Girls' Angle Bulletin

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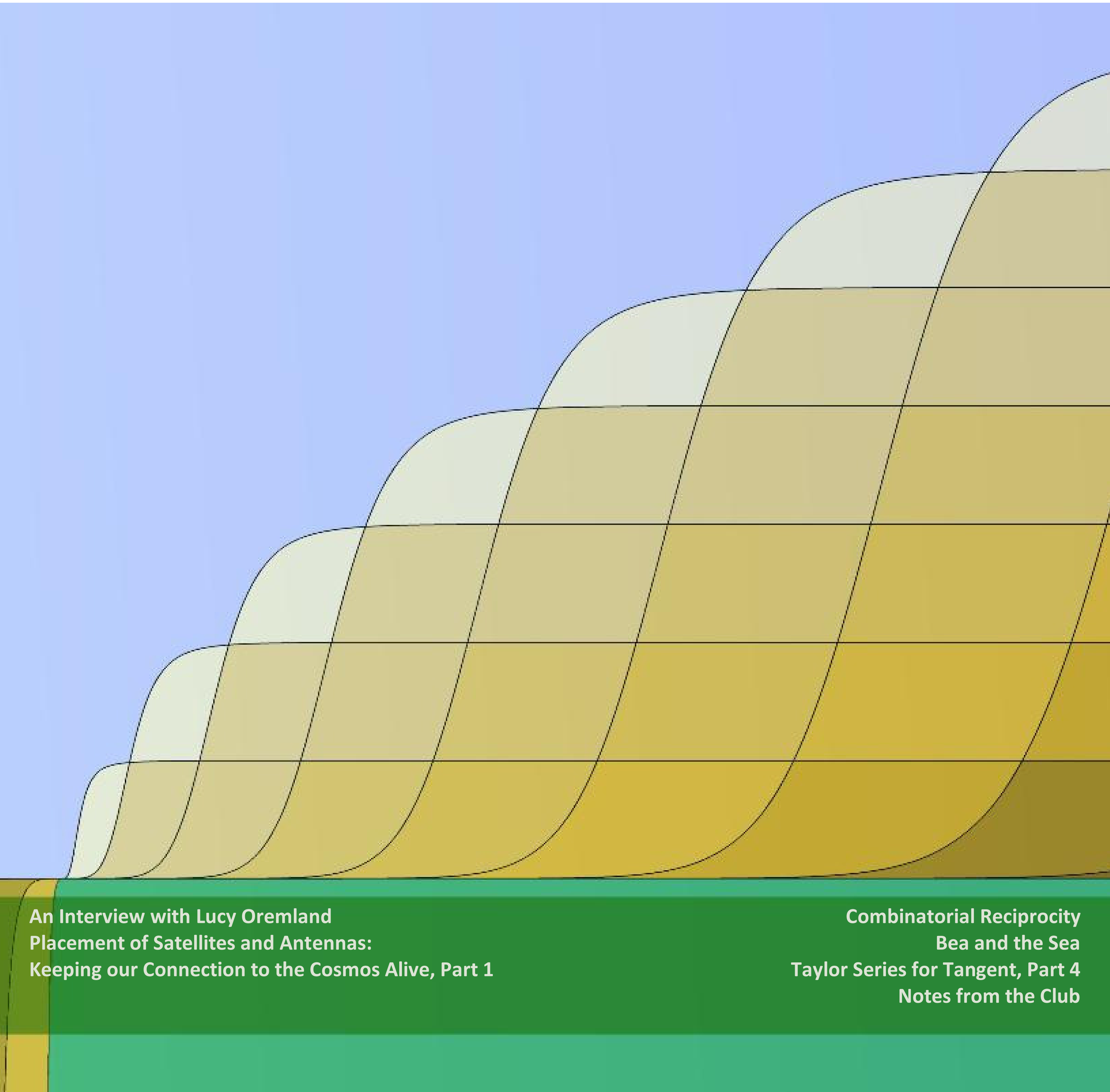
To Foster and Nurture Girls' Interest in Mathematics



From the Founder

Never dismiss your own math question! Your question is precious. It's the seed that sprouts mathematics. Think about it carefully and, before you realize it, it will give birth to more questions. As you find their answers, you'll be making math. - Ken Fan, President and Founder

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Girls' Angle Bulletin

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Jennifer Sidney
Executive Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: *Hill Functions* by C. Kenneth Fan. See page **Error! Bookmark not defined.** of our interview with Lucy Oremland.

An Interview with Lucy Oremland

Share what you (think you) know. Whether it's a more formal presentation or just helping someone out after class, explaining math to someone else has always helped me deepen my understanding in ways I couldn't do on my own.

Lucy Oremland is Associate Professor of Mathematics at Skidmore. She received her PhD in mathematics from the University of Pittsburgh. She then completed a postdoctoral fellowship at the Mathematical Biosciences Institute. She creates and analyses models of biological systems, such as the menstrual cycle. She also works to improve math education.

Ken: How would you describe your relationship to mathematics while growing up? Did you experience any setbacks in math that you had to overcome? If so, how did you overcome them?

Lucy: As a student I likely would have said that I loved math because there's always "a right answer" and that mathematics "is very black and white," but that sort of thinking no longer resonates with me. In fact, my job requires a lot of creativity, and my strongest students are often those that think outside the box and approach problems in novel ways.

I was fortunate to grow up in an environment where math and science were encouraged from an early age. I loved puzzles, and I found the step-by-step process of solving a problem using an algorithm incredibly satisfying. Mathematics was on my radar from early on because it was what I saw around me and I genuinely enjoyed it. I also loved theater and public speaking, so teaching seemed like a great way to combine my interests in a complementary and fulfilling way.

In terms of setbacks – completing graduate school was the hardest thing I've ever done. The mathematics itself was incredibly challenging and I was constantly comparing myself to my peers. To get through it, I studied hard, harder than I ever had before – I had to teach myself how to study. I learned how to learn from my mistakes. And I relied on my friends as a support network. In terms of my imposter syndrome, I can't quite say I've overcome it yet! While I still struggle in this area, repeatedly doing the work has gradually strengthened my confidence and sense of belonging.

Ken: What did you learn about studying?

Lucy: A big part of how I learned to study effectively was working with others. I noticed that we each approached problems differently – one of my friends always drew pictures, another started by looking for contradictions, and I often preferred to build from definitions. Sometimes one of those methods was the key to unlocking the problem, sometimes it was a combination, and other times we were completely stumped. But those discussions showed me that there are actually many ways to engage with a problem, and that false starts and dead ends are an essential part of the process. Our collaboration went a long way to help me get out of my own head and just try something.

I also started to develop mathematical intuition, which I had previously assumed was innate. It came from working through many, many, many problems, more than what my professor had assigned, particularly in areas where I knew I was struggling. And I didn't just work through them once, I started reflecting and looking for patterns in the problems themselves – sometimes problems seemed similar but were solved very differently, and sometimes they seemed different but used the same solution approach. While it didn't feel like it in the moment, over time I built a sensibility that

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Thank you and best wishes,
Ken Fan
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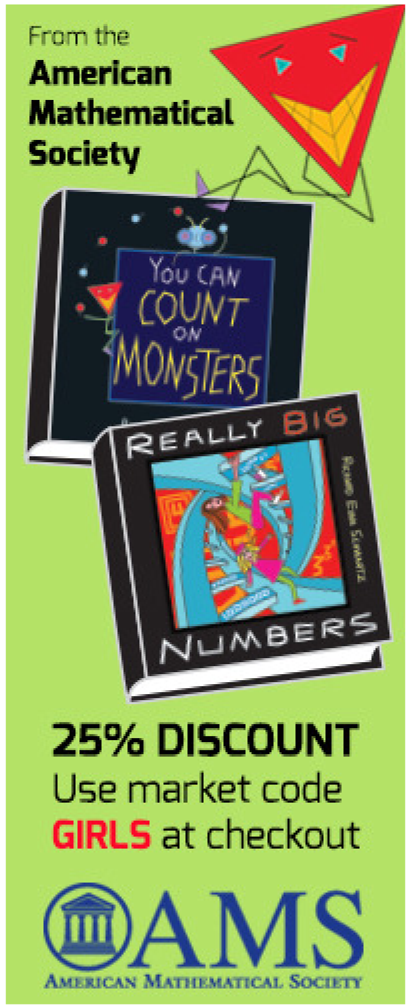

