Girls' Angle Bulletin

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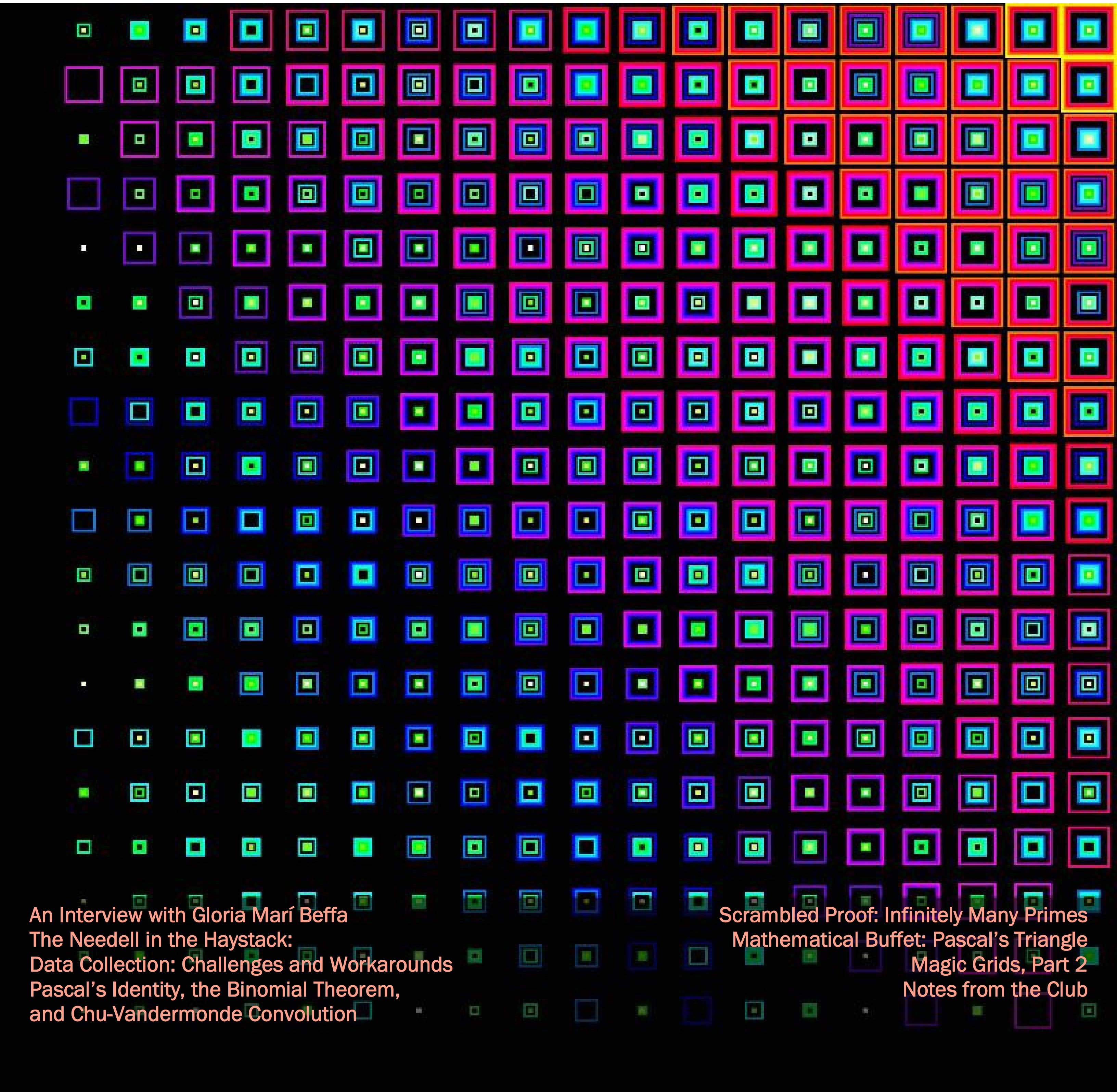
To Foster and Nurture Girls' Interest in Mathematics



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From the Founder

Can you figure out how to interpret the cover image? It would probably be impractical to create it by hand, but becomes fairly straightforward with the advent of modern computers. Do you enjoy generating math-related images? If so, we're interested! - Ken Fan, President and Founder

Girls' Angle Bulletin

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Girls' Angle welcomes submissions that pertain to mathematics.

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Jennifer Sidney
Executive Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.

On the cover: *Pascal Primed* by C. Kenneth Fan. For more on Pascal's triangle, see the articles that start on pages 12 and 18.

An Interview with Gloria Marí Beffa

Gloria Marí Beffa is Professor of Mathematics at the University of Wisconson-Madison. She also serves as the Associate Dean for the Natural, Physical and Mathematical Sciences, as well as Interim Associate Dean for Research. She also served as her Department Chair from 2014-2018. She was raised in Spain and earned her doctoral degree in mathematics at the University of Minnesota-Twin Cities under the supervision of Jack Conn.

Ken: I'd like to start by highlighting a wonderful quote of yours from another interview:

"Do not get discouraged because you are confused, do not understand, or it seems hard. We spend our lives trying to understand things, that is the beauty of it! When you are discouraged, remember that there is nothing like working hard to understand or accomplish something, and finally doing it! Focus on that and give your best, you will not regret it."

I'd like to ask you questions that expand on this important advice. Suppose someone is trying to work hard to understand a math problem, but she's drawing a complete blank. She's stuck. What can she do to get working?

Gloria: Drawing a complete blank tells you that it is the time to stand, go for a walk, think about something else. Then reach out to a good friend and try to explain the problem to them, even if they will not understand. Sometimes only trying to explain it will clear things up for you. Those who are unafraid to try to explain things they do not understand are the fastest learners, no doubt about it. Being confused is not the same as being incompetent; we are all confused much of the time by many things, and math is no different. Do not be afraid to discuss what is in your mind. If you start the discussion with "I am confused," the other will understand the explanation will be fuzzy – and that is OK.

Ken: What are good ways for us to clear up our own confusions?

Gloria: Talk to others who are working on those problems also. There is a belief that math is done alone, but that is not true! It

There is a belief that math is done alone, but that is not true! … For many of us, … math problems are best done in a group… It forces you to organize your thoughts and try to get to the origin of your confusion… There is nothing like talking and listening to others to get new ideas and clear up your ideas… We should be moving away from individualism and into a more concerted effort to create efficient groups for whom the most important task is to solve the problem and to clear up confusions – independently from who does what. This is the approach engineers use and I think math would advance much faster that way.

Dear Reader,

We're committed to producing quality math educational content and make every effort to provide this content to you for free.

We are also committed to surviving as a nonprofit!

For this issue, those who do not subscribe to the print version will be missing out on a portion of this interview with Prof. Gloria Marí Beffa and other content. We hope that you consider the value of such content and decide that the efforts required to produce such content are worthy of your financial support.

We know that mathematical interest and talent is unrelated to economic status, which is why we provide so much content for free. But we hope that those of you who are comfortable financially will help us to continue in our efforts.

So, please consider subscribing to the Bulletin. Thanks to our sponsors, subscriptions cost $36/year. With a subscription, you have also gained access to our mentors via email and the ability to influence content in this Bulletin. Visit www.girlsangle.org/page/bulletin_sponsor.html for more information.

Thank you and best wishes,
Ken Fan
President and Founder
Girls' Angle: A Math Club for Girls

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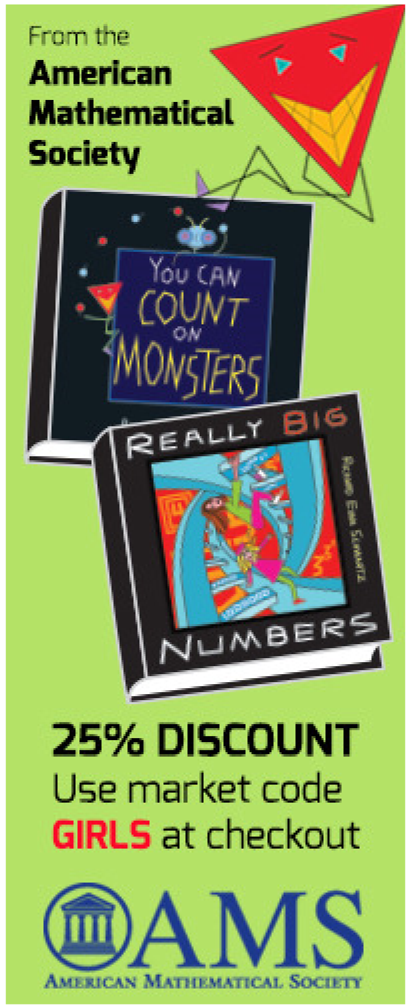

The American Mathematical Society is generously offering a 25% discount on the two book set *Really Big Numbers* and *You Can Count On Monsters* to readers of this Bulletin. To redeem, go to http://www.ams.org/bookstore-getitem/item=MBK-84-90 and use the code "GIRLS" at checkout.

