Girls' Angle Bulletin

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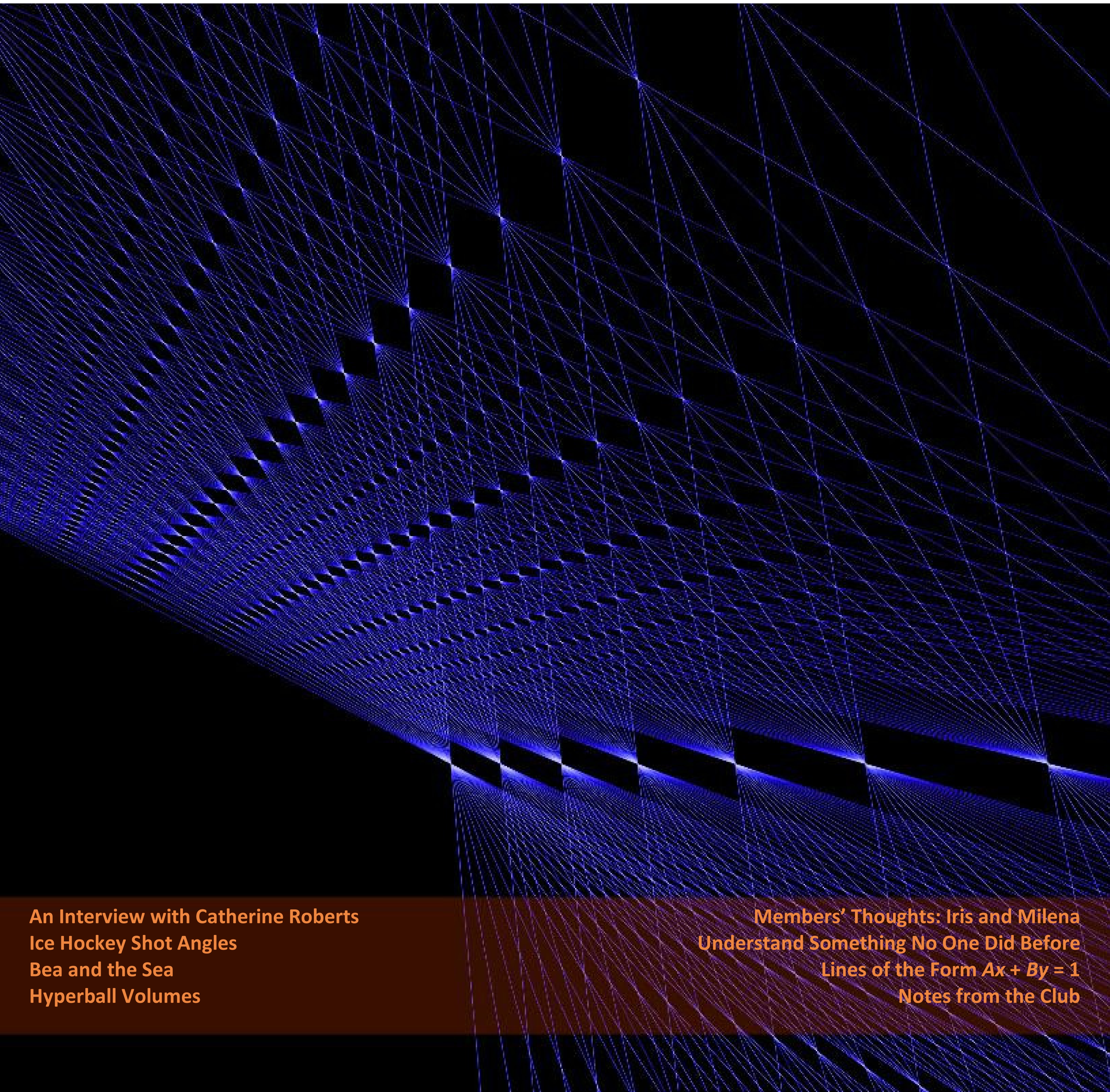
To Foster and Nurture Girls' Interest in Mathematics



From the Founder

Students of all levels rule this issue: A recent professional publication by members Milena Harned and Iris Liebman, a new comic strip by MIT grad student and Girls' Angle mentor Hanna Mularczyk, and the work of 14 BB&N students. Hope you're inspired! - Ken Fan, President and Founder

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Girls' Angle Bulletin

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Girls' Angle welcomes submissions that pertain to mathematics.

The print version of the Bulletin is printed by the American Mathematical Society.

Editors: Amanda Galtman
Jennifer Sidney
Executive Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: Detail of a Harned-Liebman line arrangement. Harned and Liebman discovered a new family of line arrangements that are devoid of polygons with more than 4 sides. See page 23.

An Interview with Catherine Roberts

Catherine Roberts is Professor of Mathematics at the College of the Holy Cross. She obtained her undergraduate degree at Bowdoin College and her PhD from Northwestern University under the supervision of W. Edward Olmstead. She is an expert on applied mathematics and mathematical modeling. Prof. Roberts was also the first woman executive director of the American Mathematical Society (AMS) from 2016-2023. She is a Fellow of the Association for Women in Mathematics.

This interview was conducted be Girls' Angle mentor and Wellesley College undergraduate Elsa Frankel.

Elsa: Please describe your current role as a professor at Holy Cross? What is your research area, and what courses are you teaching this semester?

Catherine: I have been a professor at College of the Holy Cross in Worcester, MA since 2001, aside from being away for 7 years when I served as the Executive Director for the American Mathematical Society. As an applied mathematician deeply interested in the environment, my teaching and research all connect to mathematical modeling of the environment. This semester, I am teaching courses for first-year students. One is a seminar called Environmental Reasoning and the other is a special version of Calculus 1 that includes modeling using real-world data.

Elsa: How did you find your area of specialization, mathematical modeling? What projects have you been excited about lately, and what would you want students to know about the area?

…after my first year, the chair of the department wrote a letter to me inviting me to consider majoring in math. I truly didn't see myself as a math geek—I was very well-rounded and into art, theater, and literature. Thankfully, it was easy for me to continue to pursue all my interests because Bowdoin College is a liberal arts college. I think that I did well in math because I was a natural problem-solver, and this translated well into the math classroom.

Catherine: After high school, I went to Bowdoin College in Maine where I majored in both math and art history. I also got certified to be a high school math teacher. I realized my senior year that I wanted to become a college professor, so I then went to Northwestern University near Chicago for five years to earn my doctorate, or Ph.D., in applied math and engineering sciences. I realized that my interests were in using math to solve real-world problems, rather than extend humanity's understanding of the theoretical math structures. So, this was a good fit for me. Lots of problem-solving, not many theorems and proofs. Mathematical modeling is a powerful tool for understanding our world and for making predictive estimates about the future.

My first research area was more theoretical, but when I was a professor at Northern Arizona University, I was invited to help the Grand Canyon National Park figure out a better way of scheduling white water rafting trips down the Colorado River. This project changed my life! I used to publish a new math result and share it with about a dozen people across the planet. But as I developed new results for the Grand Canyon, thousands of people were interested! I gave a talk at the Denver public library with hundreds of concerned citizens curious about how math could help us manage

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Thank you and best wishes,
Ken Fan
President and Founder
Girls' Angle: A Math Club for Girls

Content Removed from Electronic Version

Ice Hockey Shot Angles

by Nava Galperin, Marie Lee, Eliot Saad, Ajay Shroff, Oliver Song
edited by Amanda Galtman

The authors are students at Buckingham, Browne, and Nichols School. They did this work as 8th graders, inspired by a mutual love of hockey!

In this paper, we consider the problem of determining the chances of getting a shot on goal in ice hockey from various places on the ice. We look at the locus of positions on the ice that offer the same chance of success. We first analyze this problem in two dimensions by using shot angle. We then expand into the third dimension by using spherical shot angle.

The illustrations of an ice hockey rink in this paper have the same proportions as the rink in use at the 2026 Winter Olympics in Italy, which is 60 meters by 26 meters. Over this rink, we imagine an *xy*-coordinate plane with the horizontal *x*-axis placed over one of the goal lines and the *y*-axis oriented so that the other goal line has a positive *y*-coordinate.

Two Dimensions

To begin with the two-dimensional interpretation of the problem, we asked: what points on the rink have the same shot angle on goal? To standardize our problem, we scaled our axes so that the goal posts are located at (-1, 0) and (1, 0). We placed the puck at (x, y). Taking the puck and the two posts as the vertices of a triangle, let a be the length of the side corresponding to the goal. Let b and c be the lengths of the sides from the puck to the goal posts. See Figure 1.

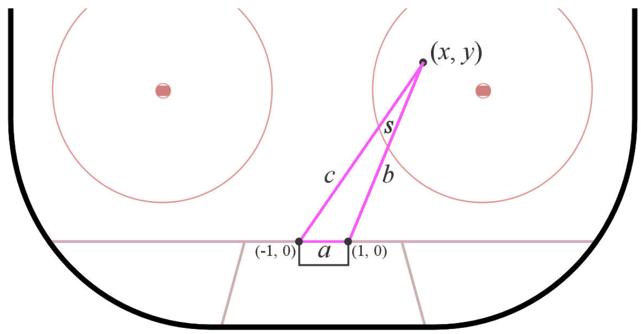

Figure 1. A hockey rink with the puck at (x, y).

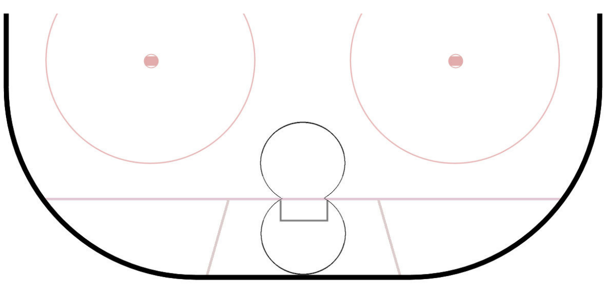

Figure 2. The 30° locus.