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Placement of Satellites and Antennas: Keeping our Connection to the Cosmos Alive, Part 1

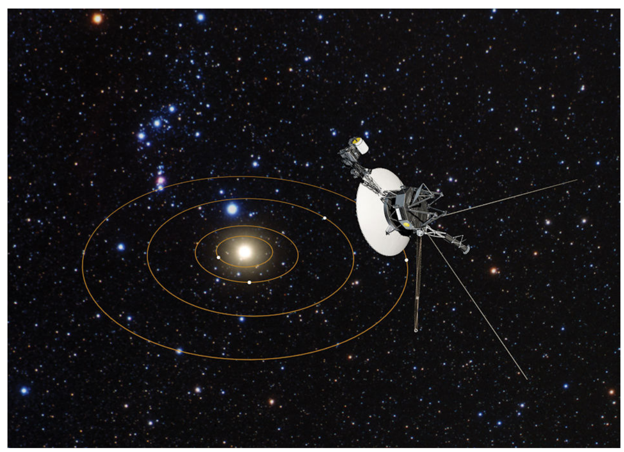
by Lilly Carrillo, Jillian Cervantes, and Dr. Pamela E. Harris | edited by Jennifer Sidney

"We used to look up at the sky and wonder at our place in the stars…"

- Cooper, *Interstellar*

Luckily, we haven't stopped wondering.

Right now, more than 15 billion miles away from the sun, a little spacecraft named Voyager 1 is still whispering to Earth. Voyager 1 was launched in 1977 carrying a golden record with whale sounds, laughter, and 90 minutes of classical music (in case anyone or anything comes across it)! In 2012, Voyager 1 entered interstellar space, which is outside our sweet little solar system. Voyager 1 doesn't only contain a message; it also helps us learn more about our solar system. It has a bunch of cool instruments on board, like a radio system that sends data back to earth. It talks to NASA through the Deep Space Network (DSN), which has giant antennas around the world. All the data and discoveries we get from these spacecraft – the pictures, the sounds, the science – come through these faint but powerful broadcasts by ensuring that information passes through the network.



Courtesy of NASA[1]

Figure 1. Voyager 1 in space.

One day, Voyager 1's power will fade, or it will drift too far into the stars, and we'll no longer hear its whispers. That is why we carefully position our antennas and spacecraft and give them just the right amount of power, so we can keep our connection to the cosmos alive for as long as possible.

This naturally leads us to a mathematics problem in the area of graph theory.

To describe a graph, we need two types of objects: vertices, which we draw as circles, and edges, which we draw as lines between vertices. We don't allow edges to start and end at the same vertex; such constructs are sometimes referred to as loops. To define **(t, r) broadcast domination**, we first recall the definition of a dominating set: this is a subset of vertices of the graph such that every vertex is either in the selected subset or there is an edge from the vertex to a vertex in the set. Given a dominating set and a vertex v in the graph, we say v is **dominated** if either it is in the dominating set or it shares an edge with a vertex in the dominating set. Whenever all of the vertices of a graph are dominated by a set, we say that the set **dominates** the graph. Recall that the domination number of a graph is the smallest number of dominating vertices that can dominate the entire graph. We are going to refer to this type of dominating as **classic domination**. We will now go into brief details of (t, r) broadcast domination and explore some challenge problems!

[1] https://science.nasa.gov/asset/hubble/voyager-1s-view-of-solar-system-artists-concept/

To begin, you may revisit "Optimal Resource Placement: From Disneyland to Dominating Sets, Part 2" in Volume 17, Number 4. In that article, we talked about domination numbers for the grid graph and posed the following problem, which we now solve:

Challenge Problem: If G is the 4-by-6 grid graph, according to Fisher's result, $\gamma(G) = 7$. Can you find an arrangement of seven vertices which dominate the 4-by-6 grid graph?

In Figure 2, we give an example arrangement. You may have come up with this arrangement, or a different one!

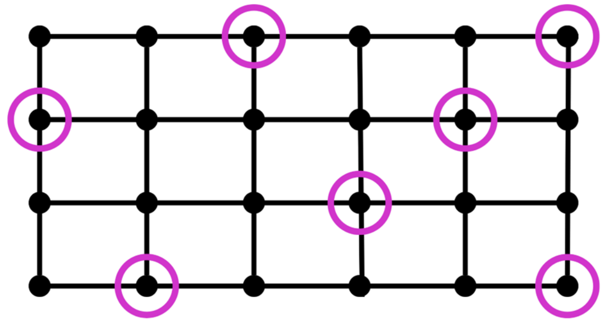


Figure 2. An answer to the challenge problem.

Now, imagine if domination were given superpowers and we have extra responsibility. In (t, r) broadcast domination, a dominating vertex can reach vertices more than one edge away. That dominating vertex has a particular signal strength, so if we increase the strength we can reach a farther distance. Just like a satellite or antenna, the closer we are to a dominating vertex, the better our signal!

A $\boldsymbol{(t, r)}$ **broadcast dominating vertex** contributes reception t to itself where t is a positive integer, and its reception decays by 1 every time it crosses an edge. Any vertex that is more than $t - 1$ edges away from a (t, r) broadcast dominating vertex receives no reception from that vertex. For example, in Figure 3 we illustrate one (t, r) broadcast dominating vertex with strength $t = 3$ in a 5-by-5 grid graph. The pink circle is the (t, r) broadcast dominating vertex. The reception that each vertex receives is shown northeast of the vertices.

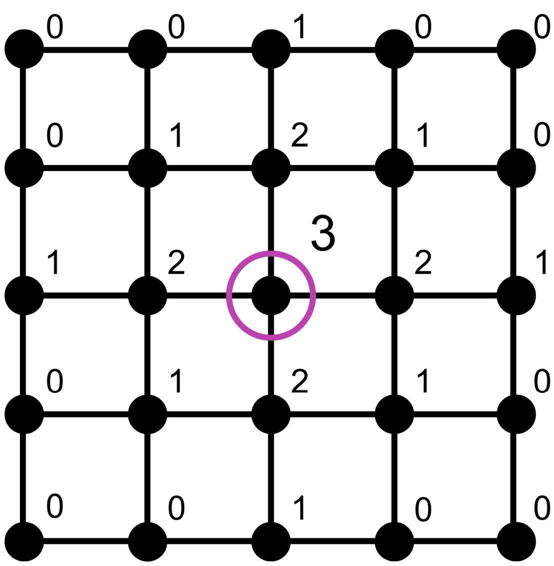


Figure 3. A dominating vertex and the signal it emits.

Saying "(t, r) broadcast dominating vertex" is quite a mouthful, so we may refer to such vertices as (t, r) satellites. If we were focusing on classic domination only, we would care about ensuring that every vertex is either a satellite or an edge away from a satellite. This is expensive and not always realistic! So instead, we require that every non-satellite vertex receives "enough" signal, a positive integer parameter, which we denote r.

You might wonder: what if a vertex is within range of two (t, r) satellites? Then the vertex receives signal from each, and it may be that it receives "extra signal" if the sum of the signal it receives from each satellite adds up to more than r.

The **reception strength** of a vertex u, denoted $r(u)$, counts how much reception u receives from all (t, r) satellites; if u is a satellite as well, then $r(u)$ also includes the reception u gives itself. In Figure 4, circled in magenta on the 3-by-4 grid are three (t, r) satellites graph that have transmission strength $t = 3$. Notice that some vertices receive reception from all three (t, r) satellites. For example, consider vertex a. Since a is two edges away from (t, r) satellite c and two edges away from (t, r) satellite f, we have $r(a) = 2$, as vertex a receives reception 1 from each of the satellites. It should be noted that vertex a does not receive any reception from the satellite at vertex l, as it is five edges away from vertex l.

A (t, r) **broadcast dominating set** T is a set of satellites of strength t that fulfill the requirement that every vertex in the graph has reception r. Note that the set of three vertices in Figure 4 is not a (3, 2) broadcast dominating set, because there are some vertices which have reception less than 2. But the three vertices in Figure 4 *are* a (3, 1) broadcast dominating set, as every vertex receives reception at least 1.