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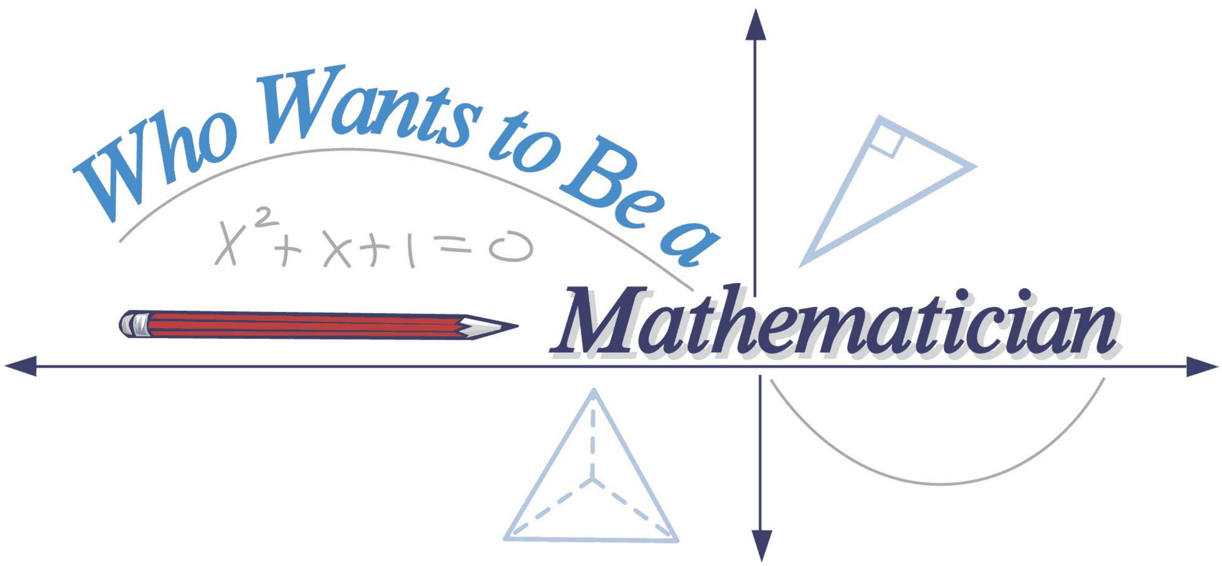

America's Greatest Math Game: Who Wants to Be a Mathematician.

The Needell in the Haystack[1]

Data Collection: Challenges and Workarounds
by Anna Ma | edited by Jennifer Sidney

Data collection has always been a major part of the scientific process. Even before the technological advances that led to the availability of big data, there were ways to collect information about the world. For example, suppose you wanted to collect information about how people travel around your city. To do this, every half hour you could look out your window and count the number of bicycles, pedestrians, and cars that you see. Over a 24-hour period you would have 48 data points that tell you the rough distribution of modes of transport used throughout the day. This is a form of data collection! Do you see any problems with this approach to collecting data? You might miss cars that drive by too quickly or you might miscount the number of cars, creating inaccuracies in your dataset. You might even fall asleep in the middle of the night, missing some data. Our data collection approach here might make sense but it's not very practical; in order to collect this data, you'd have to stay awake for a 24-hour period.

This is where technology comes in. Technology allows us to automate the data collection process. In our previous example, instead of having a human sit awake for 24 hours and record the number of cars, pedestrians, and bicycles that pass by, we can video record the street for a 24-hour period and count the different modes of transport using the video. Taking the automation one step further, instead of having a human process the video, we can *train a computer or machine* to do that for us. In other words, we can use *machine learning* for the purpose of data collection. For example, object recognition is a very common problem in machine learning. In object recognition, the goal is to be able to identify objects in an image or video. Object recognition algorithms and hardware have become so advanced that the task can be done in real time! This is exactly what self-driving cars do. See Figure 1 for an example where an object recognition algorithm aids a self-driving car in identifying other cars, bikes, and even a traffic light [1].

In our last installment, we discussed how to take information and represent it as data in the form of a vector or a matrix. Before we can do this, we must first collect the data. How is data collected? What are the implications of new technologies on the methods of collection? What kinds of challenges arise in collecting data? These are the types of questions that we will be answering in this installment of "The Needell in the Haystack." In particular, we will discuss how data can be collected, the challenges that arise when collecting large amounts of data, and some potential solutions for these challenges.

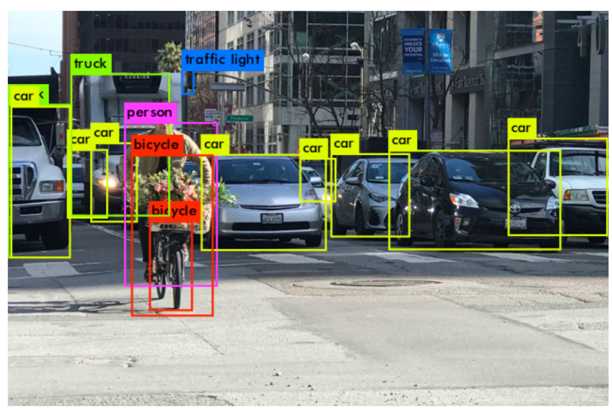

Figure 1. Object recognition of cars, trucks, people, and streetlights.

[1] This content is supported in part by a grant from MathWorks. Anna Ma is a Visiting Assistant Professor at the University of California Irvine.

Collecting Data

There are many ways that data can be collected. More traditional methods of data collection include focus groups, personal interviews, and surveys. The rise of machine learning, digitization of everyday life, and advancements in technology have given us access to more ways of collecting data. Not only are these methods new, but they also allow us to access data at rates and in volumes never before seen. Before the data deluge, in order to survey a town about the adequacy of bike lanes available, a person would have to be hired to walk around from house to house and ask residents a list of questions, or the survey could be administered via postal mail or a phone call. Now, we can cast a wide net by posting a survey online for community members to access, emailing out the survey, and asking community members to share it with their friends!

Not only does technology allow us to obtain more data more efficiently, it also allows us to access data not previously accessible. For example, wearable technologies can give information on a minute-by-minute basis about a person's heart rate. This data couldn't be collected before, at least not in a practical way. Imagine all of the data that is at our disposal now compared to before: millions of people use wearable technologies and there is a lot of data to be gained from that. However, with all this data, there are challenges that arise even to just get the data into a state where it can be processed.

Transporting Data

Transportation is an often-overlooked challenge in data collection. Although in theory we can collect as much data as we want, at the end of the day, we need to be able to transport and store this data in a computer. If data is too large to store or too difficult to transport, then the effort and time that we put into data collection may be all for naught.

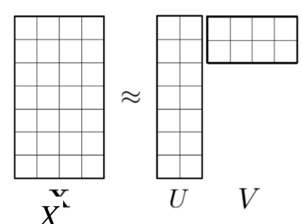

Figure 2. Visualization of matrix factorization.

For example, suppose that our survey is extremely popular and we now have a large matrix X of data to store and transport to a different computer for further analysis. Instead of keeping this entire matrix, we can try to factor the matrix. Here, the idea is similar to when you learned how to factor an equation. How do you factor the equation $x^2 - 4$? One way is to write it as the product of $x + 2$ and $x - 2$. In matrix factorization, the story is the same, except we want to write X as the product of two matrices U and V, as shown in Figure 2. Instead of saving $X \approx UV$, we transport and save the factorization of $U \in \mathbb{R}^{m \times k}$ and $V \in \mathbb{R}^{k \times n}$. It's important to note that in order for this to make sense, we would want the number of entries (which translates to the amount of memory needed) of V and X to be less than that of X, that is, $mk + kn \leq mn$. Can you find suitable values for k so that the inequality is true?