We determined a, b, and c by applying the distance formula. The value of a is 2, since that side's endpoints are located at (-1, 0) and (1, 0). The distances from (1,0) and (-1,0) to the puck are

$$\sqrt{(x-1)^2+y^2} \text{ and } \sqrt{(x+1)^2+y^2}.$$

Therefore,

$$\begin{aligned} a^2 &= 4, \\ b^2 &= (x-1)^2+y^2, \\ \text{and} \quad c^2 &= (x+1)^2+y^2. \end{aligned}$$

To determine the shot angle s, which is the angle between the sides of length b and c, we used the law of cosines, which is: $a^2 = b^2 + c^2 - 2bc \cos s$. To illustrate, we set $s = 30°$. After substituting and simplifying, we found that

$$\cos 30° = \frac{x^2 + y^2 - 1}{\sqrt{((x-1)^2 + y^2)((x+1)^2 + y^2)}}.$$

The graph of this equation is shown in Figure 2 (previous page). The graph is the union of two arcs of congruent circles that pass through the goal posts and whose centers are located at $(0, \sqrt{3})$ and $(0, -\sqrt{3})$. However, the part of the graph with negative y-coordinates does not apply, as a hockey goal is open only on one side, so we added the specification that $y > 0$.

This same process can be used for any other angle by substituting it for s. For general s, the equation is

$$\cos s = \frac{x^2 + y^2 - 1}{\sqrt{((x-1)^2 + y^2)((x+1)^2 + y^2)}}.$$

Figure 3 shows the loci of constant shot angle starting with the shot angle seen from the middle of the opposite goal line (which happens to be about 2°) and increasing by a factor of $\sqrt{2}$. That is, as one moves toward the goal, each successive ring has shot angle $\sqrt{2}$ times larger.

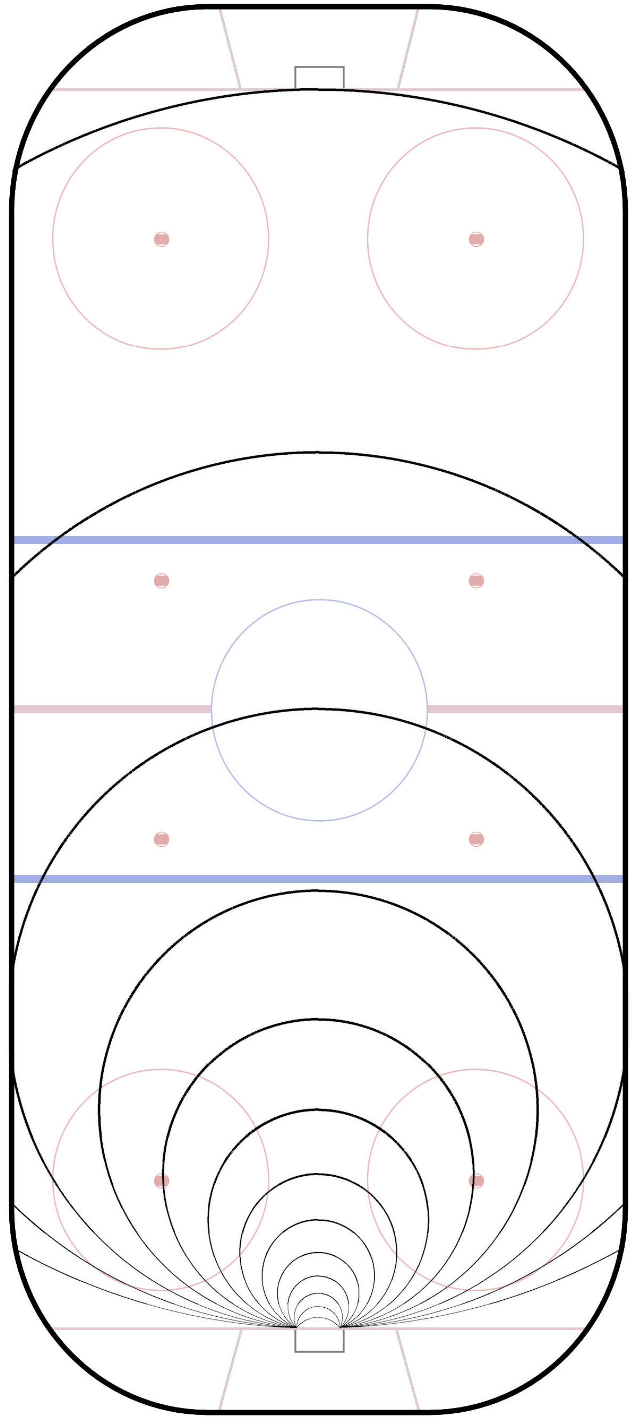

Figure 3. The loci of points with the same shot angle are arcs of circles that pass through the goal posts.

Three Dimensions

Next, we expanded to three dimensions. Our goal was to find the locations on the ice with the same chance of a successful shot on a 3D goal, and the first step was to quantify this chance. We did this by projecting the goal mouth onto a unit sphere surrounding the puck. The area of this projection, a spherical quadrilateral, essentially tells us how big the goal appears to the puck at that given position on the ice. We called this area the **spherical shot angle**.

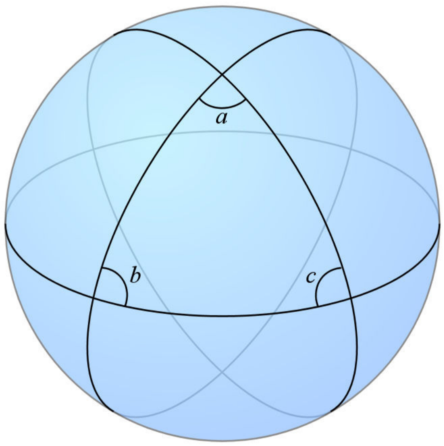

Figure 4. A spherical triangle.

To find this area, we had to find a general formula for the surface area of a spherical quadrilateral. Since a spherical quadrilateral can be split into two spherical triangles, we reduced to the problem of finding a formula for spherical triangles. A spherical triangle is formed by three great circles (i.e., circles with radius equal to that of the sphere). We labeled the interior angles of this triangle a, b, and c (see Figure 4). Each angle was formed by the intersection of two great circles and thus corresponds to its own lune (see Figure 5). A lune captures a slice of a sphere's surface area equal to the total surface area of the sphere ($4\pi r^2$) multiplied by whatever fraction of a full circle the lune's angle is. If the lune's angle in radians is θ, the area is:

$$\frac{\theta}{2\pi}(4\pi r^2) = 2\theta r^2.$$

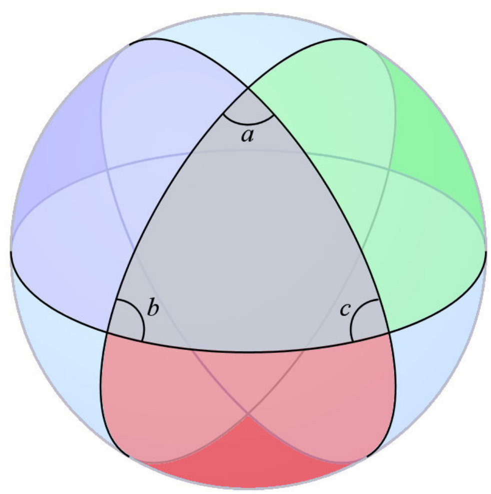

Figure 5. Corresponding lunes.

This formula for the area of a lune, in conjunction with the formula for the total area of the sphere, enabled us to find the area of the triangle. This is because the sum of the areas of all three lunes, each counted twice, gives the entire surface area of the sphere with the area of the spherical triangle counted six times (again, see Figure 5). That is,

$$4\pi r^2 = 2(2ar^2 + 2br^2 + 2cr^2) - 4t,$$

where t is the area of the spherical triangle. Rearranging terms, we found that

$$t = r^2(a + b + c - \pi).$$

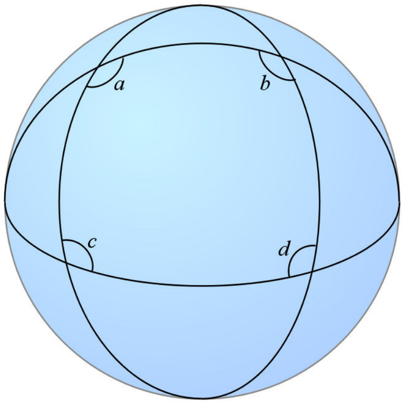

Figure 6. A spherical quadrilateral.

To find the area of a spherical quadrilateral (see Figure 6), we split the quadrilateral into two triangles by drawing a great circle through the vertices at angles b and c (see Figure 7). In one triangle, the angles were a, x, and y. In the other, the angles were d, $b - x$, and $c - y$.

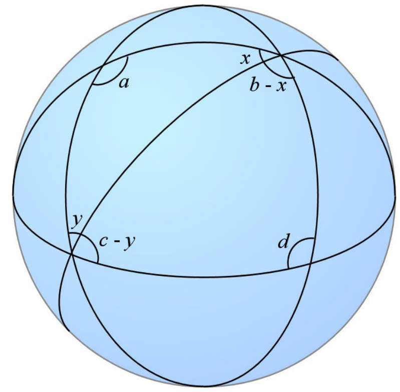

Figure 7. Splitting into two triangles.

Thus, the area of the spherical quadrilateral turned out to be

$$r^2(a + x + y - \pi) + r^2(b - x + c - y + d - \pi) = r^2(a + b + c + d - 2\pi).$$

This formula enabled us to quantify the chance that a shot makes it into the goal by determining the area of the projection of the goal onto the unit sphere centered on the puck.