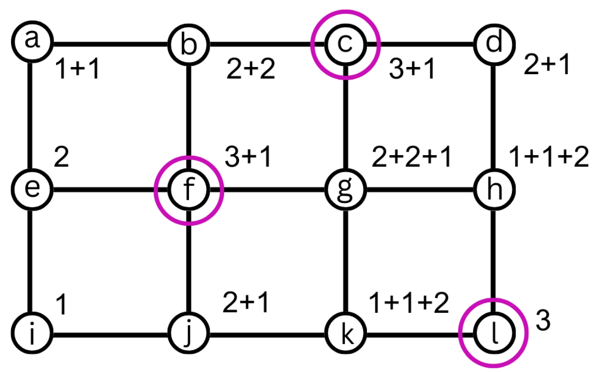
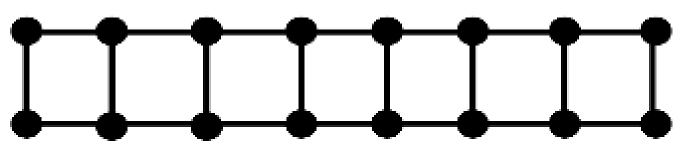


Figure 4. Three (t, r) satellites of strength $t = 3$ and the signal they emit.

But how do we know if we've chosen the smallest possible (t, r) broadcast dominating set? Just like with classic domination, we care about minimizing the number of vertices in our (t, r) broadcast dominating set. The minimum size of any (t, r) broadcast dominating set for a graph G is called the (t, r) **broadcast domination number** of G, and denoted $\gamma_{t,\,r}(G)$.

Note that the previous definitions are similar to a classic dominating set and domination number; the difference is that they now have two new parameters, t and r. Let us put these definitions into practice!

Example 1. Find a (3, 2) broadcast dominating set for the 2-by-8 grid graph below. Try to make your dominating set as small as possible!



Let's walk through one possible construction of a (3, 2) broadcast dominating set. We first place a (3, 2) broadcast dominating vertex in the second column of the grid graph (see Figure 6). We've added checkmarks to the vertices that receive a reception of 2 or greater.

Recall that any vertex with a satellite receives reception 3 from itself, so that vertex is dominated.

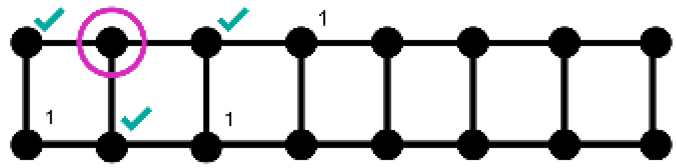


Figure 6. A 2-by-8 grid graph with one broadcast dominating vertex.

To ensure that the bottom left corner vertex receives reception 2, we place the next (3, 2) satellite as illustrated in Figure 7.

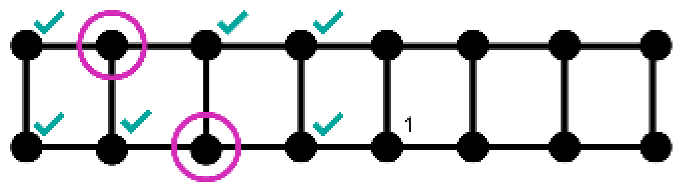


Figure 7. A 2-by-8 grid graph with two (3, 2) satellites.

Now we need to make sure every vertex in the graph has reception at least 2. Sometimes it helps to space out vertices! So we place our next (3, 2) broadcast dominating vertex as far away as we can without leaving "gaps" of vertices that have a reception less than 2. Thus, we place our next (3, 2) satellite as illustrated in Figure 8.

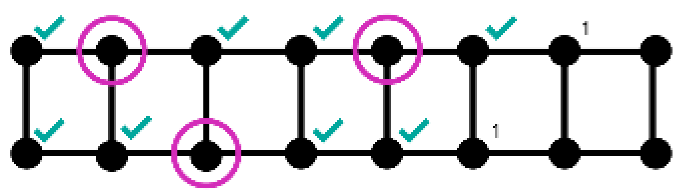


Figure 8. A 2-by-8 grid graph with three (3, 2) satellites.

Finally, we place our last (3, 2) satellite as illustrated in Figure 9. We have constructed a (3, 2) broadcast dominating set of size four. Can you construct a (3, 2) broadcast dominating set with three or fewer (3, 2) satellites?

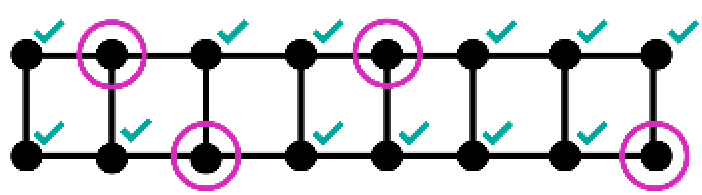


Figure 9. A 2-by-8 grid graph with a (3, 2) broadcast dominating set.

To recap our journey, we brushed up on classic domination and then learned a type of graph domination called (t, r) broadcast domination. Stay tuned for Part 2, where we explore a toy model of satellite placement on an even bigger grid graph!

Combinatorial Reciprocity[1]

by Robert Donley[2]
edited by Amanda Galtman

In this installment, we continue investigating the counting function for anti-magic squares. Specifically, we focus on **combinatorial reciprocity,** a general phenomenon for rational polytopes, although our interest lies in applications to generating functions and explicit forms of counting functions as polynomials. We recommend that you review the previous two installments and references noted there (see Volume 19, Numbers 1 and 2).

Let's review the binomial series with $n \geq 0$:

$$\frac{1}{(1-x)^{n+1}} = 1 + \binom{n+1}{1}x + \binom{n+2}{2}x^2 + \ldots + \binom{n+k}{k}x^k + \ldots.$$

Several proofs of the binomial series were noted in previous installments. We give a direct proof here, beginning with the usual proof of the binomial theorem. The binomial theorem proof will have applications to later parts of this installment.

Binomial Theorem: For $n \geq 0$, $(1 + x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \ldots + \binom{n}{n}x^n$.

Proof. We expand the left-hand side of the equation as the n-fold product

$$(1 + x) \cdots (1 + x).$$

Before simplification, there are 2^n terms in the product, each obtained by choosing a 1 or an x from each parenthetical term. If we assign a 0 to each choice of 1 and a 1 to each choice of x, then we obtain a binary string of length n, and each such string occurs as a term in the product. The coefficient of x^k is $\binom{n}{k}$ because there are $\binom{n}{k}$ binary strings of length n with exactly k 1s. □

Proof of the binomial series equation. We expand the left-hand side of the equation as

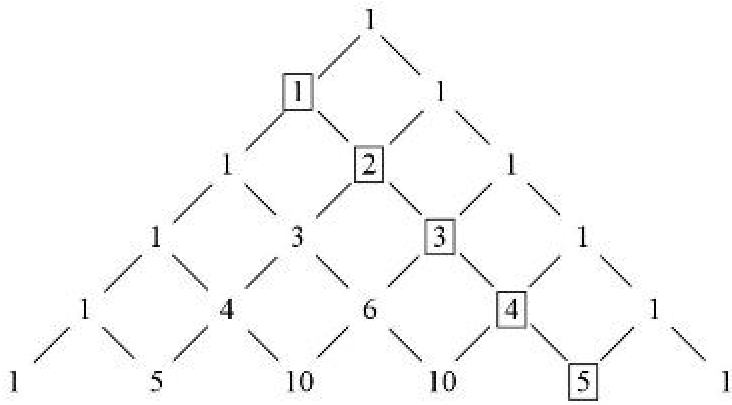
$$\frac{1}{(1-x)} \cdots \frac{1}{(1-x)} = (1 + x + x^2 + \ldots) \cdots (1 + x + x^2 + \ldots).$$

Now, instead of a binary string, each term in the product corresponds to an ordered list of $n + 1$ nonnegative integers, so the coefficient of x^k in the product is the number of weak compositions of k with $n + 1$ parts. To count such compositions, we count the number of ways to place k balls into $n + 1$ boxes. Such a choice corresponds to a binary sequence of length $k + n$; if the boxes are placed side to side, then there are n interior sides, each represented by 1, and the balls in each box are represented by zeros. To finish the proof, we count the number of binary sequences of length $n + k$ with k 0s. □

[1] This installment is 25th in a series that began in Volume 15, Number 3 and the third of a new subseries.
[2] This content is supported in part by a grant from MathWorks.