Typically when we perform matrix factorization, we assume that the matrix is low rank. To understand what it means for a matrix to be low rank, let's first interpret the rows of the $m \times n$ matrix as a set of m data points in $\mathbb{R}^n$. If X is low rank, then the data points can be represented in a lower dimensional space $\mathbb{R}^k$, where $k < n$, without losing too much information. With respect to the factorization, we can interpret the matrix $U \in \mathbb{R}^{m \times k}$ as the representation of the m data points in the smaller dimension, $\mathbb{R}^k$. The matrix V then gives us a way to go between the k-dimensional representation and the n-dimensional representation. Simply put, while the data may seem high-dimensional, it can be presented in lower dimensions.

Another way to approach the problem of transportation is to **sketch** the matrix X. You can think of a matrix sketch just as you would think of a sketch of a pet. In a sketch, you just want to get the essence of the thing that you're trying to draw. For example, if I wanted to sketch out a cat, I might only capture the original shape of the cat's face, his ears, his whiskers, and maybe his legs and tail. Although the sketch is not the original fully-colored and detailed cat, we can still deduce that it's a cat. Relating this back to matrix sketching, the original matrix might encode an image of my cat (who is sitting next to me right now). The sketch of my matrix would capture only the important features of my cat, e.g., outlines of his tail, nose, ears, and legs. I want to accomplish two things with this sketch: (1) to be able to demonstrate that I have a pet cat, and (2) to not have to encode unnecessary details about my cat.

More generally, the sketch of a matrix $X \in \mathbb{R}^{m \times n}$ is another matrix $Y \in \mathbb{R}^{m_1 \times n_1}$ such that the sketching matrix accomplishes two tasks. We want the matrix Y to be smaller (in dimension) than the matrix X so that it takes up less memory, that is, we want $m_1 n_1 < mn$. We also want the sketch to contain important information about X. If it does, then we should be able to come up with an algorithm that will allow us to recover the important information about X from its sketch Y. One way to sketch a matrix is to multiply it by a random matrix of size $m_1 \times m$. Using matrix sketching and matrix factorization, we can transport and store the original data set while saving on computation power and storage.

Dealing with Missing Data

It's common for data to be missing. Data can be missing as a result of a broken sensor (e.g., traffic light camera is broken and can't record video) or some other kind of error. Missing data can even be intentional. For example, suppose we have a set of 30 survey questions. Instead of asking all of our participants each of the 30 questions, we might decide to ask a random sample of these 30 questions to reduce the chances of **survey fatigue**. You have probably experienced survey fatigue if you were ever asked to fill out a long survey. You stop reading the questions carefully and answering the questions accurately because your mind has become tired out by too many questions.

	9				7		1	
5				6				3
2		1	5					
	5	2				3		
				1				
	3	6				2		
9		5	1					
7				8				4
	2				4		5	

Figure 3. Sudoku puzzle.

What should we do about missing data? The answer is twofold. Some algorithms are designed to handle missing data. In a previous installment, we talked about the missing stochastic gradient descent algorithm, an algorithm that can overcome the fact that data is missing when solving a linear system.

Another way to deal with missing data is to fill in the missing data. There are naive ways to complete the data, such as filling in the data with zeros. Another way to more cleverly fill in data is to use a math technique known as matrix completion. Have you ever played Sudoku? Sudoku is a number game in which one is given a partially filled-in 9-by-9 matrix. See Figure 3 for an example [2]. The objective is to fill in the matrix with its missing values, the numbers 1 through 9, subject to some constraints. In particular, all 9 numbers 1 through 9 must appear in every row, every column, and each of the nine 3×3 submatrices marked off by thickened lines. Matrix completion is similar in the sense that we want to fill in the missing entries of our matrix given some constraints, such as that the matrix be of as low rank as possible. The matrix completion problem typically involves solving the following optimization problem:

$$\min_{A} rank(A)$$
$$\text{such that } A_{i,j} = X_{i,j}, \text{ for } (i,j) \in \Omega,$$

where $(i, j) \in \Omega$ denotes the set containing the indices of known entries of X. Thus, the matrix completion problem boils down to finding as simple of a representation as possible (minimizing the rank), while agreeing on the data that is available to us (constraining $A_{i,j} = X_{i,j}$, for $(i, j) \in \Omega$).

Dealing with Corrupt Data

Even if data is available, it is not impervious to errors. We refer to errors in data as **corruptions**. Sources of corruption include human error (random mistakes), adversarial errors (intentional mistakes made by external actors), and limitations to measurement/data collection tools. An example of human error is when a person incorrectly counts the number of cars that passed by. An adversarial corruption would be when a person (for example, a person who wanted to argue that there weren't a lot of cars on the road) goes in and changes the number of cars you recorded on purpose (say, decreases every count by 3-5 cars). And of course, there are always limitations to the tools we use to collect data. For example, digital cameras have a limited color spectrum; information in shadow may be completely lost if the camera cannot detect it and instead fills in the shadow with black.

In addition to these sources of corruption, the physical device in which the data is stored can experience **data degradation** over time, causing data to become corrupt. Data degradation describes the gradual process of data corruption due to non-critical failures in the physical storage device. For example, data printed on traditional paper will literally rot over time. Have you ever looked at an old sheet of printed paper and noticed that the ink was starting to fade away? This is an example of data degradation. Data stored on hard drives can still experience data degradation due to extreme fluctuations in temperature or imperfect insulation of the device. It's also been posited that cosmic rays can affect data storage devices [3]. Even having a small number of **bit flips** (data becoming corrupt) can ruin a perfectly good data set. Figure 4 shows a picture of my cat and the same picture after a number of bit flips. On the left is the original picture. Next to it is the resulting picture after 1 random bit flip, after that 4 random bit flips, and on the far right is a whopping 67 random bit flips. You can try it yourself using the link from [4].

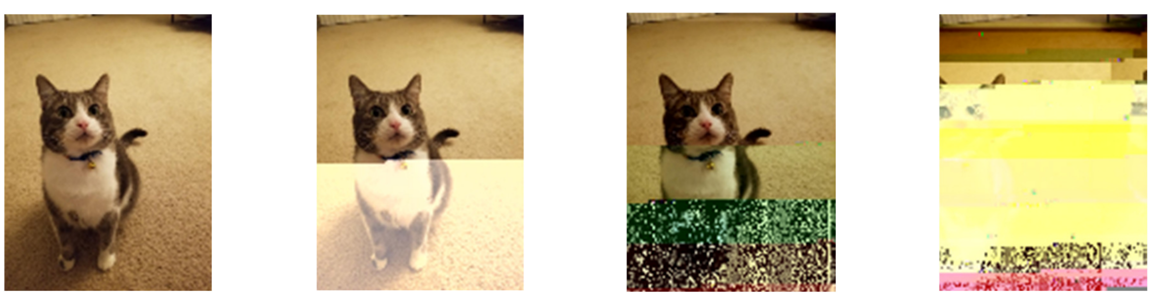

Figure 4. Images of my cat with 0, 1, 4, and 67 random bit flips (left to right).

If corrupted data can be identified and isolated, then this problem can be treated in the same way as the missing data problem where the corrupt data is removed. Perhaps we just take the top quarter of the photo in Figure 4 since only the bottom half is corrupt in the images with fewer corruptions. In this case it's easy to visually see where the corruption may lie; more often than not, though, corrupt data cannot be identified and we need to use other methods that can overcome corruptions. In the non-adversarial case, corruptions are often modeled as random perturbations. Figure 5 demonstrates the impact of increasing the size of perturbations on a data set arranged as a circle in $\mathbb{R}^2$. In the far left, we have the original data set without any corruptions. In the middle figure, we can see that the data set still has a circular shape but is not perfect. The last figure shows what happens when the corruptions disrupt the data set too much. In both the center and right subfigures, the corruptions are random, but in the right-hand figure, the corruptions are larger in magnitude.