We then had to find the interior angles of the projection of the goal mouth onto the unit sphere centered on the puck. The projection of each of the four sides of the goal mouth onto the unit sphere was contained in the intersection of the plane that contains that side and the puck with the surface of the unit sphere. These four intersections formed the sides of a spherical quadrilateral. Its interior angles were the dihedral angles between planes corresponding to sides that shared an endpoint. So, our next step was to find a general formula for the dihedral angle between two such planes.

Here's how we went about this: Consider the planes $ax + by + cz = 0$ and $dx + ey + fz = 0$ in the xyz-coordinate space, where a, b, c, d, e, and f are constants. Then the vectors (a, b, c) and (d, e, f) are perpendicular to these planes, respectively. The angle formed by these two vectors is the dihedral angle between the two planes. (Actually, the two vectors form angles with two different angle measures, say D, and $\pi - D$. When we computed the spherical shot angle, we had to make sure we used the correct one of these angle measurements.)

The angle formed by the two vectors (a, b, c) and (d, e, f) can be obtained by using the dot product (which amounts to using the law of cosines again). We found that $\cos D$, up to sign, is

$$\frac{ad + be + cf}{\sqrt{a^2 + b^2 + c^2}\sqrt{d^2 + e^2 + f^2}}.$$

Then it was time to get specific. Our spherical angle was defined by specific planes, each of which contains a side of the goal mouth and passes through the puck. Note that the angle between each of the side posts and the ice is always 90°, so the dihedral angle between those planes that contain a side post and the ice sheet itself is 90°.

To compute the remaining two angles in our spherical quadrilateral, we needed the equations of the planes that contain the puck and one of the side posts or the crossbar (the horizontal bar crossing the top of the goal mouth). In the following equations for each of these three planes, from the perspective of the puck, we placed the right post through the point (-*B*, 0, 0) and the left post through the point (*B*, 0, 0). We let *A* be the height of the goal, and we placed the puck at (P_x, P_y, 0), with $P_y > 0$.

Plane	Equation
Contains crossbar	$\frac{1}{P_y} y + \frac{1}{A} z = 1$
Contains right side post	$-\frac{1}{B} x + \frac{B+P_x}{BP_y} y = 1$
Contains left side post	$\frac{1}{B} x + \frac{B-P_x}{BP_y} y = 1$
Ice sheet	$z = 0$

Applying the dihedral angle formula gave us the following formulas for top left and right angles of the projection, respectively:

$$\arccos\left(-\frac{A(B-P_x)}{\sqrt{A^2+P_y^2}\sqrt{P_y^2+\left(B-P_x\right)^2}}\right) \text{ and } \arccos\left(-\frac{A(B+P_x)}{\sqrt{A^2+P_y^2}\sqrt{P_y^2+\left(B+P_x\right)^2}}\right).$$

Using the formula for the area of the spherical quadrilateral with these four angles, we found that the spherical shot angle, which we called *S*, is

$$S = \arccos\left(-\frac{A(B-P_x)}{\sqrt{A^2+P_y^2}\sqrt{P_y^2+\left(B-P_x\right)^2}}\right) + \arccos\left(-\frac{A(B+P_x)}{\sqrt{A^2+P_y^2}\sqrt{P_y^2+\left(B+P_x\right)^2}}\right) - \pi.$$

Notice that $\frac{A}{\sqrt{A^2+P_y^2}}$ is the sine of the angle of elevation of the crossbar (i.e., the sine of the dihedral angle between the ice plane and the plane that contains the puck and the crossbar), which we called *E*.

Also, $-\frac{B-P_x}{\sqrt{P_y^2+(B-P_x)^2}}$ and $-\frac{B+P_x}{\sqrt{P_y^2+(B+P_x)^2}}$ are cos θ_L and cos θ_R, respectively, where θ_L and θ_R are as indicated in Figure 8. Thus,

$$S = \arccos(\sin E \cos \theta_L) + \arccos(\sin E \cos \theta_R) - \pi.$$

When the puck is very close to the goal line, i.e., when y is positive but small, sin E is very close to 1, and S is closely approximated by $\theta_L + \theta_R - \pi = s$. Here, s is the shot angle. In other words, when y is small, the spherical angle is closely approximated by the shot angle, and the locus of points of constant spherical shot angle asymptotically approaches the circular arc corresponding to the locus of constant shot angle $s = S$. However, farther from the goal line, where sin E is no longer almost 1, the locus of points of constant spherical shot angle looks like a somewhat squashed circle. In Figure 9 (next page), we show some of these loci superimposed upon a diagram of an ice hockey rink.

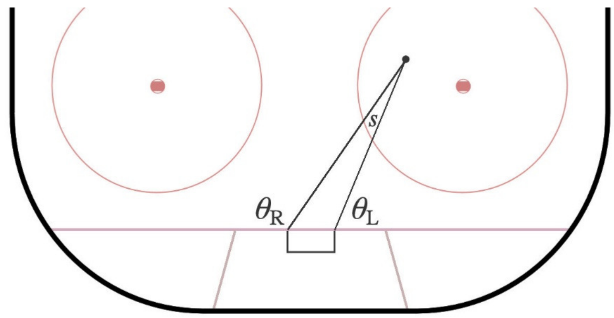

Figure 8. In this diagram, the "*s*" is lower case and corresponds to the 2D shot angle.

For a given spherical shot angle S, we determined the location of the puck that enjoys that spherical shot angle along the line running between the centers of the two goals. To do that, we solved for P_y in our equation for spherical shot angle with $P_x = 0$:

$$S = 2\arccos\left(-\frac{AB}{\sqrt{A^2+P_y^2}\sqrt{P_y^2+B^2}}\right) - \pi.$$

Rearranging terms, we found $\frac{AB}{\sqrt{A^2+P_y^2}\sqrt{P_y^2+B^2}} = \sin\frac{S}{2}$.

After further rearrangement, we obtained a quadratic in P_y^2 whose solution is

$$P_y = \sqrt{\frac{-(A^2+B^2)+\sqrt{(A^2+B^2)^2+(2AB\cot\frac{S}{2})^2}}{2}}.$$

The spherical shot angle S grows much faster than the shot angle s as you skate directly towards the goal. Heuristically, this is because the shot angle is roughly inversely proportional to the distance from the goal, whereas spherical shot angle is inversely proportional to the square of the distance from the goal.

Can you rigorously prove some of the facts for which we provided only a heuristic argument?

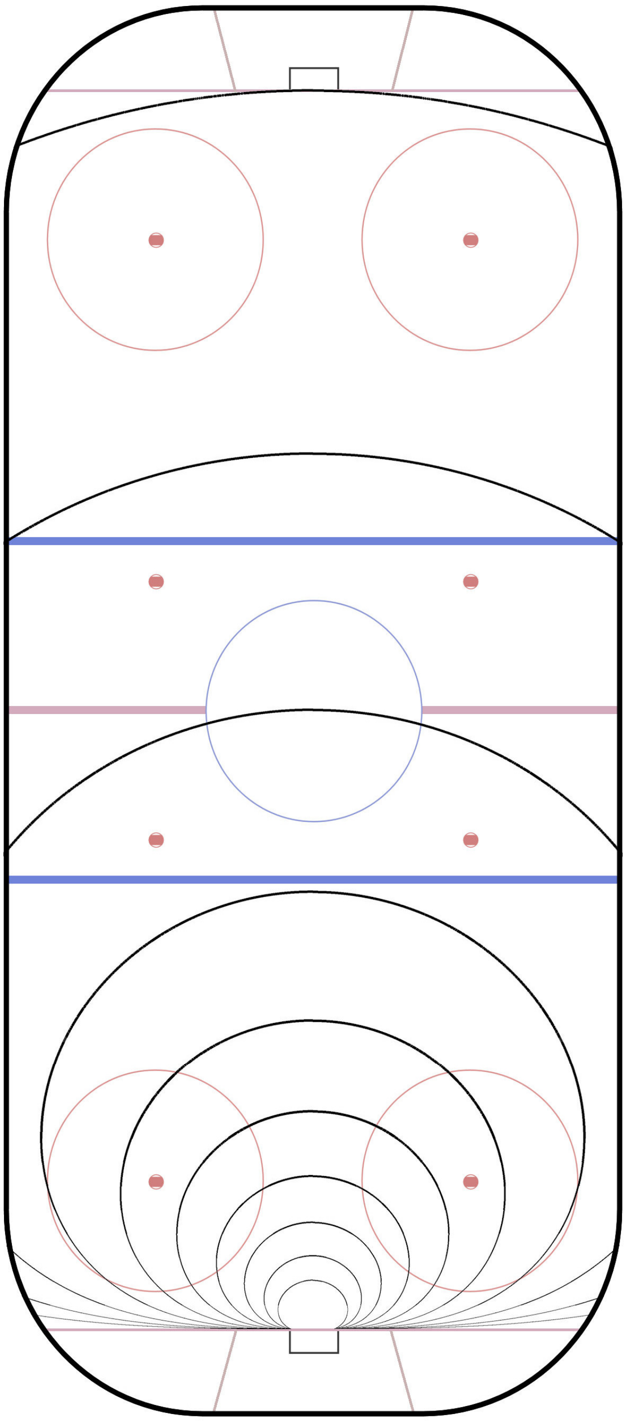

Figure 9. The loci of points with the same spherical shot angle. The farthest locus passes through the midpoint of the opposite goal. Successively closer loci correspond to multiplying the spherical shot angle by 2.