The binomial theorem calculates the rows of Pascal's triangle. The binomial series calculates the diagonals of Pascal's triangle. Either diagonal of 1s corresponds to the $n = 0$ case, the geometric series.

Exercise: Verify that the nth binomial series corresponds to the nth diagonal of Pascal's Triangle. That is, verify that the indices of the binomial coefficients are correct.



Exercise: Recall that multiplying a generating function $a_0 + a_1x + a_2x^2 + \ldots$ by the geometric series $\frac{1}{(1-x)} = 1 + x + x^2 + x^3 + \ldots$ yields the generating function for the partial sums of a_k. How are the binomial series for n and $n + 1$ related?

The previous exercise is just a restatement of the Hockey Stick Rule for Pascal's triangle in terms of the binomial series.

Exercise: Prove the Hockey Stick Rule by iterating Pascal's identity.

Exercise (calculus required): Give another proof of the binomial series equation by differentiating both sides of the geometric series equation n times.

Next, we consider symmetry properties of polynomials. Suppose $h > 0$. For a given function $f(x)$, the graph of $f(x - h)$ is the graph of $f(x)$ shifted to the right by h. As a guiding example, every quadratic polynomial can be put into the standard form $g(x) = a(x - h)^2 + k$, where a is a nonzero constant, (h, k) is the vertex of the associated parabola, and $x = h$ is its axis of symmetry. Note that $f(x)$ is an even polynomial in $x - h$.

Exercise: Convert the quadratic function $f(x) = 3x^2 - 12x + 16$ to standard form, graph the parabola, and find the axis of symmetry.

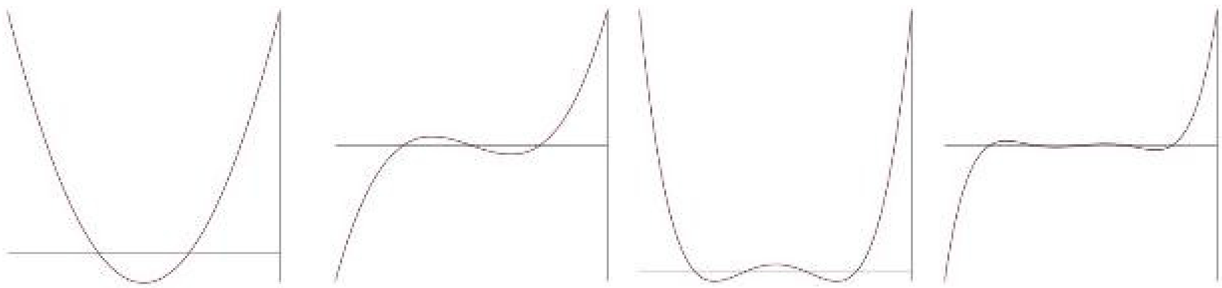
Exercise: Express $f(x) = 2x - 6$ as an odd function of $x - h$, and draw its graph. For the linear function $f(x) = mx + b$, express $f(x)$ as an odd function of $x - h$ for some h. What notable item is h?

Exercise: Use the binomial theorem to express $f(x) = x^3 - 3x^2 + 6x - 4$ as an odd function of $x - 1$. Prove that the graph of any cubic polynomial is symmetric about some point (h, k). That is, for the polynomial $f(x) = ax^3 + bx^2 + cx + d$ with nonzero a, prove that $f(x) - k$ is an odd polynomial in $x - h$ for some constants h and k.

As seen in the previous installment, we can rewrite the binomial coefficient $\binom{L+n}{n}$ as a polynomial in either variable. As a polynomial in L, we have

$$Q_n(L) = \frac{(L+n)!}{L!n!} = \frac{(L+n)(L+n-1)\cdots(L+1)}{n!},$$

and the graphs of n = 2, 3, 4, and 5 are as follows:



Of course, $Q_n(L) = 0$ for L = -1, …, -n. If we center the graphs about the y-axis, which appears on the right in each figure, we see that each shifted function is even or odd. More precisely, each polynomial is even or odd about the vertical line $x = -\frac{n+1}{2}$, and, in turn, we can write each polynomial as an even or odd function of $L + \frac{n+1}{2}$. For instance,

$$Q_2(L) = \frac{1}{2}((L + \frac{3}{2})^2 - \frac{1}{4}) \text{ and } Q_3(L) = \frac{1}{6}(L + 2)((L + 2)^2 - 1).$$

Exercise: Verify the formulas for $Q_2(L)$ and $Q_3(L)$, and express $Q_4(L)$ and $Q_5(L)$ as an even or odd polynomial about the corresponding axis. Give corresponding formulas for general n.

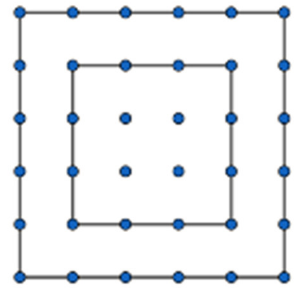
Furthermore, if we replace L with -L in $Q_n(L)$, we obtain $\binom{L-1}{n}$

$$Q_n(-L) = (-1)^n \frac{(L-n)(L-n+1)\cdots(L-1)}{n!} = (-1)^n \binom{L-1}{n} = (-1)^n Q_n(L - n - 1).$$

With the polynomial interpretation, we can extend the binomial coefficient notation to allow any real number in the upper index. Thus, as polynomials in L,

$$\binom{-L+n}{n} = (-1)^n \binom{L-1}{n}.$$

This equation is useful for calculations and is a fundamental example of **combinatorial reciprocity**. That is, if we have a counting formula represented by a polynomial, replacing the variable with its negative might produce a new counting formula. For example, consider the square with side length L with one corner at (0, 0) in the first quadrant of the xy-plane. The number of points with integer coordinates in the square is $\rho(L) = (L + 1)^2$. Changing the sign leads to $\rho(-L) = (L - 1)^2$, the number of integer points strictly inside the square.



This phenomenon is explained generally by Ehrhart theory, which considers the broader context of rational polytopes. Our approach to the subject is less geometric, with a focus on basic counting formulas, the binomial series, and generating functions. For an in-depth treatment of the subject, we recommend the advanced book *Combinatorial reciprocity theorems*[3] by Matthias Beck and Raman Sanyal.

Exercise: The binomial coefficient $\binom{L+n}{n}$ counts the number of ways to place L balls into $n + 1$ boxes. To prove the reciprocity equation up to sign, count such placements with the additional condition that each box contains at least one ball.

As noted above, $\binom{L+n}{n}$ counts the number of weak compositions of L with $n + 1$ parts. The answer to the exercise is the corresponding number of strict compositions, as given in the reciprocity equation. The excluded compositions have at least one vanishing part. Also, note that there are no strict compositions of L with $n + 1$ parts for $L \leq n$.

Another approach to reciprocity is through recursive sequences. For instance, consider the binomial series when $n = 2$. Numerically, we have

$$F_2(x) = \frac{1}{(1-x)^3} = 1 + 3x + 6x^2 + 10x^3 + 15x^4 + \ldots.$$

If we clear the denominator in the first equation, we obtain $(1 - x)^3 F_2(x) = 1$, which yields the following recursive description for $a_k = \binom{k+2}{2}$:

$$a_k = 3a_{k-1} - 3a_{k-2} + a_{k-3}, \text{ for } k > 0, \text{ and } a_{-2} = a_{-1} = 0,\ a_0 = 1.$$

The recursion is implemented by hand using discrete convolution. To calculate the coefficient of x^k without reference to x, we reverse the coefficients of $1 - 3x + 3x^2 - x^3$ and position the 1 over a_k. Then we multiply down the columns, set the sum of the products to 0, and solve, as implemented below.

-1	3	-3	1
0	0	1	a_1

	-1	3	-3	1
0	0	1	3	a_2

		-1	3	-3	1
0	0	1	3	6	a_3

[3] Beck, M. and R. Sanyal. Combinatorial reciprocity theorems. Graduate studies in mathematics 195, American Mathematical Society: Providence, RI, 2018. Beta version at https://matthbeck.github.io/papers/crt.pdf.

