each.

It might seem like we have found all the symmetries, but there is one missing! We have checked translations, reflections, and rotations, but we have not checked compositions of these. A **glide reflection** is a reflection across a line followed by a translation parallel to that line. Since we know translations must be to the left or right, the line of reflection must also run left to right, that is, be a horizontal line.

Summer Fun!



Now, on to the last frieze:



7. Show that this frieze pattern has the same types of symmetries as the previous one (even though the specific distances, line, and center placement might differ).

Since these two frieze patterns have the same types of symmetries, we say they are the same type. So far we have seen three different types of symmetry groups. But there are more!

8. For each of the below problems, design a frieze pattern that has the listed symmetries and no other symmetries (that aren't combinations of the listed ones).

a. Translation and reflection across a vertical line only
b. Translation and glide-reflection only
c. Translation, glide-reflection, and reflection across a horizontal line only
d. Translation, reflection across horizontal and vertical lines, and rotation only

Bonus Explorations

9. Are there any symmetry groups not given in the 7 above cases? If so, what are they? If not, why?

10. Next time you're out and about, hunt down and classify some frieze patterns.

Summer Fun!

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 38 - Meet 12 May 7, 2026	Mentors:	Elisabeth Bullock, Elsa Frankel, Clarise Han, Shauna Kwag, Kira Lewis, Yaqi Li, Sophia Liao, Hanna Mularczyk, AnaMaria Perez, Maya Robinson, Amanda Tran, Anlan Xu

We held our traditional end-of-session math collaboration. This session's math collaboration was designed by Yaqi Li and AnaMaria Perez. Try your hand at solving some of the event's math problems:

> Emma, Jenny, Nia and Ursla are trying to figure out the seasons the others are born in. They know they are all born on different days of the same year, and that this year is not a leap year. Suppose that all days of the year are equally likely to be a birthday.
>
> Nia starts by saying: "For any one of you, the probability that you are born in a given season is different for every season."
>
> Emma replies: "Now I know I am born after you."
>
> Ursula says to Nia: "I can't tell which of us is older, but it's more likely that I am older."
>
> Nia says: "Now I know there are two seasons in which it is equally likely Jenny is born."
>
> Jenny says: "There is exactly a 10% chance that Emma is born in the same season as I am."
>
> Emma concludes: "Although I am not born at the changing of seasons, I am confident I am not the youngest person here."
>
> In what season was each person born?

- AnaMaria Perez

There are 30 girls going on a class trip to Miami on a train with 30 seats. The first girl who boards the train loses her ticket and sits at a random seat. For each of the following girls who boards the train, if their seat is taken, then they randomly choose a seat, otherwise they sit at their assigned seat. What is the probability that the last girl boarding will sit at their assigned seat?

- Yaqi Li

Calendar

Session 38: (all dates in 2026)

January	29	Start of the thirty-eighth session!
February	5	
	12	
	19	No meet
	26	
March	5	
	12	
	19	
	26	No meet
April	2	Ila Fiete, MIT
	9	
	16	
	23	No meet
	30	
May	7	

Session 39: (all dates in 2026)

September	10	Start of the thirty-ninth session!
	17	
	24	
October	1	Avery Vilandrie, Shiftsmart
	8	
	15	
	22	
	29	
November	5	
	12	
	19	
	26	Thanksgiving - No meet
December	3	

Girls' Angle has run nearly 200 Math Collaborations. Math Collaborations are fun, fully collaborative, math events that can be adapted to a variety of group sizes and skill levels. We now have versions where all can participate remotely. We have now run four such "all-virtual" Math Collaboration. If interested, contact us at girlsangle@gmail.com. For more information and testimonials, please visit www.girlsangle.org/page/math_collaborations.html.

Girls' Angle can offer custom math classes over the internet for small groups on a wide range of topics. Please inquire for pricing and possibilities. Email: girlsangle@gmail.com.

Girls' Angle: A Math Club for Girls

Membership Application

Note: If you plan to attend the club, you only need to fill out the Club Enrollment Form because all the information here is also on that form.

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address (the Bulletin will be sent to this address):

Email:

Home Phone: ____________________________ Cell Phone: ____________________________

Personal Statement (optional, but strongly encouraged!): Please tell us about your relationship to mathematics. If you don't like math, what don't you like? If you love math, what do you love? What would you like to get out of a Girls' Angle Membership?

The $50 rate is for US postal addresses only. **For international rates, contact us before applying.**

Please check all that apply:

- ☐ Enclosed is a check for $50 for a 1-year Girls' Angle Membership.
- ☐ I am making a tax-free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com.

Girls' Angle

A Math Club for Girls

Girls' Angle
Club Enrollment

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

The club is where our in-person mentoring takes place. At the club, girls work directly with our mentors and members of our Support Network. To join, please fill out and return the Club Enrollment form. Girls' Angle Members receive a significant discount on club attendance fees.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-12. We welcome *all girls* (in grades 5-12) regardless of perceived mathematical ability. There is no entrance test. Whether you love math or suffer from math anxiety, math is worth studying.

How do I enroll? You can enroll by filling out and returning the Club Enrollment form.

How do I pay? The cost is $20/meet for members and $30/meet for nonmembers. Members get an additional 10% discount if they pay in advance for all 12 meets in a session. Girls are welcome to join at any time. The program is individually focused, so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is based in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org/page/calendar.html or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Peirce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science, worked in the mathematics educational publishing industry, and taught at HCSSiM. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high-level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. In this way, we hope to help girls become solvers of the yet unsolved.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: Club Enrollment Form

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Please fill out the information in this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. Names:

____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: Girls roughly in grades 5-12 are welcome. Although we will work hard to include every girl and to communicate with you any issues that may arise, Girls' Angle reserves the discretion to dismiss any girl whose actions are disruptive to club activities.

Personal Statement (optional, but strongly encouraged!): We encourage the participant to fill out the optional personal statement on the next page.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: _______________________

(Parent/Guardian Signature)

Participant Signature: ____

Members: Please choose one.	Nonmembers: Please choose one.
☐ Enclosed is \$216 for one session (12 meets)	☐ I will pay on a per meet basis at \$30/meet.
☐ I will pay on a per meet basis at \$20/meet.	☐ I'm including \$50 to become a member, and I have selected an item from the left.

☐ I am making a tax-free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Personal Statement (optional, but strongly encouraged!): This is for the club participant only. How would you describe your relationship to mathematics? What would you like to get out of your Girls' Angle club experience? If you don't like math, please tell us why. If you love math, please tell us what you love about it. If you need more space, please attach another sheet.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: ___________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____