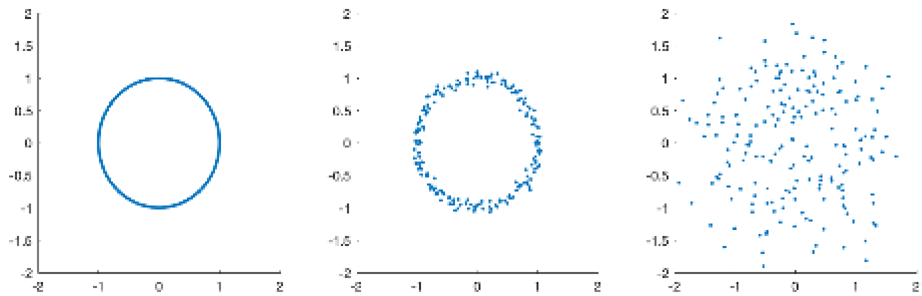

Figure 5. Impact of increasing the size of perturbations on a data set arranged as a circle.

Notice in the example provided in Figure 5 that every single data point was corrupted. In this case, removing the corrupt data is not an option. Instead of trying to remove corruptions from data, it's typically assumed that data will be corrupted by some noise and that modern algorithms should have a robustness, meaning that they should not be sensitive to such corruptions. Looking at Figure 5, can you think of a way we would possibly remove the corruption? What if you didn't have access to the left subfigure (i.e., you didn't know the points were supposed to create a perfect circle)?

Moving Forward with Data

After collecting, transporting, and processing the data (through matrix completion or other techniques), the next step is to get a broad-strokes overview of what your data looks like. To accomplish this, we typically turn to **data visualization** techniques. You may be familiar with some, such as box whisker plots and histograms. But there are many more that can come in handy, particularly when you have large-scale data. Stay tuned for the next installment of "The Needell in the Haystack" to learn what we can do to visualize data that lives in high dimensions!

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[1] How Pixel Differences Can Affect Sensors in Self-driving Cars, https://centralcoastdatascience.org/projects/all/2021/how-pixel-differences-can-affect-sensors-self-driving-cars

[2] Sudoku 4,646, Medium, https://www.theguardian.com/lifeandstyle/2019/dec/18/sudoku-4646-medium

[3] T. J. O'Gorman, The effect of cosmic rays on the soft error rate of a DRAM at ground level, *IEEE Transactions on Electron Devices* **41.4** (1994), 553-557.

[4] A. Consroe, Image Bit Flips, https://aconz2.github.io/2019/07/25/Image-Bit-Flips.html

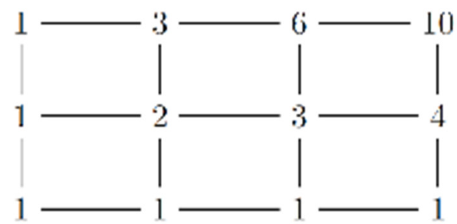
Pascal's Identity, the Binomial Theorem, and Chu-Vandermonde Convolution

by Robert Donley[2] | edited by Amanda Galtman

In last issue's article, *Shortcuts to Counting*, we described several counting methods and tools:

- The matching rule,
- The factorial $N!$,
- k-permutations $P(n, k)$, and
- Combinations $C(n, k)$.

We also showed that the number of paths between opposite corners of an M-by-N rectangle, provided that the path always advances toward the goal, is given by $C(M + N, M)$. Let's orient the rectangle in the coordinate plane so that opposite corners have coordinates (0, 0) and (M, N) and indicate the number of paths to each node of the grid.



In this article, we expand on these ideas to obtain neat facts about Pascal's triangle.

Consider the grid above on the right. For a path of our type to end at (M, N), it must go through one of the nodes $(M, N - 1)$ and $(M - 1, N)$, and no path goes through both. Thus, the number of paths from (0, 0) to (M, N) is divided, and we have

$$C(M + N, M) = C(M + N - 1, M) + C(M + N - 1, M - 1).$$

Rewriting in general form gives

Pascal's identity: $C(n, k) = C(n - 1, k) + C(n - 1, k - 1)$ when $n \geq 1$, $0 \leq k \leq n$.

We recall that we defined $C(n, 0) = 1$ and, if either index is negative, or $k > n$, $C(n, k) = 0$.

Exercise: Verify Pascal's identity in the grid above: each entry is the sum of the entry below and the entry to the left.

Exercise: When $n, k > 0$, verify Pascal's identity by expanding each $C(n, k)$ on the right-hand side using factorials. What happens when one of n or k equals 0?

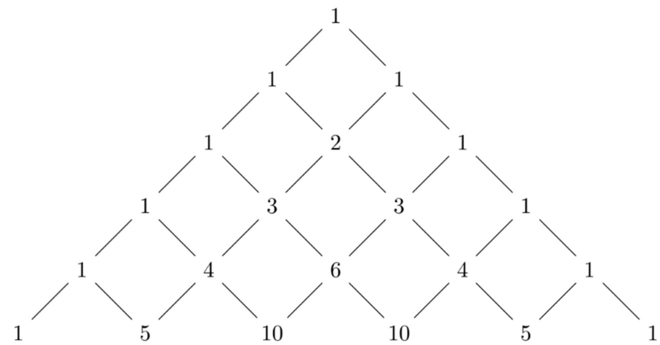
Exercise: Recall that the combination $C(n, k)$ counts the number of ways to choose k balls from n balls. Interpret Pascal's identity in the context of choosing balls.

Pascal's Triangle

If we work out several rectangular grids, we'll find ourselves recomputing the same numbers over and over. However, note that the number of paths to a node depends only on the part of the

[2] This content is supported in part by a grant from MathWorks.

rectangle below and left of the node. This means we can organize all the path numbers into one giant array, known as Pascal's triangle. It's called Pascal's *triangle* because traditionally, the numbers are organized into the shape of an infinite triangle that can be obtained from our rectangular array by rotating it clockwise through 135°.



The first few rows of Pascal's triangle.

After rotating the array, notice that Pascal's identity says that each entry is the sum of the entry just above and to the left with the entry just above and to the right.

The triangle is organized into rows. Because the top entry corresponds to (0, 0), let's agree to call the top row the zeroeth row. Then the nth row of Pascal's triangle consists of the numbers

$$C(n, 0), C(n, 1), C(n, 2), \ldots, C(n, n).$$

Instead of thinking of Pascal's triangle as many iterations of Pascal's identity, we organize these sums as an operation that passes from one row to the next. All consecutive sums of two entries in a given row are obtained by the sum of the row with its shifted self. For instance, we add the fourth row to itself, with a shift, to obtain the fifth row:

$$\begin{array}{cccccc} 1 & 4 & 6 & 4 & 1 & \\ & 1 & 4 & 6 & 4 & 1 \\ \hline 1 & 5 & 10 & 10 & 5 & 1 \end{array}$$

Perhaps this "addition" reminds you of carrying out the multiplication algorithm when you multiply a decimal number by 11, except that there is no carrying between columns. To avoid carries, we could work in a base that is bigger than the sums, but that would work only up to a certain number of rows because the entries in Pascal's triangle become arbitrarily large. So, instead of thinking in terms of a specific base, we can think of the entries as coefficients of a polynomial! The above operation corresponds to multiplying a polynomial by $1 + x$:

$$(1 + 4x + 6x^2 + 4x^3 + x^4)(1 + x) = 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5.$$

The zeroeth row corresponds to the polynomial $p(x) = 1$. Since we pass from row to row by multiplying by $1 + x$, we now have two ways to express the polynomial that represents the nth row of Pascal's triangle:

Binomial theorem: For $n \geq 0$, we have

$$(1 + x)^n = C(n, 0) + C(n, 1)x + C(n, 2)x^2 + \ldots + C(n, n-1)x^{n-1} + C(n, n)x^n.$$

"Binomial" refers to the two terms in $1 + x$, and the combination numbers $C(n, k)$ are also known as **binomial coefficients**.

To save space, we'll also denote $C(n, k)$ by $C_{n,k}$.

Exercise: In the binomial theorem, set $x = 1$ and interpret the resulting identity numerically. Verify it for the 4th and 5th rows of Pascal's triangle. Repeat this exercise with $x = -1$. For $x = 1$, do you see an interpretation in terms of paths?