We repeat this process by continuing to the right, but we can also continue to the left, creating a new sequence with generating function $F_2'(x) = a_{-1}x + a_{-2}x^2 + \ldots$. For instance, the convolution template at right gives $a_{-3} = 1$.

-1	3	-3	1	
a_{-3}	0	0	1	3

Exercise: Use discrete convolution to compute a_{-4} and a_{-5}.

We obtain the same sequence! In fact, when read backwards, the recursive equation is unchanged up to a sign factor. Also, since $a_{-3} = 1$, the initial conditions are the same up to a shift in the indices.

On the other hand, if we use positive exponents in $F_2'(x)$, then, as a convolution, the recursive equation for a_{-k} should be modified by replacing x with $1/x$. With a sign change to be explained, we have

$$F_2'(x) = -F_2(1/x).$$

Now reciprocity follows from

$$-F_2(1/x) = \frac{-1}{(1-1/x)^3} = \frac{x^3}{(1-x)^3} = x^3 F_2(x).$$

Likewise, it holds that $F_n'(x) = -F_n(1/x)$. In general, the negative sign is the correction to a_{-n-1}, depending on whether the coefficient of x^{n+1} in $(1-x)^{n+1}$ is -1 or 1. When n is odd, $a_{-n-1} = -1$.

Exercise: Repeat the calculations with the discrete convolution for $n = 0, 1, 3, 4$, and verify that the formulas for $F_n'(x)$ match the recursive sequences to the left.

With partial fractions and the binomial and geometric series, we have all the tools we need to expand a general rational function $r(x) = \frac{p(x)}{q(x)}$ into a formal power series, assuming we can factor $q(x)$ completely into powers of linear factors.

Exercise: Expand $G_0(x) = \frac{1}{1-2x}$ and $G_1(x) = \frac{1}{(1-2x)^2}$. Describe the corresponding sequences a_k, both in closed form and recursively. Extend each sequence to the left recursively, and compare each $G'(x)$ with $-G(1/x)$. Repeat for each sequence of partial sums of a_k.

Our interest lies mainly in cases where $q(x) = (1-x)^{s+1}$ for some positive integer s. Then, no exponential terms are needed, and we consider examples where the corresponding polynomials $h(L)$ retain some reciprocity from the binomial series. That is, for some s,

$$h(-1) = h(-2) = \ldots = h(-s) = 0 \text{ and } h(-L) = \pm h(L-s-1).$$

As before, the latter condition implies that $h(L)$ can be represented as an even or odd polynomial in $L + \frac{s+1}{2}$.

Example: In the previous installment, we encountered the family of generating functions

$$F_m(x) = \frac{(1+x)^{m-1}}{(1-x)^{m+1}}.$$

For each fixed m, the recursive sequences to the left are given in the columns of the table at the end of the installment. The corresponding polynomials $\rho_{m,2}(L)$ are even or odd in $L + 1$, have degree m, and satisfy $\rho_{m,2}(-L) = (-1)^m \rho_{m,2}(L-2)$.

Next, we revisit the counting function for anti-magic squares from the previous two installments.

Example: Recall that the number of anti-magic squares of size n with index L is given by

$$\rho_n(L) = \binom{L+2n-1}{2n-1} - \binom{L+n-1}{2n-1}$$

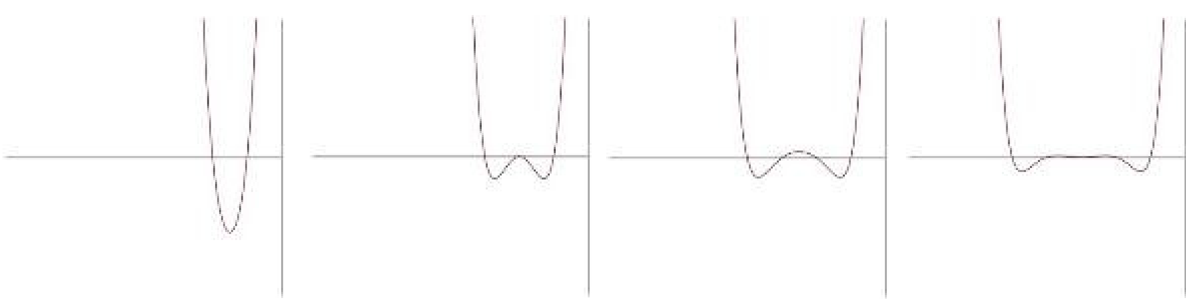
with corresponding generating function

$$F_n(x) = 1 + \rho_n(1)x + \rho_n(2)x^2 + \ldots = \frac{1-x^n}{(1-x)^{2n}}.$$

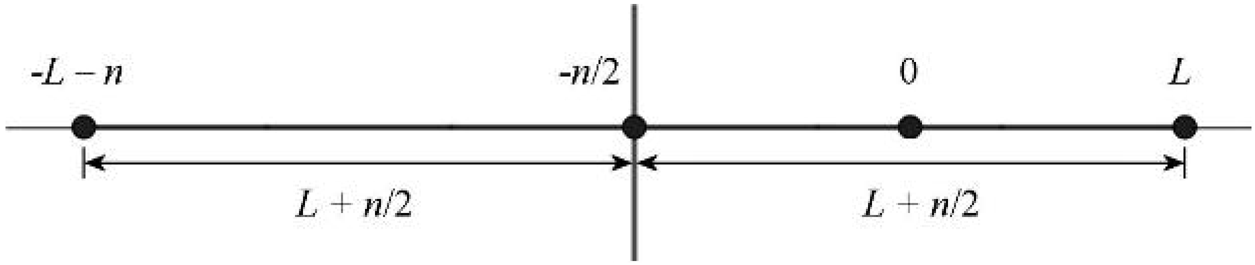
If we replace L with $-L$ in $\rho_n(L)$, we obtain

$$\begin{aligned}
\rho_n(-L) &= \binom{-L+2n-1}{2n-1} - \binom{-L+n-1}{2n-1} \\
&= \binom{(-L)+2n-1}{2n-1} - \binom{(-L-n)+2n-1}{2n-1} \\
&= -\binom{L-1}{2n-1} + \binom{L+n-1}{2n-1} \\
&= \rho_n(L-n).
\end{aligned}$$

Thus, for all n, $\rho_n(L)$ is an even polynomial in $L + n/2$. For $n = 3, 4, 5, 6$, we have the graphs of $\rho_n(L)$ below.



Furthermore, the following diagram explicitly describes the reflection about $x = -n/2$.



Let's find an explicit formula for $\rho_4(L)$ with an eye towards the general formula. Now

$$\rho_4(L) = \binom{L+7}{7} - \binom{L+3}{7} = \frac{1}{7!}((L+1)\cdots(L+7)-(L-3)\cdots(L+3)).$$

If we factor out $(L + 1)(L + 2)(L + 3)$ and write each factor in terms of $L + 2$, then we obtain the difference between

$$((L + 2) + 2)((L + 2) + 3)((L + 2) + 4)((L + 2) + 5)$$

and

$$((L + 2) - 2)((L + 2) - 3)((L + 2) - 4)((L + 2) - 5).$$

If we let $x = L + 2$ and expand, we obtain

$$(x^4 + e_1x^3 + e_2x^2 + e_3x + e_4) - (x^4 - e_1x^3 + e_2x_2 - e_3x + e_4) = 2(e_1x^3 + e_3x),$$

where $e_k = e_k(2, 3, 4, 5)$ is the kth elementary symmetric polynomial evaluated at (2, 3, 4, 5). To define this polynomial, we revisit the main idea in the proof of the binomial theorem on page 12.

Consider the product $P(x) = (x + r_1)(x + r_2)(x + r_3)(x + r_4)$. We could expand $P(x)$ directly using polynomial multiplication, but the elementary symmetric functions $e_k(r_1, r_2, r_3, r_4)$ give a direct approach. To calculate the coefficient of x^k, we consider all choices of k factors that contribute an x, so that each remaining factor contributes an r_i. There are $\binom{4}{k}$ ways to choose, but instead of a contribution of 1 in the binomial theorem, each contribution is now a product of $4 - k$ of the r_i values. For example, the coefficient of x^2 equals

$$e_2(r_1, r_2, r_3, r_4) = r_1r_2 + r_1r_3 + r_1r_4 + r_2r_3 + r_2r_4 + r_3r_4.$$

The symmetric property means that this expression is unchanged if we permute the indices in any manner. Alternatively, to construct a given e_k, we list the first k of the r_i values and then add a term for each permutation of the indices exactly once. As a convention, we set $e_0 = 1$.