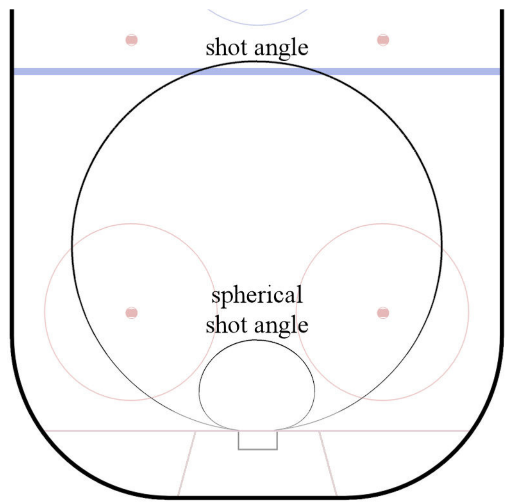

Figure 10. Comparing spherical shot angles and shot angles.

The indicated shot angle locus is the set of locations where a puck that is shot in a random direction uniformly selected from the half circle of directions that have a negative y-component toward the goal has a 3% chance of scoring. The indicated spherical shot angle locus is the set of locations where a puck that is shot in a random direction uniformly selected from the quarter sphere of directions that have a component up from the ice and with a negative y-component toward the goal has a 3% chance of scoring.

The locus of spherical shot angles is much smaller due to the fact that when you move into the third dimension, not only can you miss wide (i.e., to the side of the goal), but you can also miss by shooting high (i.e., over the goal).


BEA AND THE SEA
ISSUE 1, MARCH 2026
BY HANNA MULARCZYK
EDITED BY MABEL YE
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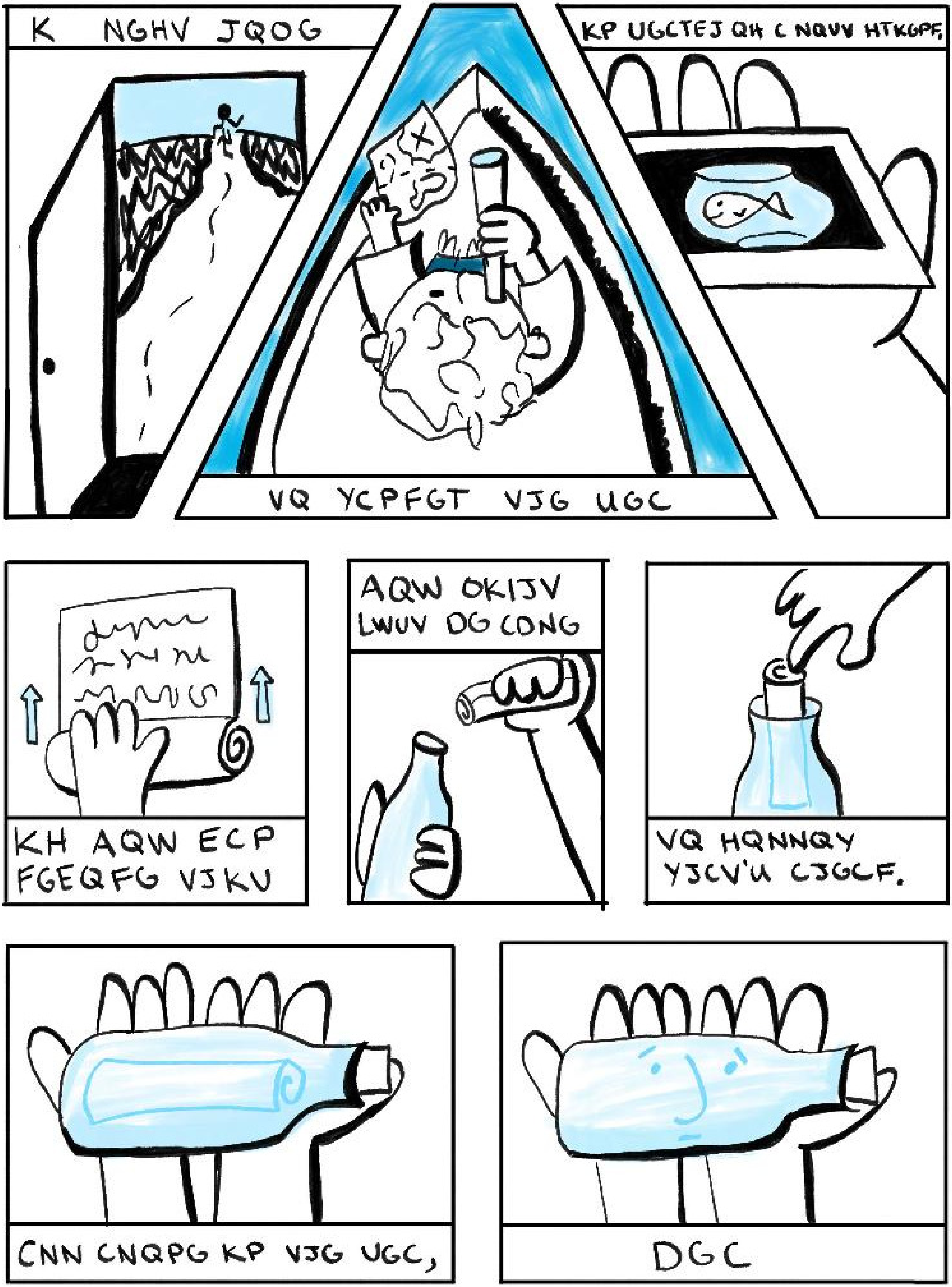
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Hyperball Volumes

by Ken Fan | edited by Amanda Galtman

The 8th graders are Aras Acemoglu, Zachary Davis, Alex Inker, Evan Keri, Eleanor Kuo, Estella Mo, Juliana Pierry Barbic, Sophia Shao, and Eleanor Swallow-Crumbley.

Nine 8th graders at the Buckingham Browne and Nichols Middle School had just returned from a journey exploring the k-dimensional faces of the n-dimensional hypercube and were brainstorming for another math adventure. It wasn't long before they were back asea, propelled by their latest question:

What is the volume of the d-*dimensional hyperball of radius 1?*

Theirs was a journey that captures the story of all math in microcosm: A long, wondrous, demystification that comes in fits and starts.

As they began thinking about it, they realized that a path to the answer was more obscure than they had initially thought. They realized that they knew the answer for $d = 2$ only because they had all learned the formula πr^2 for the area of a circular disc with radius r by rote. One of the students explained the well-known heuristic argument where you slice up the circle into a bunch of thin pie slices and rearrange these into a shape roughly resembling an r-by-πr rectangle, then assert that "as the slices become thinner, the quasi-rectangle gets closer and closer to being a true rectangle." It's possible to turn this argument into a rigorous proof, but most people don't.

You might feel that finding the volumes of higher-dimensional hyperballs is hopeless, knowing that your knowledge of the two-dimensional disc is merely a memorized fact.

Well, in mathematics, you already have everything you need to answer questions like this: your own mind.

Using their minds, these 8th graders made tangible progress, though a lot of mystery remains.

Let's follow their progress and see what they figured out.

"What do we even mean by volume in *d*-dimensional space?"

They thought about area and how it gives the equivalent number of unit squares needed to fill up a planar region. And they thought about volume, which tells how many unit cubes are needed to fill up a space. They decided to use the unit d-dimensional hypercube as the unit of volume.

"Why is the area of a circular disc equal to πr^2?"

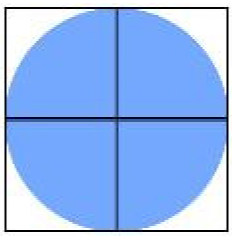
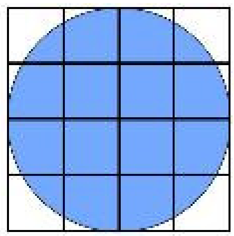
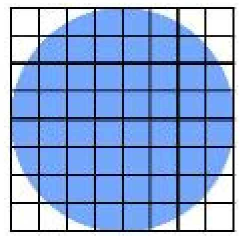
They turned their attention to the 2D disc.

They realized that the form of the formula for the area of a circle, a constant times the square of the radius, reflects the fact that all circles are similar to each other. That is, a circle of radius r is a scaled-up version of a circle of radius 1, by a scale factor of r. When a figure is scaled up by a certain factor, its area grows by the square of the scale factor. So if A is the area of a disc with radius 1, then Ar^2 is the area of a disc of radius r.

Usually, π is defined not in terms of area, but as a ratio of distances, specifically, the ratio of the circumference of a circle to its diameter. It then becomes a theorem that the constant A (the area of the unit disc) is the same as this ratio of lengths. However, the students decided to define π as the area of the unit disc and worry later about showing that this π coincided with the ratio of the circumference to the diameter.

They turned their attention to finding the numerical value of π.

To get an estimate for the numerical value of π, they went back to their definition of area in terms of unit squares. They saw that they could fit a unit disc snugly inside 4 unit squares as shown in the leftmost figure below.

This shows that $\pi < 4$. But they wanted a better estimate, so they decided to split each square into 4 smaller squares, each of area 1/4. By counting the number of squares fully contained within the circle and the number of squares that overlap with the circle, they found that $1 < \pi < 4$ (middle figure above). And to improve this further, they turned their 4-by-4 grid of squares into an 8-by-8 grid of squares of area 1/16, and found $2 < \pi < 15/4$. Better, but still rather poor! They continued quadrupling the number of squares in their grid until it was 128 squares by 128 squares. Here are the estimates they got for π:

Grid Size	2×2	4×4	8×8	16×16	32×32	64×64	128×128
Lower Bound	0	1	2	2 9/16	2 55/128	3 1/128	3 77/1024
Upper Bound	4	4	3 3/4	3 1/2	3 11/32	3 65/256	3 51/256

As decimal numbers, the last approximation puts π between 3.0751953125 and 3.19921875, a significant improvement but not accurate enough to build a decent bike tire.