Exercise: It is often useful to have the binomial theorem in the form

$$(a+b)^n = C_{n,0}a^n b^0 + C_{n,1}a^{n-1}b^1 + C_{n,2}a^{n-2}b^2 + \ldots + C_{n,n-1}a^1 b^{n-1} + C_{n,n}a^0 b^n.$$

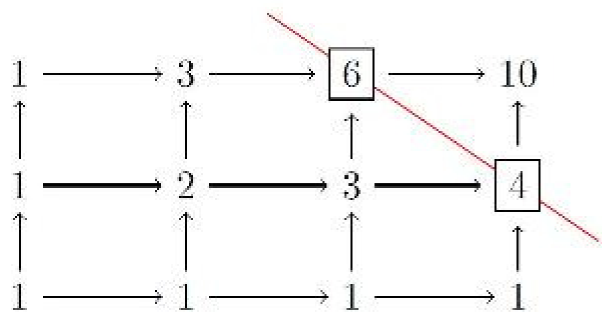
To obtain this form, in our version of the binomial theorem, set $x = b/a$ and multiply both sides by a^n. What do you notice about the sum of the exponents of a and b in any term on the right-hand side of the identity?

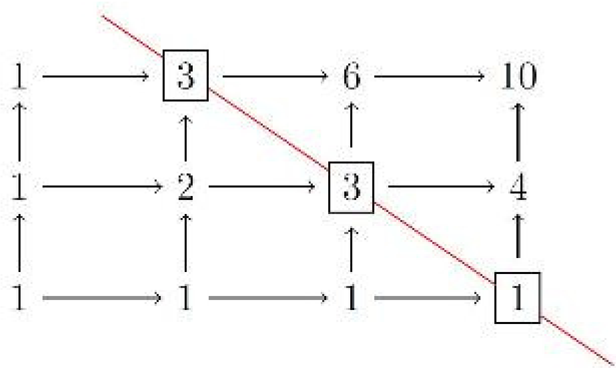
Exercise: In the product $(a + b)^n = (a + b)(a + b) \cdots (a + b)$, each monomial of the form $a^{n-k}b^k$ corresponds to a selection of one of the two terms in each factor $a + b$. Can you directly relate each such monomial with a path in some grid?

Chu-Vandermonde Convolution

For an advanced form of Pascal's identity, we now explain Chu-Vandermonde convolution.

In the path interpretation of Pascal's identity, cutting the rectangle with the diagonal just before the upper right-hand corner separates all paths into two types. If we take any parallel diagonal of the rectangle, a similar situation occurs: each path must go through some node on the diagonal, and no path goes through more than one such node.





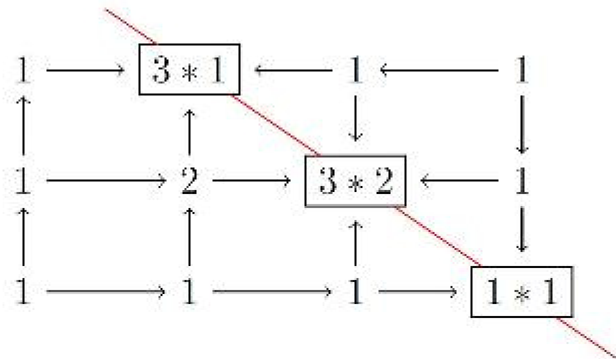
To obtain a counting formula, we have to control several bookkeeping issues. First, let's label diagonals counting from right to left, so that the summands in Pascal's identity are on the 1st diagonal (see figure above). Then the Kth diagonal contains the node $(M - K, N)$. All other nodes on the diagonal are of the form $(M - K + i, N - i)$ when still in the rectangle. (The figure at left shows the 2nd diagonal.)

Now consider how many paths go through the node $(M - K + i, N - i)$. Any such path has two parts: a path from $(0, 0)$ to $(M - K + i, N - i)$, and the continuation to (M, N). Since these choices of beginning and ending paths are independent, the Matching Rule says that the total number of paths that pass through the node $(M - K + i, N - i)$ is equal to the product of the number of paths from $(0, 0)$ to $(M - K + i, N - i)$ and the number of paths from $(M - K + i, N - i)$ to (M, N). Using our original derivation of path numbers, the number of paths from $(0, 0)$ to $(M - K + i, N - i)$ is equal to $C(M + N - K, N - i)$. For the second count, we rotate the grid by 180°. Under this rotation, paths from $(M - K + i, N - i)$ to (M, N) become paths from (0, 0) to $(K - i, i)$, of which there are $C(K, i)$. Thus the number of paths through $(M - K + i, N - i)$ is equal to

$$C(M + N - K, N - i)C(K, i).$$

And the sum of these terms over all $0 \le i \le k$ is the total number of paths $C(M + N, M)$.

Alternatively, a path from (0, 0) to (M, N) is just the concatenation of two paths, one starting at (0, 0) and going only up or right, the other starting at (M, N) and going only down or left. The two paths meet somewhere on the diagonal.



We re-index in preparation for some further applications that will be described shortly:

Chu-Vandermonde convolution: Fix m, n and k with $0 \le k \le m, n$. As i ranges from 0 to k, the sum of all products of the form $C_{m,\, k-i}C_{n,\, i}$ equals $C_{m+n,\, k}$.

Exercise: Verify that Pascal's identity corresponds to Chu-Vandermonde convolution when $n = 1$.

Exercise: Use Chu-Vandermonde convolution to show that

$$C_{n,0}^2 + C_{n,1}^2 + C_{n,2}^2 + \ldots + C_{n,n-1}^2 + C_{n,n}^2 = C_{2n,n}.$$

Exercise: Using the binomial theorem, deduce Chu-Vandermonde convolution another way by considering the coefficient of x^k in the product $(1 + x)^m(1 + x)^n$.

Chu-Vandermonde convolution has a natural interpretation in terms of combinations. As we saw in the previous article, the number of ways to choose k balls from $m + n$ balls is $C(m + n, k)$. On the other hand, split the balls into two groups with m and n balls, respectively. Every choice of k balls has zero or more from the first group and zero or more from the second. If we index the number from the second group by i, then we must choose $k - i$ balls from the first group. Since the choices are independent, the Matching Rule applies, and the formula follows.

Chu-Vandermonde convolution also has an interpretation using Pascal's triangle. For convenience, we first configure Pascal's triangle in a rectangular grid.

Choose m, n, and k as before, and locate $C(m+n, k)$ in left-justified Pascal's triangle. (It is in row $m+n$ and column k, if we remember to start counting rows and columns from 0.) Section off the rectangle with corners at this node and (0, 0). For any two rows in this rectangle, we define the

convolution product: If the entries in the chosen rows of the rectangle are $(a_0, a_1, \ldots, a_k)$ and $(b_0, b_1, \ldots, b_k)$, the convolution product is

$$a_0 b_k + a_1 b_{k-1} + \ldots + a_{k-1} b_1 + a_k b_0.$$

1					
1	1				
1	2	1			
1	3	3	1		
1	4	6	4	1	
1	5	10	10	5	1

That is, we reverse the entries of the second chosen row, multiply entries in the same position, then add up these products.

Chu-Vandermonde Convolution in Pascal's Triangle: Fix m, n, and k as above. For $0 \le i \le m+n$, the convolution product of row i with row $m+n-i$ equals $C_{m+n,k}$.

Chu-Vandermonde convolution is not a single identity but a family of identities, each summarized by a convolution product, as illustrated in the example below.

Example: Suppose $m = 3$, $n = 2$, and $k = 2$. Then $C(5, 2) = 10$. The convolution products of the shaded rows correspond to the following three convolutions, respectively:

$$1 \times 10 + 0 \times 5 + 0 \times 1 = 1 \times 6 + 1 \times 4 + 0 \times 1 = 1 \times 3 + 2 \times 3 + 1 \times 1.$$

1	0	0
1	1	0
1	2	1
1	3	3
1	4	6
1	5	10

1	0	0
1	1	0
1	2	1
1	3	3
1	4	6
1	5	10

1	0	0
1	1	0
1	2	1
1	3	3
1	4	6
1	5	10

Exercise: Verify as above when $m = 3$ and $n = 3$ and 4.