With our numbers of interest,

$$e_2(2, 3, 4, 5) = 2 \times 3 + 2 \times 4 + 2 \times 5 + 3 \times 4 + 3 \times 5 + 4 \times 5 = 71.$$

Exercise: For $k = 1, 3, 4$, find $e_k(r_1, r_2, r_3, r_4)$. Then verify that $e_1 = 14$ and $e_3 = 154$.

Exercise: Use elementary symmetric polynomials to expand $(x + 2)(x + 3)(x + 4)$. What changes if we instead expand $(x - 2)(x - 3)(x - 4)$?

Exercise: Completely expand $\binom{L+4}{4}$, $\binom{L+3}{4}$, and $\binom{L+2}{4}$ as polynomials in L using elementary symmetric functions.

Exercise: If we have s variables, what are $e_1(r_1, \ldots, r_s)$ and $e_s(r_1, \ldots, r_s)$? For general k and s, calculate e_k when each $r_i = 1$. What polynomial results when each $r_i = 1$?

We now have a formula for $\rho_4(L)$ that exhibits the even property:

$$\rho_4(L) = \frac{2}{7!}(L+2)^2((L+2)^2 - 1)(14(L+2)^2 + 154).$$

Exercise: Repeat the preceding derivation for $\rho_6(L)$ and $\rho_8(L)$. For $n = 6$, the corresponding values are $e_1 = 33$, $e_3 = 3{,}135$, and $e_5 = 24{,}552$, while, for $n = 8$, we have

$$e_1 = 60,\ e_3 = 22{,}680,\ e_5 = 1{,}155{,}420,\ e_7 = 7{,}893{,}840.$$

Exercise: We might hope e_1 divides each e_k with odd k as above, but this property fails when $n = 10$. If you have access to a math computer program with symmetric functions, verify the values of e_k above and the statement about $n = 10$.

Exercise: Suppose $n = 2m$. Derive the general polynomial formula for $\rho_n(L)$ as an even polynomial in $L + m$. The formula should have three parts: the $2/(2n - 1)!$ factor, the factor for zeros, and the polynomial with coefficients e_k, which is always even in $L + m$. Express the arguments for e_k in terms of m.

Exercise: Prove that a polynomial of odd degree has at least one real root. In the previous exercise, prove that the polynomial with coefficients e_k has no real roots.

Exercise: Suppose $n = 2m + 1$. Derive the general polynomial formula for $\rho_n(L)$ as an even polynomial in $2L + n$.

Exercise: Find an alternative derivation for $\rho_4(L)$ from the reduced expression

$$F_4(x) = \frac{1 + x + x^2 + x^3}{(1-x)^7}.$$

Exercise: Using only $F_n(x)$, prove that $\rho_n(L)$ is a polynomial of degree $2n - 2$.

Exercise: Repeat the exercise sequence for $\rho_n(L)$ with the generating function $F_n(x) = \frac{1+x^n}{(1-x)^{2n}}$.

Example: In the installment "Hypergraphs for semi-magic squares" (Volume 18, Number 4), we considered the counting function $H_4(L)$ for semi-magic squares of size 4. In this case, we noted that $H_4(-L) = -H_4(L - 4)$ and that the generating function is given by

$$F(x) = \frac{1+14x+87x^2+148x^3+87x^4+14x^5+x^6}{(1-x)^{10}}.$$

The corresponding polynomial is

$$H_4(L) = \frac{11}{11340}L^9 + \frac{11}{630}L^8 + \frac{19}{135}L^7 + \frac{2}{3}L^6 + \frac{1109}{540}L^5 + \frac{43}{10}L^4 + \frac{35117}{5670}L^3 + \frac{379}{63}L^2 + \frac{65}{18}L + 1,$$

and we can show that it is an odd polynomial in $L + 2$ by writing

$$H_4(L) = \frac{32}{9!}(L+2)((L+2)^2 - 1)(11(L+2)^6 + 23(L+2)^4 + 128(L+2)^2 + 360).$$

Exercise: From the odd polynomial for $H_4(L)$, verify that $H_4(-3) = H_4(-2) = H_4(-1) = 0$, $H_4(0) = 1$, $H_4(1) = 24$, $H_4(2) = 282$, and $H_4(3) = 2008$. If available, use a math computer program to expand $F(x)$ in series form and to verify that the polynomials for $H_4(L)$ are equal.

Exercise: Explain the reciprocity and vanishing properties of $H_4(L)$ in terms of permutation matrices and semi-magic squares with only positive entries.

Finally, we revisit two examples from past installments with geometric terms.

Exercise: Recall that the generating function for the Fibonacci sequence is

$$F(x) = \frac{x}{1-x-x^2}.$$

With $a_0 = 0$ and $a_1 = 1$, the recursive equation is given by $a_{k+1} = a_k + a_{k-1}$ for $k > 1$. Calculate the recursive sequence to the left. Explain the behavior of this sequence by rewriting the recursive equation and the initial condition to the left and then comparing $F(1/x)$ with $F(-x)$. Use partial fractions to derive a Binet formula for $F(1/x)$.

Exercise: From the installment "Generating Functions for Partitions" (Volume 16, Number 5), recall that the counting function for partitions with at most two parts has generating function

$$F_2(x) = \frac{1}{(1-x)(1-x^2)}.$$

Calculate the recursive sequence to the left, and describe reciprocity and vanishing in this case. Repeat for partitions with at most three parts.

Exercise: Suppose the generating function for the sequence a_k is represented by the rational function $F(x) = \frac{p(x)}{q(x)}$. Find conditions on $F(x)$ such that a reciprocity formula holds for the recursive sequence a_{-k}. First, consider the case where $q(x) = (1-x)^{n+1}$ for some n. For the general case, consider both the recursive equation for a_k, which targets $q(x)$, and the initial conditions, which target $p(x)$.


BY HANNA MULARCZYK
EDITED BY MABEL YE
BEA AND THE SEA
"FOCUS ON THE LOCUS"
ISSUE 2, MAY 2026
WE'RE ALMOST AT OUR FIRST DESTINATION!
HOPEFULLY NOTHING STOPS US FROM GETTING
!
ZU BEE
WHAT IS THAT GROOVY MUSIC YONDER?
ZU -BEE
ZU BEE
I MUST GET CLOSER
ZU BEE
?
!
GRAB
NOT SO FAST, BIRDIE!
THAT'S A SIREN
HER SONG IS SO IRRE-SISTABLE
ZU BEE ZU BEE ZU
ZU BEE ZU
ONCE YOU HEAR IT, YOU HAVE NO CHOICE...
ZU BEE ZU
BEE ZU
...BUT TO CRASH DIRECTLY INTO HER ROCKS!
SADLY FOR US,
I LEFT MY NOISE CANCELLING HEADPHONES AT HOME.


* BIRDIE POV *
WE MUST CAREFULLY SAIL AROUND HER
IF WE STAY 100 METERS AWAY, WE CAN'T HEAR HER SONG.
100 m
* BEA POV *
HOW DO WE NAVIGATE THAT?
EASY! THE POINTS 100m AWAY EXACTLY FORM A CIRCLE OF RADIUS 100m
100m
PHEW!
NOW THERE ARE TWO!
IF THEY WERE FAR APART ENOUGH, WE COULD GO STRAIGHT DOWN THE MIDDLE.
100m
100m
BUT ACCORDING TO MY CALCULATIONS, THEY ARE ONLY 100m APART.
THEIR VOLUMES ADD.
ZU
BEE
TO BE SAFE, THE SUM OF OUR DISTANCES TO EACH SIREN SHOULD BE 200m
b
a
a+b=200m
SO OUR PATH AROUND THEM IS AN ELLIPSE
YOU'RE GOOD AT THIS!