"Is there a mathematical formula that expresses these estimates?"

They decided to try to write down a mathematical expression for their computation. Perhaps, they thought, by writing the formula down, they might see ways to evaluate it.

They created two functions, $L(n)$ and $U(n)$, that accept as input the value n, indicating the size of their grid. They took the grid to be 2^{n+1} by 2^{n+1}. The function $L(n)$ is the area of the grid squares fully contained inside the circle, and $U(n)$ is the area of the grid squares that contain parts of the circle. That way, $L(n) < \pi < U(n)$.

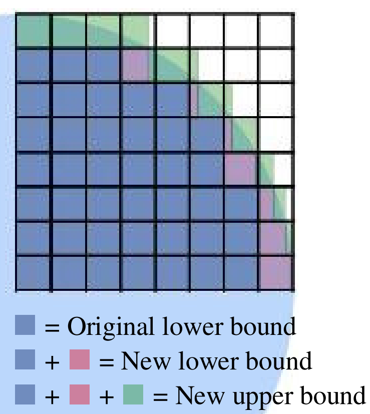
Because of the symmetry, they decided to focus solely on the upper right quadrant of the circle, then quadruple their counts to get the full counts.

Within the quadrant, which was cordoned off with two perpendicular radii, a grid square could be identified by specifying its column and row number, with the first column being just to the right of the vertical radius and the first row sitting just above the horizontal radius. Then the function could be written as a double sum over rows r and columns c. For each value of r and c between 1 and 2^n, inclusive, they wanted a way to add the area of the grid square if the grid square was fully contained within the quadrant or overlapped with the quadrant, depending on whether they were computing $L(n)$ or $U(n)$, respectively.

They decided that the way they could determine if a grid square was fully contained within the circle was to compute the distance of its farthest vertex from the circle's center. If this distance was less than or equal to 1, the grid square was fully contained inside. Similarly, if the distance from the center of the circle to its nearest vertex was at least 1, then the grid square sat outside the circle. Otherwise, the grid square straddled the boundary of the circle.

However, they could also see that the grid squares they were counting formed an edifice of stacked, contiguous rows of squares. Therefore, rather than determine whether each individual grid square should be counted or not, they decided to determine the leftmost and rightmost grid squares in each row and turn their double sum (over rows *and* columns) into a single sum (over just rows). Within the quadrant, the leftmost grid squares were always right up against the vertical radius. In row r, the coordinates of the rightmost grid square's vertex farthest from the circle's center was $(c/2^n, r/2^n)$, where c was the largest integer such that $(c/2^n)^2 + (r/2^n)^2 \le 1$. Rearranging, they found that

$$c = \left\lfloor \sqrt{2^{2n} - r^2} \right\rfloor,$$



where "$\lfloor x \rfloor$" is the "floor of x," i.e., x rounded down to the nearest integer. Thus, they found that row r had c grid squares fully contained in the quadrant. Similarly, they found

$$d = 1 + \left\lfloor \sqrt{2^{2n} - (r-1)^2} \right\rfloor$$

grid squares that overlap with the quadrant.

But wait a second! The expression $\sqrt{2^{2n} - r^2}$ tells where the line running along the top of the rth row of grid squares intersects with the circle, in units of grid square side lengths. Even though this may contain a fractional number of squares, the row rectangle that extends to the right from the vertical axis this entire distance would still be entirely contained in the circle and, generally, include more area than the complete grid squares in that row—so why not add up the areas of these rectangles instead? (See the figure above.)

What's more, $\sqrt{2^{2n} - r^2}$ also gives the length (in units of grid square side lengths) of the smallest rectangle in row $r + 1$ that contains all of the circle in that row of the quadrant. Therefore, by using these snug rectangles, they'd get a better area estimate with less computation! In fact, the containing rectangle in row $r + 1$ is congruent to the snugly contained rectangle in row r. So the only difference between the lower bound and upper bound (within the quadrant) would be the area of the smallest containing rectangle in row 1. This area is always $(2^n)(1/2^{2n}) = 1/2^n$. Thus,

$$L(n) = \frac{4}{2^{2n}} \sum_{r=1}^{2^n} \sqrt{2^{2n} - r^2}$$

and

$$U(n) = L(n) + 1/2^n.$$

Unfortunately, they couldn't see how to simplify $L(n)$ any further. They decided to write a computer program to numerically compute $L(n)$. Here's the computer's output:

n	**Lower approximation**	**Upper approximation**
1	1.7320508075688772	3.732050807568877
2	2.4957090681024408	3.4957090681024408
3	2.839819144357173	3.339819144357173
4	2.9982530378277414	3.2482530378277414
5	3.0726024228771305	3.1976024228771305
6	3.108046912779686	3.170546912779686
7	3.1251557923966145	3.1564057923966145
8	3.13349308295723	3.14911808295723
9	3.1375849027196545	3.1453974027196545
10	3.13960364153902	3.14350989153902
11	3.1406034029114425	3.1425565279114425
12	3.141099886357759	3.142076448857759
13	3.14134692692469	3.14183520817469
14	3.141470022526393	3.141714163151393
15	3.1415314201779934	3.1416534904904934
16	3.1415620659177144	3.1416231010739644
17	3.1415773700187652	3.1416078875968902
18	3.141585015433538	3.1416002742226006
19	3.141588835794821	3.141596465189352
20	3.1415907451460074	3.141594559843273
21	3.1415916995283926	3.1415936068770254
22	3.1415921766157315	3.141593130290048
23	3.141592415122798	3.1415928919599563
24	3.14159253436396	3.141592772782539
25	3.1415925939779767	3.1415927131872663
26	3.1415926237845815	3.1415926833892263
27	3.141592638689265	3.1415926684915876
28	3.141592646138097	3.141592661039258
29	3.141592649864554	3.1415926573151345
30	3.141592651726471	3.1415926554517615
31	3.141592652658422	3.1415926545210673
32	3.141592653123865	3.1415926540551875
33	3.141592653357096	3.1415926538227574
34	3.1415926534732996	3.1415926537061303
35	3.1415926535371694	3.1415926536535848
36	3.141592653582341	3.1415926536405485

Thus, to 10-digit accuracy, π = 3.1415926536…accurate enough to compute the surface area of the Earth to within the size of Joan Lorentz Park, the welcoming green in front of the main branch of the Cambridge Public Library.

If you've learned about integration, perhaps you recognized that in the expression for $L(n)$, these students reinvented the Riemann sum.

“What’s the volume of a 3D ball?”

Having freed their minds from unit squares and unit cubes, they next wanted to determine the volume of a unit ball. They sliced it up into $2N$ thin, parallel slabs for some positive integer N. Each circular slab snugly contains a cylinder of height $1/N$ and radius as large as possible. Since a cylinder of height h and radius r has volume $\pi r^2 h$, they found the following formula for an estimate of the volume of the unit ball:

$$2\sum_{r=1}^{N}\pi\left(\sqrt{1-(r/N)^2}\right)^2\frac{1}{N}.$$

(The factor of 2 appears because the sum approximates the volume of a hemisphere.)

Unlike the sum for the circle, the students were able to simplify this expression by using the fact that the sum of the first N perfect squares is $N(N + 1)(2N + 1)/6$:

$$\begin{aligned}
2\sum_{r=1}^{N}\pi\left(\sqrt{1-(r/N)^2}\right)^2\frac{1}{N} &= 2\sum_{r=1}^{N}\pi\left(1-\frac{r^2}{N^2}\right)\frac{1}{N}\\
&= 2\pi\sum_{r=1}^{N}\frac{N^2-r^2}{N^3}\\
&= 2\pi\left(\sum_{r=1}^{N}\frac{1}{N}-\sum_{r=1}^{N}\frac{r^2}{N^3}\right)\\
&= 2\pi\left(1-\frac{N(N+1)(2N+1)}{6N^3}\right)\\
&= 2\pi\left(\frac{2}{3}-\frac{1}{2N}-\frac{1}{N^2}\right)\\
&= \frac{4\pi}{3}-\frac{\pi}{N}-\frac{2\pi}{N^2}
\end{aligned}$$

As N gets larger and larger, both π/N and $2\pi/N^2$ become smaller and smaller…we can pick N so that π/N and $2\pi/N^2$ are as small as we desire. Therefore, we can pick N to make this expression as close to $4\pi/3$ as we wish, without ever going over.

In this way, the 8th graders proved that the volume of a ball of radius r is $4\pi r^3/3$, where π is the area of the unit disc!

“What about the hypervolume of the 4D hyperball?”

The four-dimensional unit hyperball centered at the origin consists of the points (x, y, z, w) in 4D space that satisfy

$$x^2 + y^2 + z^2 + w^2 \le 1.$$

By analogy with their method for finding the area and volume of the disc and ball, they sliced this hyperball with parallel, equally spaced, hyperplanes that split the w-axis contained inside the sphere (from (0, 0, 0, -1) to (0, 0, 0, 1)) into $2N$ segments of length $1/N$. What does the piece of the hyperball look like?

The hyperplane corresponding to $w = c$, where c is a constant between -1 and 1, intersects the hyperball in the points (x, y, z, c) that satisfy the equation

$$x^2 + y^2 + z^2 \le 1 - c^2.$$

That's the equation for a 3D ball! So as they swept the hyperplane $w = c$ from $c = 0$ to $c = 1$, the intersections were balls of ever smaller radii. Between the hyperplanes $w = (k - 1)/N$ and $w = k/N$, they found a shape that looks roughly like a 4D cylinder with bases that are 3D balls. The base in the hyperplane $w = (k - 1)/N$ is bigger than the base in $w = k/N$, and the curved side of this piece of the hyperball bulges outward.