Exercise: Chu-Vandermonde convolution is a special case of a more general phenomenon. Fix a rectangle of any size and place any sequence of numbers in the top row. Fill in the remainder of the rectangle with the same pattern as Pascal's identity: extend the first entry down the first column, and obtain all other values by adding the entry above to the entry above and to the left. What happens when you apply the convolution product and telescope rows as above? Can you explain this using polynomials? See what patterns you can find when the first row is

$$1, -1, 0, 0, 0, \ldots, 0.$$

Scrambled Proof: Infinitely Many Primes

by Lightning Factorial

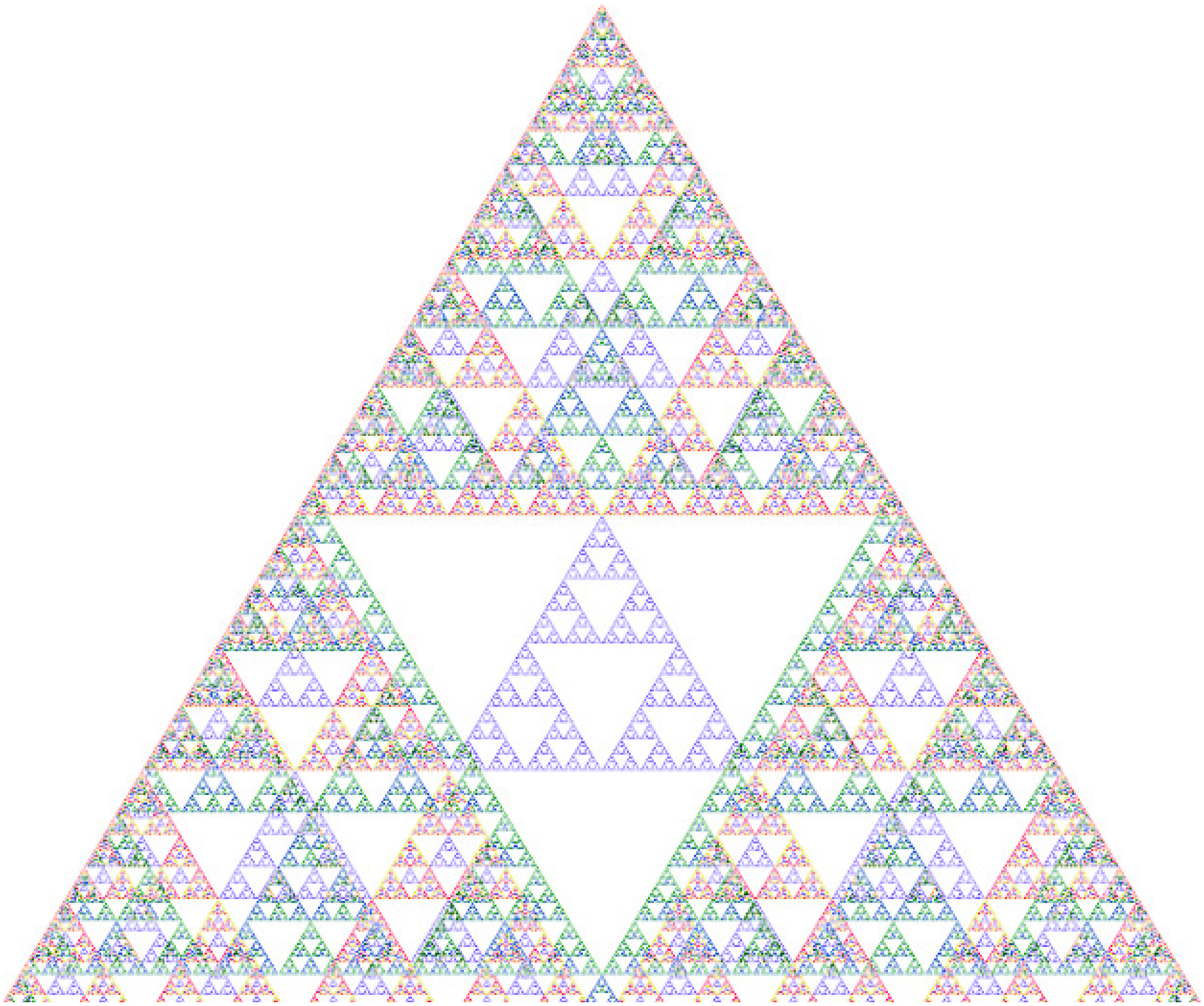
Here are 17 statements all scrambled up. Unscramble them to form a coherent proof.

Therefore, Y must be composite.
Because Y is one more than the product of the prime numbers, it is bigger than all the p_k.
Proof.
That is a contradiction!
We can then label them $p_1, p_2, p_3, \ldots, p_n$, where p_k is the kth prime number and n is an integer.
We conclude that there are infinitely many prime numbers.
If Y is divisible by p_k, then since X is divisible by p_k, it must be that $Y - X$ is divisible by p_k.
Therefore, our assumption that there are only finitely many prime numbers must be false.
Proposition. There are infinitely many prime numbers.
Let X be the product of all the prime numbers p_k, $1 \leq k \leq n$.
Suppose, to the contrary, that there are only finitely many prime numbers.
Therefore, Y is not divisible by p_k for any k, $1 \leq k \leq n$.
But we have already shown that Y is not divisible by any prime number p_k.
Let $Y = X + 1$.
Note that X is divisible by p_k for all k, $1 \leq k \leq n$.
Therefore, Y cannot be composite.
However, $Y - X = 1$, and no prime number divides evenly into 1.

Mathematical Buffet

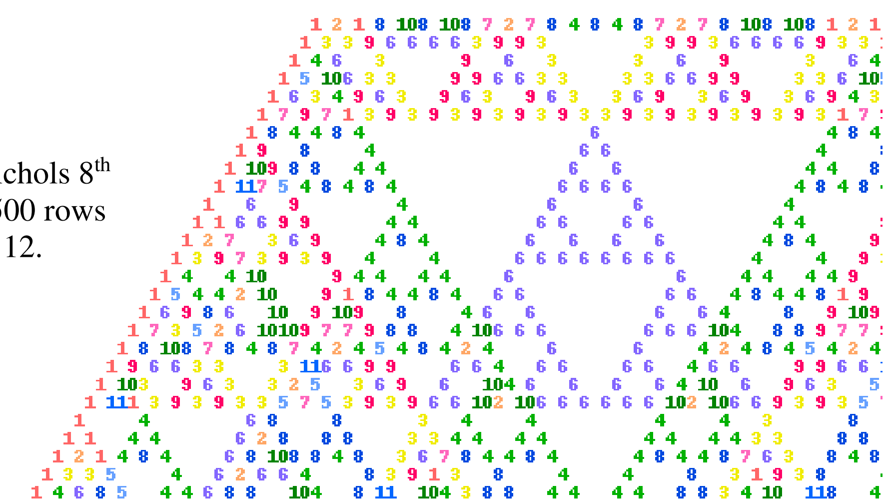
Pascal's Triangle

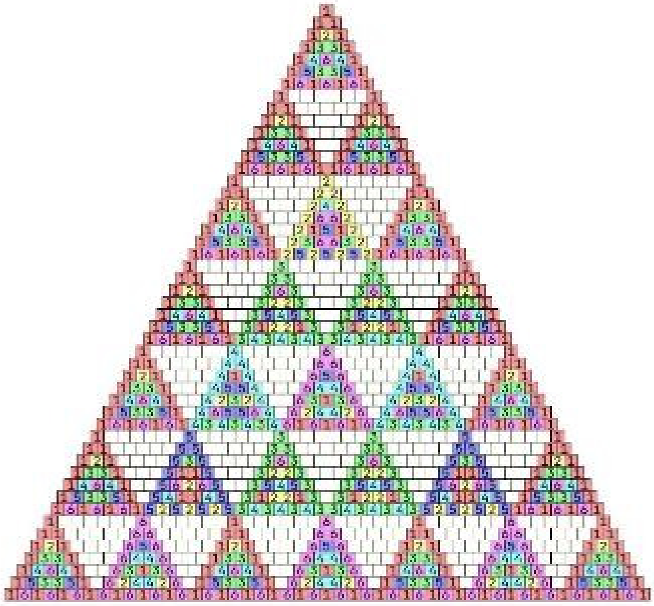
Here we present different renditions of various features of Pascal's triangle.