DON'T GET TOO COMFY
AHEAD THERE'S A LINE OF SIRENS ACROSS FROM ONE
WE HAVE JUST ENOUGH ROOM TO SAIL BETWEEN
IF OUR PATH IS A PARABOLA, WE STAY EXACTLY BETWEEN THE SIRENS.
NOW WE CAN RELAX
ZU
BEE
NO!
NO!
A WHOLE LINE STRAIGHT AHEAD!
I CAN'T TURN IN TIME!
IT WAS NICE KNOWING YA, BIRDIE.
!
CLUTCH
room
PLOP
HEY! YOU'RE GOOD AT THIS TOO!
QED

Taylor Series for Tangent, Part 4

by Ken Fan | edited by Jennifer Sidney

As Emily and Jasmine walk to Cake Country, they continue discussing the math.

Jasmine: If we didn't know that $c_{k,n}$ and $d_{k,n}$ ultimately come from the tangent function, I don't know how we would have been able to prove $c_{0,n} = d_{0,n} + d_{1,n} + d_{2,n} + \ldots + d_{n-1,n}$ for odd n. The induction approach we tried looked awfully messy!

Emily: I'm okay with not having an algebraic proof. I actually really like our way of showing it! What nags at me is that we've proven an identity that only works for odd n. It makes me think that we've got only half of some picture.

Jasmine: Hm. Maybe!

The nth derivative of $\tan(x)$, with respect to x, is

$$\frac{p_n(2\sin(x))}{\cos^{n+1}(x)},$$

where $p_n(x)$ is defined recursively by $p_1(x) = 1$ and

$$p_{n+1}(x) = \frac{(n+1)xp_n(x) + (4 - x^2)p'_n(x)}{2},$$

for $n \geq 1$, where $p'_n(x)$ denotes the derivative of $p_n(x)$ with respect to x. Let $c_{k,n}$ be the coefficient of x^k in $p_n(x)$ and let $d_{k,n} = c_{k,n}/2^{(n-k-1)/2}$. We also define $c_{-1,n}$ to be 0. We have

$$c_{k,n+1} = \frac{n-k+2}{2}c_{k-1,n} + 2(k+1)c_{k+1,n},$$

$$d_{k,n+1} = \frac{n-k+2}{2}d_{k-1,n} + (k+1)d_{k+1,n}.$$

Emily: Actually, I believe that $d_{k,n}$ is the coefficient of x^k in the polynomial $p_n(\sqrt{2}x)/\sqrt{2}^{n-1}$ for *all* positive integers n, not just odd n.

Jasmine: Yes, that's true for all n.

Emily: And so it must be true, by setting x equal to 1, that the sum of the $d_{k,n}$, over k while fixing n, is equal to $p_n(\sqrt{2})/\sqrt{2}^{n-1}$, for all n.

Jasmine: Yes!

Emily: And, for all n, we also know that $p_n(\sqrt{2})\sqrt{2}^{n+1} = \tan^{(n)}(\pi/4)$. So the sum of the $d_{k,n}$, over k, for all n must be $\tan^{(n)}(\pi/4)/2^n$.

Jasmine: Left!

Emily: Huh?

Jasmine: Cake Country's this way. That way's your house.

Emily: Oh, right. All this is the same situation we found when n is odd. The only difference is that we don't know how to connect these numbers to the $c_{k,n}$.

Jasmine: We just know that it's definitely not $c_{0,n}$, since for even n, $c_{0,n} = 0$.

Emily: What happens if we take the trick you used to isolate the odd terms in the expansion of $\tan(\pi/4 + x)$ and we apply it to isolate the even terms?

Jasmine: Yes, that should inform us what the sum of the $d_{k,n}$ (over k) is for even n!

Emily and Jasmine arrive at Cake Country and seat themselves at their favorite booth. Instead of ordering, they pull some scratch paper out of their backpacks and carry on with the math.

Emily: To isolate the even terms, I guess we should take the average of $\tan(\pi/4 + x)$ and $\tan(\pi/4 - x)$. Actually, to account for the powers of 2, we can take the average of $\tan(\pi/4 + x/2)$ and $\tan(\pi/4 - x/2)$. That is,

$$\frac{\tan(\frac{\pi}{4}+\frac{x}{2})+\tan(\frac{\pi}{4}-\frac{x}{2})}{2} = \tan(\pi/4) + \frac{\tan^{(2)}(\pi/4)/2^2}{2!}x^2 + \frac{\tan^{(4)}(\pi/4)/2^4}{4!}x^3 + \ldots.$$

Jasmine: We can probably simplify that left-hand side with trig identities.

$$\begin{aligned}
\frac{\tan(\frac{\pi}{4}+\frac{x}{2})+\tan(\frac{\pi}{4}-\frac{x}{2})}{2} &= \frac{\frac{\tan(\pi/4)+\tan(x/2)}{1-\tan(\pi/4)\tan(x/2)}+\frac{\tan(\pi/4)-\tan(x/2)}{1+\tan(\pi/4)\tan(x/2)}}{2} \\
&= \frac{\frac{1+\tan(x/2)}{1-\tan(x/2)}+\frac{1-\tan(x/2)}{1+\tan(x/2)}}{2} \\
&= \frac{(1+\tan(x/2))^2+(1-\tan(x/2))^2}{2(1-\tan(x/2))(1+\tan(x/2))} \\
&= \frac{2+2\tan^2(x/2)}{2(1-\tan^2(x/2))} \\
&= \frac{1+\tan^2(x/2)}{1-\tan^2(x/2)}
\end{aligned}$$

Emily: Here we can use the half-angle identity for tangent, $\tan(x/2) = \dfrac{1-\cos x}{\sin x}$.

$$= \frac{1+\left(\frac{1-\cos x}{\sin x}\right)^2}{1-\left(\frac{1-\cos x}{\sin x}\right)^2}$$

$$= \frac{\sin^2 x + (1 - 2\cos x + \cos^2 x)}{\sin^2 x - (1 - 2\cos x + \cos^2 x)}$$

$$= \frac{2 - 2\cos x}{2\cos x - 2\cos^2 x)}$$

$$= \frac{1}{\cos x}$$

Jasmine: How about that! So if we denote by $\sec^{(n)}(x)$ the nth derivative of secant of x, with respect to x, then for even n, we have

$$d_{0,\,n} + d_{1,\,n} + d_{2,\,n} + \ldots + d_{n-1,\,n} = \sec^{(n)}(0).$$

Emily: I guess we can combine our observations. If we let $D_n = \sum_{k=0}^{n-1} d_{k,n}$ and we let

$$f(x) = \tan(x) + \sec(x) = \frac{1 + \sin x}{\cos x} = \tan(\pi/4 + x/2),$$

then $D_n = f^{(n)}(0)$, for all integers $n \geq 1$. This feels more complete.

Jasmine: It does, and it's all very interesting. Yet it doesn't seem to help us get any closer to finding a quick way to compute the Taylor series for the tangent function. We still have no way to compute either $c_{0,\,n}$ or D_n other than recursively, unless the Taylor series for $f(x)$ is easy to compute…

Mr. ChemCake: You two have been working hard! Here's some fuel for your minds … on the house!

Emily: Wow! Thanks, Mr. ChemCake!

Emily and Jasmine each pick up what looks like a very dense dark brownie with some kind of off-white frosting and try a bite. But it's not a brownie at all; it tastes like a meat wafer!

Jasmine: That's different! Actually, I kind of like it! What is it?

Mr. ChemCake: It's pemmican … shredded meat, tallow, and salt. It's what Native Americans ate for sustenance during the winter. I've been experimenting to see if I can give it more of a cake-like texture.

Emily: Hopefully it'll help us make some progress.

Mr. ChemCake: Let me know. But that should give you enough energy for at least a few more hours of math!

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 38 - Meet 5 March 5, 2026	Mentors:	Elisabeth Bullock, Jessica Charalel, Elsa Frankel, Kira Lewis, Yaqi Li, Sophia Liao, Hanna Mularczyk, Maya Robinson, Amanda Tran

Our meets see members thinking about all kinds of mathematics. There are so many topics within mathematics and so many topics to which mathematics can be applied. Just at this meet, members were thinking about the mathematics behind Rubik's cube, Platonic solids (especially, the icosahedron and dodecahedron), symmetry groups, entanglement puzzles, power series and the exponential function, determinants and volumes, random object selection, representations of numbers by quadratic forms, the game of sprouts, logarithms, and partitions. If you want to get involved with mathematics, there are almost certainly many topics that you'd find interesting to think about.