They approximated this hypervolume by the volume of the hypercylinder whose bases are both congruent to the smaller ball contained in $w = k/N$. This hypercylinder has volume

$$\frac{4}{3}\pi\left(\sqrt{1-(k/N)^2}\right)^3 \frac{1}{N}.$$

Thus, the 8th graders found the approximation

$$2\sum_{k=1}^{N}\frac{4}{3}\pi\left(\sqrt{1-(k/N)^2}\right)^3 \frac{1}{N}$$

for the hypervolume of the 4D unit hyperball. As N becomes larger and larger, this approximation gets better and better.

Unfortunately, unlike with the 3D ball, they couldn't figure out how to simplify this expression or analyze it to understand what number it would approach as N grows. So, they turned to the computer, programming it to compute this sum using $N = 1{,}000{,}000$. The computer output: 4.934798011754494. What number is this!

One of the students noted that the area of a unit disc, π, and the volume of the unit ball, $4\pi/3$, are both rational multiples of π, so perhaps this number is also a rational multiple of π. They divided the number by the computer's built-in approximation for π, hoping to find a number that looked very close to a rational number. The computer replied: 1.5707949934615695. What number does that look like?

One of the students looked at the number and said, "$\pi/2$!"

And so they conjectured that the hypervolume of the 4D unit hyperball is $\pi^2/2$, from which it would follow that the hypervolume of the 4D hyperball of radius r is $\pi^2 r^4/2$.

"What about 5D hyperballs and higher?"

The 8th graders realized that they could adapt the process they used to go from the second to the third dimension and the third to the fourth dimension to guess formulas for 5D hyperballs and hyperballs of higher dimension. In fact, they could use almost the same computer program. All they would have to do, to go from dimension d to $d + 1$, is replace the hypervolume formula for bases of hypercylinders with the formula they guessed for dimension d. If the formulas for the hypervolumes were all rational multiples of some power of π, they'd likely be able to guess the correct formulas.

For example, in the formula they used to compute the hypervolume of the 4D unit hyperball,

$$2\sum_{k=1}^{N}\frac{4}{3}\pi\left(\sqrt{1-(k/N)^2}\right)^3\frac{1}{N},$$

the shaded part uses the formula for the volume of a 3D ball. Replacing this shaded part with the formula for the 4D-hypervolume of the 4D-hyperball turns the expression into an approximation for the 5D-hypervolume of the 5D-hyperball:

$$2\sum_{k=1}^{N}\frac{1}{2}\pi^2\left(\sqrt{1-(k/N)^2}\right)^4\frac{1}{N}.$$

For $N = 1{,}000{,}000$, the computer computed: 5.263784079112532. Dividing this by the square of the built-in approximation of π, the computer returned: 0.5333328333333747. That's awfully close to the rational number 8/15. So they guessed that the hypervolume of the 5D-hyperball of radius r is $8\pi^2 r^5/15$. (If you know about limits, we can say that the volume of the unit 5D-hyperball is the limit of the above expression, as N tends to infinity. Also, note that this particular sum's limit can be computed because

$$2\sum_{k=1}^{N}\frac{1}{2}\pi^2\left(\sqrt{1-(k/N)^2}\right)^4\frac{1}{N}=2\sum_{k=1}^{N}\frac{1}{2}\pi^2\left(1-\frac{2k^2}{N^2}+\frac{k^4}{N^4}\right)\frac{1}{N}$$

and this can be simplified using the sum of squares and fourth powers formulas.)

Continuing in this manner, here's what they guessed for hyperballs of dimension up to 8:

Dimension	2	3	4	5	6	7	8
Volume formula	πr^2	$\frac{4\pi}{3}r^3$	$\frac{\pi^2}{2}r^4$	$\frac{8\pi^2}{15}r^5$	$\frac{\pi^3}{6}r^6$	$\frac{16\pi^3}{105}r^6$	$\frac{\pi^4}{24}r^8$

Staring at these formulas, they began to see patterns.

For $2D$-dimensional hyperballs, the volume seemed to be $\frac{\pi^D}{D!}r^{2D}$.

For $(2D + 1)$-dimensional hyperballs, the volume seemed to be $\frac{(2^{2D+1}D!)\pi^D}{(2D+1)!}r^{2D+1}$.

In fact, they guessed the correct formulas! Although they haven't yet *proven* that these formulas are correct, having specific formulas is exciting progress.

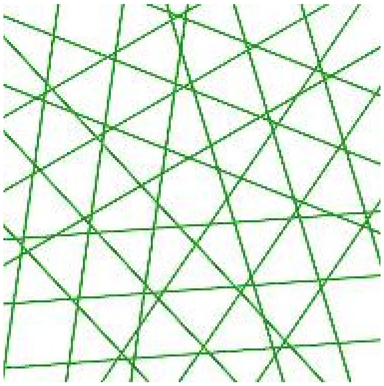
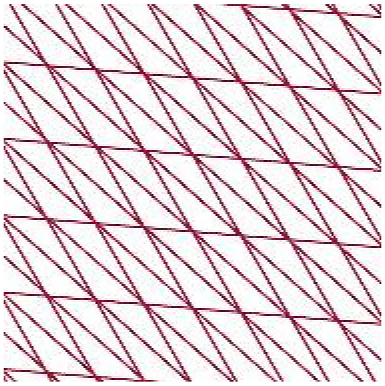
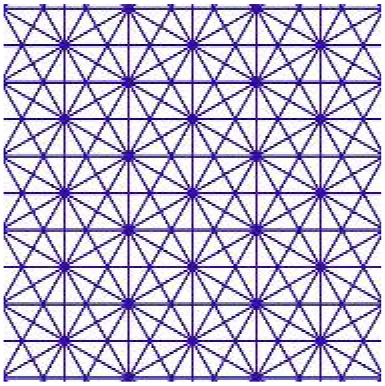
At this point, they decided to put the problem aside and turn to other topics. But I'm sure they'll revisit hyperballs again, because the allure of mystery is powerful. Do you feel it?

Members' Thoughts

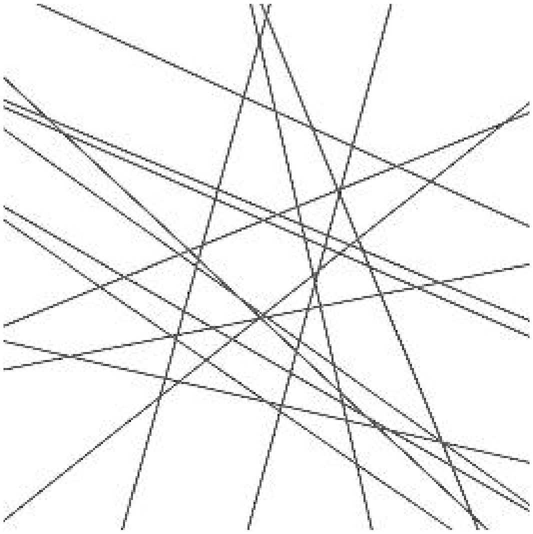
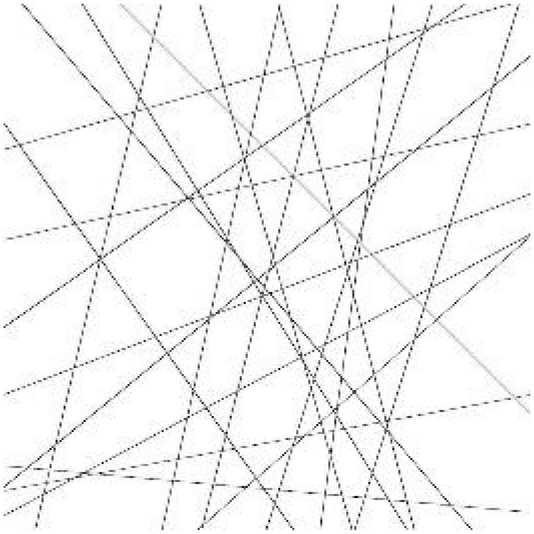
Iris and Milena Understand Something No One Did Before

by Ken Fan | edited by Jennifer Sidney

It was the summer of 2019, just before Milena Harned and Iris Liebman began high school. Iris and Milena were brainstorming for a math project and discovered that they had a mutual affinity for lines. They liked how intriguing patterns could be formed entirely out of lines, like these:

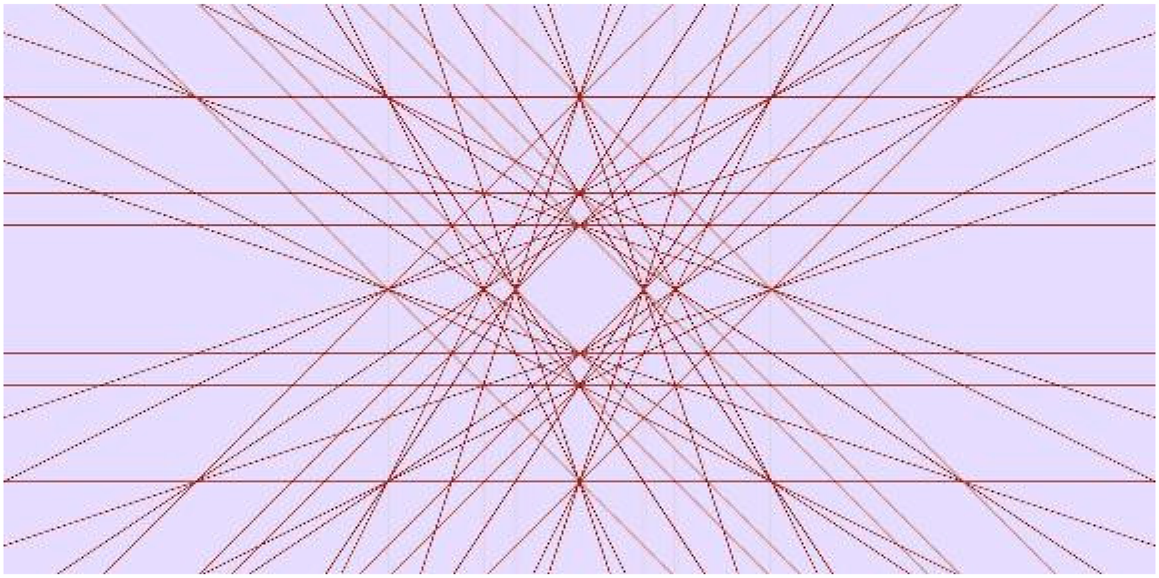
They even considered random arrangements of lines, like these:

How many pentagons would you expect to see in a pattern consisting of n lines drawn at random in some way? There are several in the above examples.