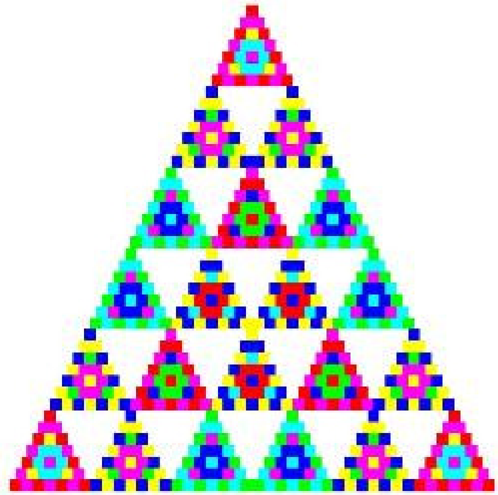
Buckingham Browne and Nichols 8th grader Sofia Egan presents 500 rows of Pascal's triangle, modulo 12.

At right is a detail.





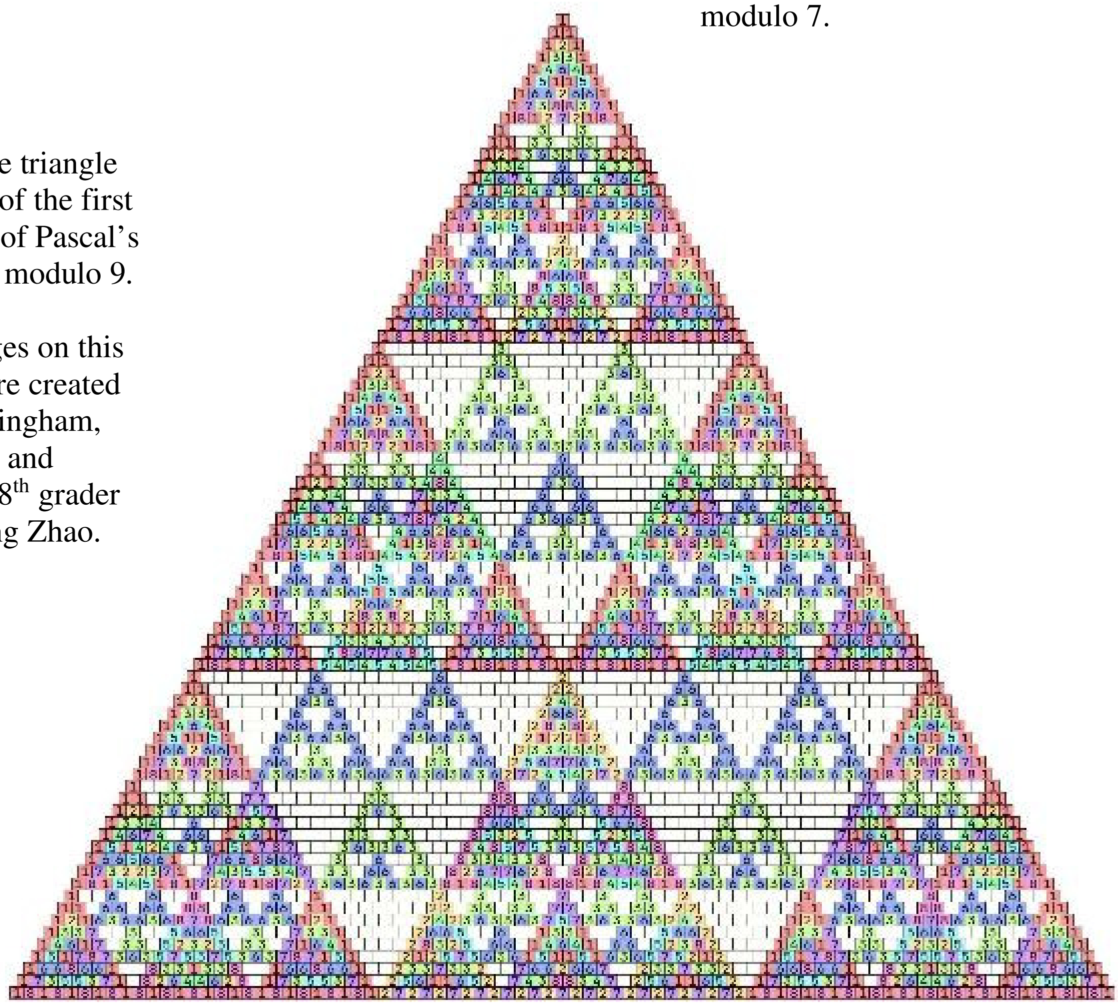
The first 49 rows of Pascal's triangle, modulo 7.

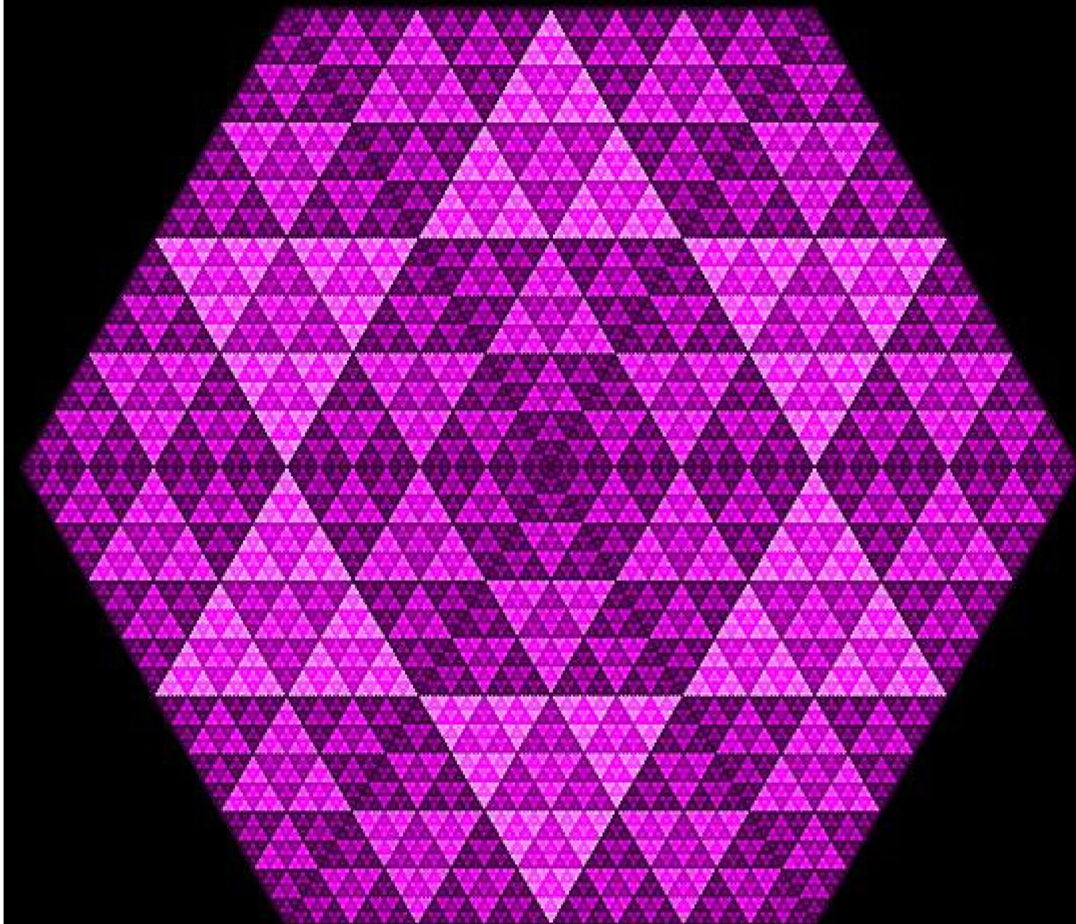


This image depicts a layer in Pascal's pyramid, specifically, the coefficients of the terms in the expansion of $(a + b + c)^{41}$, modulo 7.