Session 38 - Meet 6 March 12, 2026	Mentors:	Elisabeth Bullock, Elsa Frankel, Clarise Han, Shauna Kwag, Hanna Mularczyk, Maya Robinson, Amanda Tran

Try to forget everything you know about geometric shapes. How would you then go about classifying and categorizing them? Do you think there is a sensible way to put all geometric objects into a sensible list? What relationships would you take note of that exist between geometric objects? Are there any equivalence relations that you would want to define to reduce the scope of the task? Would your categories exist on a continuum, or would it consist of a discrete set of classes? Would there be infinitely many categories? Would there be a way to parametrize all the geometric shapes?

Session 38 - Meet 7 March 19, 2026	Mentors:	Elisabeth Bullock, Shauna Kwag, Hanna Mularczyk, Maya Robinson, Amanda Tran

If you're interested in trying your hand at creating math, but are drawing a blank about what kind of math to create, try to start with a math problem that you enjoy and then modify it. Past math competitions are a good source of problems. For example, using a random number generator to pick one of the 15 problems on the latest AIME 1 competition, this problem was selected:

Find the number of functions π mapping the set $A = \{1, 2, 3, 4, 5, 6\}$ onto A such that for every $a \in A$, $\pi(\pi(\pi(\pi(\pi(\pi(a)))))) = a$. First, try to solve the problem. If you do this, you'll see that the problem is asking for is the number of permutations in S_6 that have an order that divides evenly into 6. Then, think of ways to modify this question: How many permutations are there in S_6 that have order exactly 6? More generally, how many permutations are there in S_n that have order m? What can you say about the positive integer m for which there exists an element of order m in S_n? If you pick a random permutation, with each permutation equally likely, what is its expected order?

Session 38 - Meet 8 April 2, 2026	Mentors:	Elisabeth Bullock, Jessica Charalel, Elsa Frankel, Clarise Han, Kira Lewis, Hanna Mularczyk
	Visitor:	Ila Fiete, MIT McGovern Institute

Prof. Ila Fiete's presentation last fall (see the *Notes from the Club* in Volume 19, Number 1) was so popular, that our members made it crystal clear that they would love to hear from her again. We're grateful to Prof. Fiete for agreeing to visit again this spring!

Ila began with a brief recap of her fall presentation. She then introduced us to the hippocampus and the entorhinal cortex, brain regions of particular interest in navigation. The hippocampus is where new, experienced memories, are formed. She mentioned the intriguing case of Henry Molaison, who damaged his hippocampus and could not form memories. Another example, which Ila did not mention but the reader might be interest in is the case of Clive Wearing. Mr. Wearing was a musician and can still play pieces he learned in his childhood from memory, but he cannot remember that you visited 10 minutes ago.

She then described place cells and compass (or "head direction") cells and described various experiments that reveal what role these cells play in navigation. Members asked numerous excellent questions, such as:

- Why do some of the compass cells produce a really clean, Gaussian-like, signal, whereas others produce messy signals. (Ila: nobody knows.)

- If a rat is placed in an identical room, will its place cells remap or preserve its current mapping? (Ila: translocating into an identical room, there's no remapping…but if the rat travels from one room to an identical room through a hallway, it does remap.)

Session 38 - Meet 9 April 9, 2026	Mentors:	Elisabeth Bullock, Jessica Charalel, Elsa Frankel, Kira Lewis, Hanna Mularczyk, Amanda Tran

Our youngest members created a nifty way to transform English words into pretty logograms. If you were to invent a way to turn English words into unique pictures, what would "Girls' Angle" look like in your system?

Session 38 - Meet 10 April 16, 2026	Mentors:	Elisabeth Bullock, Jessica Charalel, Shauna Kwag, Sophia Liao, Hanna Mularczyk, Maya Robinson, Amanda Tran

For what positive integers k does the set of k-cycles in the symmetric group S_n ($n \geq k$) generate S_n? What subgroup of S_n is generated by its set of k-cycles?

Session 38 - Meet 11 April 30, 2026	Mentors:	Elisabeth Bullock, Elsa Frankel, Clarise Han, Shauna Kwag, Kira Lewis, Yaqi Li, Maya Robinson, Amanda Tran, Anlan Xu

Pick one of your favorite functions. Can you figure out a way to define it using a functional relationship?

Calendar

Session 37: (all dates in 2025)

September	11	Start of the thirty-seventh session!
	19	
	25	
October	2	
	9	Ila Fiete, MIT
	16	
	23	
	30	
November	6	
	13	
	20	
	27	Thanksgiving - No meet
December	4	

Session 38: (all dates in 2026)

January	29	Start of the thirty-eighth session!
February	5	
	12	
	19	No meet
	26	
March	5	
	12	
	19	
	26	No meet
April	2	Ila Fiete, MIT
	9	
	16	
	23	No meet
	30	
May	7	

Girls' Angle has run nearly 200 Math Collaborations. Math Collaborations are fun, fully collaborative, math events that can be adapted to a variety of group sizes and skill levels. We now have versions where all can participate remotely. We have now run four such "all-virtual" Math Collaboration. If interested, contact us at girlsangle@gmail.com. For more information and testimonials, please visit www.girlsangle.org/page/math_collaborations.html.

Girls' Angle can offer custom math classes over the internet for small groups on a wide range of topics. Please inquire for pricing and possibilities. Email: girlsangle@gmail.com.

Girls' Angle: A Math Club for Girls

Membership Application

Note: If you plan to attend the club, you only need to fill out the Club Enrollment Form because all the information here is also on that form.

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address (the Bulletin will be sent to this address):

Email:

Home Phone: ____________________________ Cell Phone: ____________________________

Personal Statement (optional, but strongly encouraged!): Please tell us about your relationship to mathematics. If you don't like math, what don't you like? If you love math, what do you love? What would you like to get out of a Girls' Angle Membership?

The $50 rate is for US postal addresses only. **For international rates, contact us before applying.**

Please check all that apply:

- ☐ Enclosed is a check for $50 for a 1-year Girls' Angle Membership.
- ☐ I am making a tax-free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com.

Girls' Angle

A Math Club for Girls

Girls' Angle
Club Enrollment

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

The club is where our in-person mentoring takes place. At the club, girls work directly with our mentors and members of our Support Network. To join, please fill out and return the Club Enrollment form. Girls' Angle Members receive a significant discount on club attendance fees.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-12. We welcome *all girls* (in grades 5-12) regardless of perceived mathematical ability. There is no entrance test. Whether you love math or suffer from math anxiety, math is worth studying.

How do I enroll? You can enroll by filling out and returning the Club Enrollment form.

How do I pay? The cost is $20/meet for members and $30/meet for nonmembers. Members get an additional 10% discount if they pay in advance for all 12 meets in a session. Girls are welcome to join at any time. The program is individually focused, so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is based in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org/page/calendar.html or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Peirce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science, worked in the mathematics educational publishing industry, and taught at Hasim. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high-level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. In this way, we hope to help girls become solvers of the yet unsolved.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: Club Enrollment Form

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Please fill out the information in this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. Names:

____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: Girls roughly in grades 5-12 are welcome. Although we will work hard to include every girl and to communicate with you any issues that may arise, Girls' Angle reserves the discretion to dismiss any girl whose actions are disruptive to club activities.

Personal Statement (optional, but strongly encouraged!): We encourage the participant to fill out the optional personal statement on the next page.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: _______________________
(Parent/Guardian Signature)

Participant Signature: ____

Members: Please choose one.	Nonmembers: Please choose one.
☐ Enclosed is $216 for one session (12 meets)	☐ I will pay on a per meet basis at $30/meet.
☐ I will pay on a per meet basis at $20/meet.	☐ I'm including $50 to become a member, and I have selected an item from the left.

☐ I am making a tax-free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Personal Statement (optional, but strongly encouraged!): This is for the club participant only. How would you describe your relationship to mathematics? What would you like to get out of your Girls' Angle club experience? If you don't like math, please tell us why. If you love math, please tell us what you love about it. If you need more space, please attach another sheet.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: ___________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ___________________________________