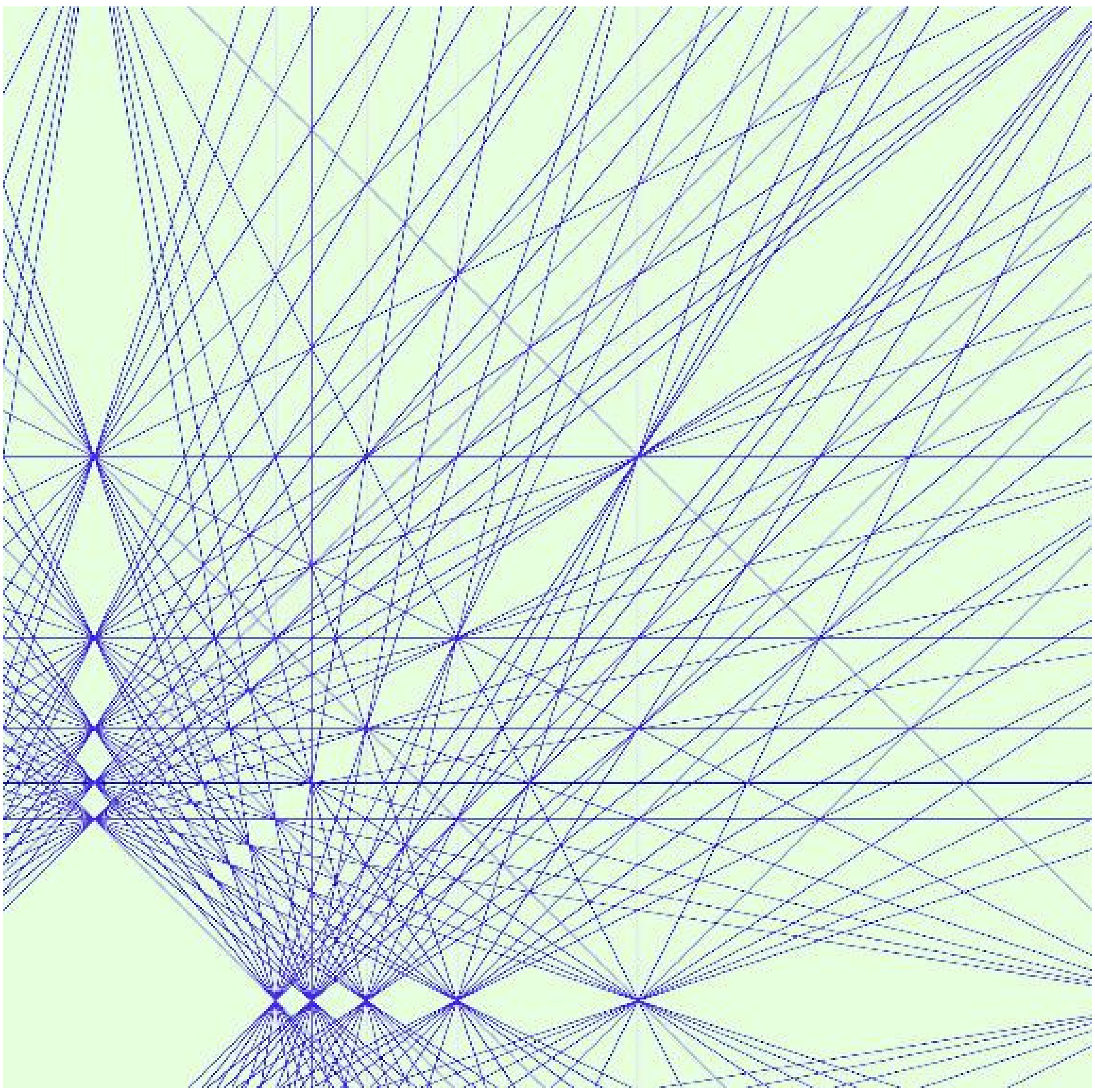
Iris and Milena singled out the following set of lines: Fix a positive integer N. Consider all lines of the form $Ax + By = 1$, where $-N \leq A \leq N$, $-N \leq B \leq N$ and A and B are integers not both zero.

Let's look at a couple of examples: $N = 3$ and $N = 6$. In general, there are $4N^2 + 4N$ lines pointing in many different directions in this pattern, so these examples have 48 and 168 lines, respectively. For $N = 6$, we zoom in on the first quadrant. (See images on the next page.)



$N = 3$

$N = 6$



Do you see a theorem in Iris and Milena's line patterns?

When $N = 3$, there are 653 bounded regions, and when $N = 6$, there are 8,409 bounded regions, of which 2,108 intersect the first quadrant. Let's call these line arrangements **HL arrangements**.

Here's what Iris and Milena observed: Every single polygon formed by the pattern of lines in an HL arrangement is either a triangle or a quadrilateral; there are no polygons with more than four sides!

How did they see this?

Who knows? That's their secret, their vision. If they hadn't noticed this, how long would it be before this amazing fact came to light? Quite possibly forever! After all, people have been thinking about lines in the plane since at least the time of Euclid, so this nifty fact eluded detection for over two thousand years.

How did they prove this?

To learn more about that, I highly recommend reading their paper, "An Unexpected Class of 5+gon-free Line Patterns," published in *La Matematica*, the journal of the Association for Women in Mathematics (see Volume 5, 18 (2026)). It is well written, and the only math you need to know to understand it is from a first-year school algebra course.

But here are some remarks:

Iris and Milena actually solved a more general problem. Let S be a finite subset of points in the coordinate plane, excluding the origin. To a point $p = (A, B)$ in S, there corresponds the line L_p defined by the equation $Ax + By = 1$.

Suppose R is a region bounded by some of the lines L_p with p in S. Suppose that p and q are in S and correspond to lines L_p and L_q that contain consecutive sides of R, if you travel around the boundary of R in the clockwise direction. Iris and Milena figured out how to determine the point r in S that corresponds to the next side around R as you continue traveling clockwise around the boundary (see Theorem 3.12 in their paper).

This result is applicable to any arrangement of a finite number of lines in the plane since every finite set of lines corresponds to a finite set of lines of the form $Ax + By = 1$; however, it may be necessary to translate the pattern slightly so that none of the lines pass through the origin.

One consequence of this "next side" theorem is that if the origin happens to be contained inside a polygon, the lines that form the boundary of this origin-containing region correspond exactly to the vertices of the convex hull of S. (Note that the convex hull of a finite set of points is a convex polygon.) In the case of HL arrangements, this means that the lines bounding the origin form a square diamond: $Nx + Ny = 1$, $Nx - Ny = 1$, $-Nx + Ny = 1$, and $-Nx - Ny = 1$.

To prove their "next side" theorem, Iris and Milena found it useful to keep track of where the origin was as you walk clockwise about R. Since lines of the form $Ax + By = 1$ never pass through the origin, as you traverse a side of R, the origin will always be either on the left or on the right, never dead ahead or behind. So for each side, they kept track of whether the origin was to the left or to the right, as you travel about R in the clockwise direction.

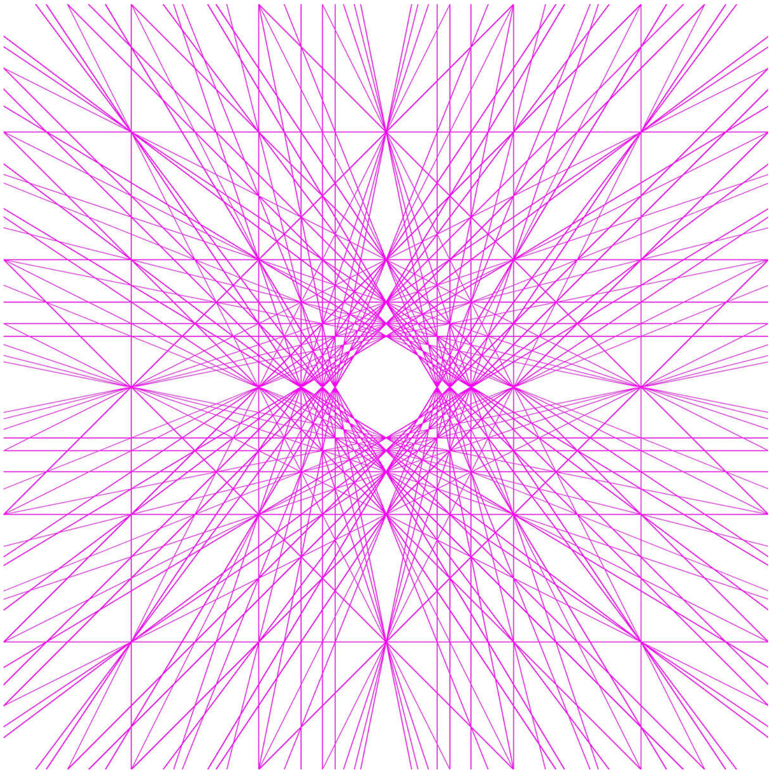
Assuming R does not contain the origin, notice that in walking a full lap about its boundary, you will see the origin switch from your left to your right or your right to your left exactly twice, because in one full lap, you will turn through a full circle of directions exactly once. If R contains the origin, then the origin will always be to your right. (Recall that we always travel clockwise around the region.)

When Iris and Milena applied their "next side" theorem to their HL arrangements, they were able to show that the origin can never be on your same side (your left or your right) for three consecutive sides. The only way that can happen is if every region in their lattice line arrangement has four or fewer sides!

In the above proof sketch, we skipped many important details that are addressed in their paper. For example, Iris and Milena's result actually applies to a more general family of line arrangements. Also, they gave counterexamples to various generalizations of their results by providing examples of line arrangements closely related to their lattice line arrangements, but for which there are polygons with more than four sides. For these details, check out their paper!

A Conjecture

Initially, Iris and Milena thought that if you take any convex set C in the plane and let S be the set of points (A, B) in C, where A and B are integers not both zero, the resulting line pattern L_p, for p in S, would also be devoid of polygons with more than four sides. However, this turned out to be false, and they gave a counterexample in their paper. But when they tried this in the case of C being a circle centered at the origin, the resulting line patterns did seem to contain only triangles and quadrilaterals, with the exception of the region containing the origin; but they couldn't prove it. Can you settle it one way or the other?



The lines L_p where $p = (A, B)$, where A and B are integers such that $0 < A^2 + B^2 < 6^2$.

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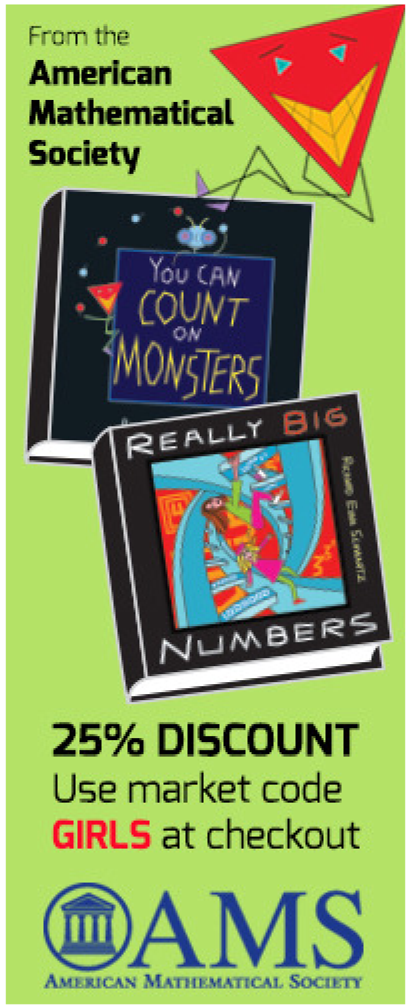

The American Mathematical Society is generously offering a 25% discount on the two book set *Really Big Numbers* and *You Can Count On Monsters* to readers of this Bulletin. To redeem, go to http://www.ams.org/bookstore-getitem/item=MBK-84-90 and use the code "GIRLS" at checkout.

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 38 - Meet 1 January 29, 2026	Mentors:	Elisabeth Bullock, Elsa Frankel, Shauna Kwag, Kira Lewis, Sophia Liao, Hanna Mularczyk, Maya Robinson, Dora Woodruff

We love the way the math just picks right up after the longish winter break! Modular arithmetic frequently arises at Girls' Angle meets, and no wonder, because you can take any polynomial equation with integer coefficients and then try to solve it modulo n for any integer $n > 1$. Can you think of a polynomial equation (possibly in more than one variable) which has a solution modulo n for every integer $n > 1$, but does not have a solution over the integers?

Session 38 - Meet 2 February 5, 2026	Mentors:	Elisabeth Bullock, Shauna Kwag, Kira Lewis, Yaqi Li, Sophia Liao, Hanna Mularczyk, Jennifer Park, Amanda Tran

One way to think of real numbers is as a way to define an addition and multiplication for points of a line. In a similar way, the complex numbers show how to define an addition and multiplication for points of a plane. Specifically, if (a, b) and (x, y) are points in a coordinate plane, their sum can be defined as $(a + x, b + y)$ and their product can be defined as $(ax - by, ay + bx)$. Can you define an addition and multiplication for points in higher dimensional spaces?

Session 38 - Meet 3 February 12, 2026	Mentors:	Elisabeth Bullock, Elsa Frankel, Kira Lewis, Amanda Tran

Can you find all the solutions to the Diophantine equation

$$a_1^2 + a_2^2 + a_3^2 + a_4^2 + a_5^2 + a_6^2 + a_7^2 + a_8^2 + a_9^2 = r^2?$$

It's a generalization of the Pythagorean Diophantine equation $a^2 + b^2 = c^2$.

Session 38 - Meet 4 February 26, 2026	Mentors:	Elisabeth Bullock, Jessica Charalel, Elsa Frankel, Clarise Han, Kira Lewis, Sophia Liao, Hanna Mularczyk, Maya Robinson, Dora Woodruff

At the club, a common problem-solving principle appeared in several different corners of the room, namely, the idea that if one has a series of related, but increasingly complicated problems, one can often use the solution to one of the problems to assist in finding the solution to the next. For example, suppose you want to find a solution to $x^2 = 3 \pmod{11^n}$ for each positive integer n. For large n, a brute force search through the possibilities might take a while. (For example, when $n = 11$, the solutions are ±73,834,823,298.) However, suppose you know that $x_k^2 = 3 \pmod{11^k}$. Can you use this to help you find a solution to $x_{k+1}^2 = 3 \pmod{11^{k+1}}$?

Calendar

Session 37: (all dates in 2025)

September	11	Start of the thirty-seventh session!
	19	
	25	
October	2	
	9	Ila Fiete, MIT
	16	
	23	
	30	
November	6	
	13	
	20	
	27	Thanksgiving - No meet
December	4	

Session 38: (all dates in 2026)

January	29	Start of the thirty-eighth session!
February	5	
	12	
	19	No meet
	26	
March	5	
	12	
	19	
	26	No meet
April	2	Ila Fiete, MIT
	9	
	16	
	23	No meet
	30	
May	7	

Girls' Angle has run nearly 200 Math Collaborations. Math Collaborations are fun, fully collaborative, math events that can be adapted to a variety of group sizes and skill levels. We now have versions where all can participate remotely. We have now run four such "all-virtual" Math Collaboration. If interested, contact us at girlsangle@gmail.com. For more information and testimonials, please visit www.girlsangle.org/page/math_collaborations.html.

Girls' Angle can offer custom math classes over the internet for small groups on a wide range of topics. Please inquire for pricing and possibilities. Email: girlsangle@gmail.com.

Bea and the Sea, deciphered message: "I left home / to wander the sea / in search of a lost friend / if you can decode this / you might just be able / to follow what's ahead. / All alone in the sea, / Bea"

Girls' Angle: A Math Club for Girls

Membership Application

Note: If you plan to attend the club, you only need to fill out the Club Enrollment Form because all the information here is also on that form.

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address (the Bulletin will be sent to this address):

Email:

Home Phone: ____________________________ Cell Phone: ____________________________

Personal Statement (optional, but strongly encouraged!): Please tell us about your relationship to mathematics. If you don't like math, what don't you like? If you love math, what do you love? What would you like to get out of a Girls' Angle Membership?

The $50 rate is for US postal addresses only. **For international rates, contact us before applying.**

Please check all that apply:

- ☐ Enclosed is a check for $50 for a 1-year Girls' Angle Membership.
- ☐ I am making a tax-free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com.

Girls' Angle

A Math Club for Girls

Girls' Angle
Club Enrollment

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

The club is where our in-person mentoring takes place. At the club, girls work directly with our mentors and members of our Support Network. To join, please fill out and return the Club Enrollment form. Girls' Angle Members receive a significant discount on club attendance fees.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-12. We welcome *all girls* (in grades 5-12) regardless of perceived mathematical ability. There is no entrance test. Whether you love math or suffer from math anxiety, math is worth studying.

How do I enroll? You can enroll by filling out and returning the Club Enrollment form.

How do I pay? The cost is $20/meet for members and $30/meet for nonmembers. Members get an additional 10% discount if they pay in advance for all 12 meets in a session. Girls are welcome to join at any time. The program is individually focused, so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is based in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org/page/calendar.html or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Peirce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science, worked in the mathematics educational publishing industry, and taught at HCSSiM. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high-level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. In this way, we hope to help girls become solvers of the yet unsolved.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: Club Enrollment Form

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Please fill out the information in this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. Names:

____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: Girls roughly in grades 5-12 are welcome. Although we will work hard to include every girl and to communicate with you any issues that may arise, Girls' Angle reserves the discretion to dismiss any girl whose actions are disruptive to club activities.

Personal Statement (optional, but strongly encouraged!): We encourage the participant to fill out the optional personal statement on the next page.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: _______________________

(Parent/Guardian Signature)

Participant Signature: ____

Members: Please choose one.	Nonmembers: Please choose one.
☐ Enclosed is $216 for one session (12 meets)	☐ I will pay on a per meet basis at $30/meet.
☐ I will pay on a per meet basis at $20/meet.	☐ I'm including $50 to become a member, and I have selected an item from the left.

☐ I am making a tax-free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Personal Statement (optional, but strongly encouraged!): This is for the club participant only. How would you describe your relationship to mathematics? What would you like to get out of your Girls' Angle club experience? If you don't like math, please tell us why. If you love math, please tell us what you love about it. If you need more space, please attach another sheet.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____ Date: ________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____________________________________