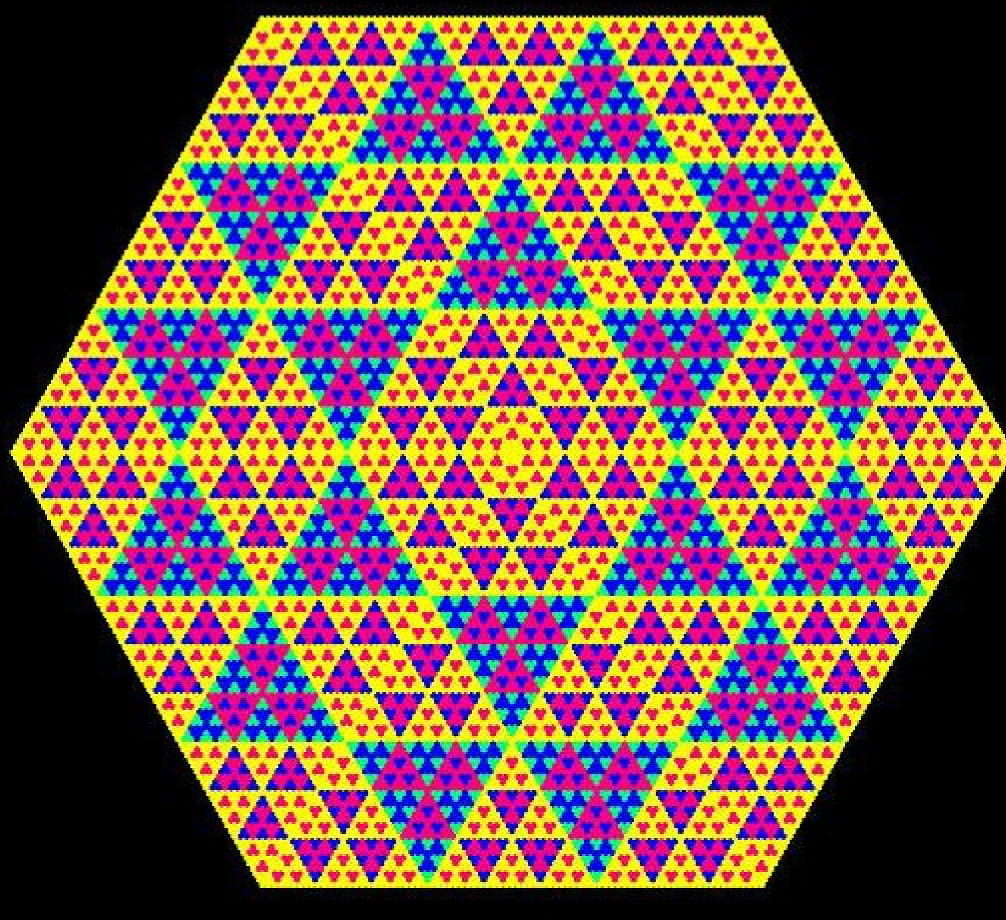
The large triangle consists of the first 81 rows of Pascal's triangle, modulo 9.

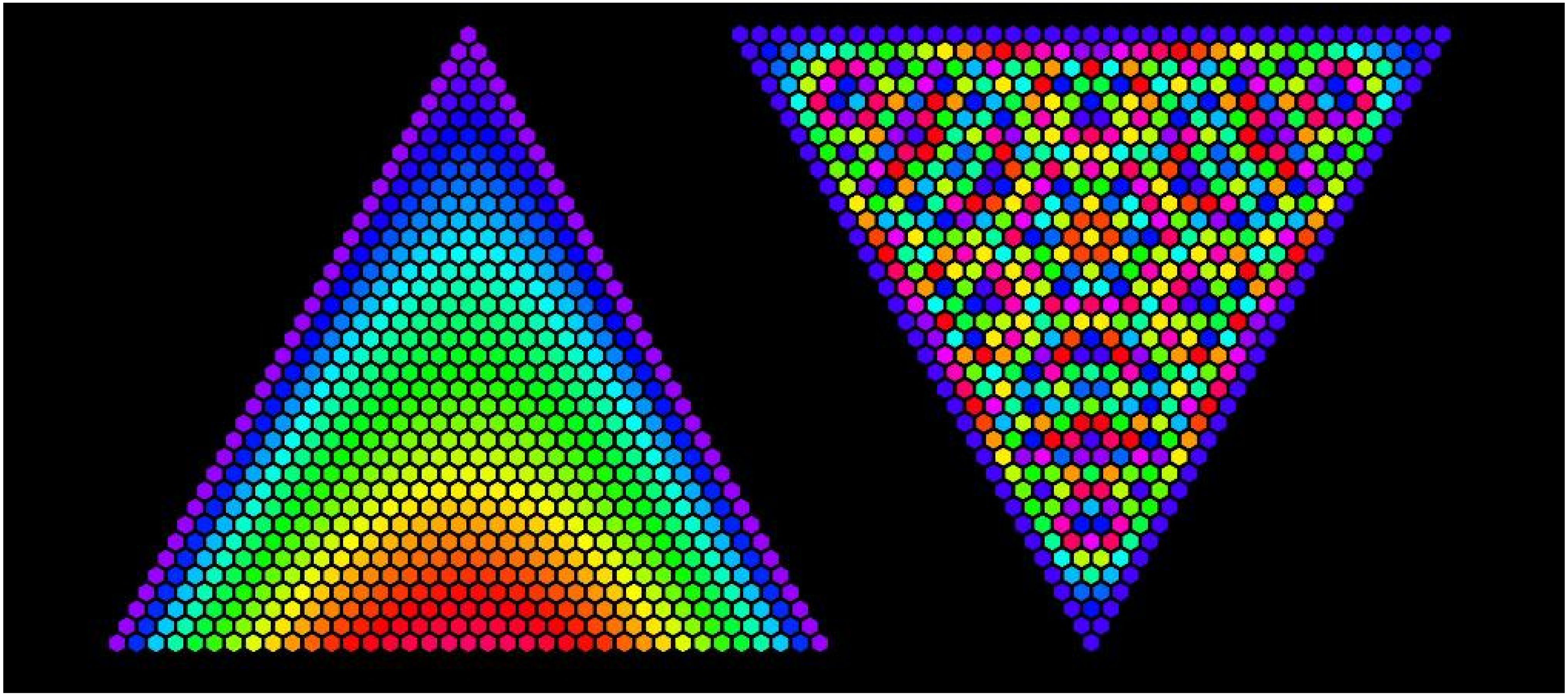
All images on this page were created by Buckingham, Browne, and Nichols 8th grader Yancheng Zhao.





The top image is built from 6 copies of the first 128 rows of Pascal's triangle, where the color scheme reflects the highest power of 2 that divides each entry. The bottom image is built from 6 copies of the first 81 rows of Pascal's triangle, where the color scheme reflects the highest power of 3 that divides each entry.





On the left are the first 37 rows of the unmodified Pascal′s triangle. On the right is the same 37 rows flipped over (for space considerations) where the color scheme is determined by the square of each entry, modulo 37.

In his paper *Sign-reductions,* p-*adic valuations, binomial coefficients modulo* p^k *and triangular symmetries*, Mihai Prunescu explains triangular symmetries that arise when certain natural functions are applied to the entries in Pascal′s triangle. The image above right and the images on the previous page exhibit symmetries explained in his paper. Try to prove these symmetries yourself!

Images on this and the previous page created by Girls′ Angle staff.

Magic Grids, Part 2

by Ken Fan | edited by Jennifer Sidney

Emily: With 4-by-4 magic grids, there are 16 entries. That's more than half of the letters in the alphabet!

Jasmine: There are also going to be more constraints.

Emily: That's true. We get constraints for every subsquare, because the corner entries of any subsquare must add up to the magic sum, which we might as well continue calling S. So there's 1 constraint for the 4 corners of the 4-by-4 square itself, 4 constraints for each of the 3-by-3 subsquares, and 9 constraints for each of the 2-by-2 subsquares, for a total of $1 + 4 + 9 = 14$ equations.

Jasmine: These equations aren't independent of each other, though, as we saw with the 3-by-3 magic grids. Some will automatically hold if others hold.

Emily: Gosh, there are a lot of variables and equations!

Jasmine: Actually, every 3-by-3 subgrid of a 4-by-4 magic grid is a 3-by-3 magic grid, so we can use what we found for 3-by-3 magic grids to help us find 4-by-4 magic grids.

Emily: That spares us from having to pick 16 variable names from the alphabet! Another simplification would be to focus on finding the grids with magic sum 0. Just as in the 3-by-3 case, we can subtract a constant from every entry in any 4-by-4 magic grid to produce a magic grid with magic sum 0; conversely, all the magic grids with magic sum S can be obtained by adding the constant $S/4$ to all the entries of some magic grid with magic sum 0.

Jasmine: Good idea! Let's focus on the case $S = 0$ for now.

Emily: So we can just plunk what we found for 3-by-3 magic grids with magic sum 0 into the upper left 3-by-3 subgrid of a 4-by-4 magic grid like so:

A	B	C	?
$-A + D$	$-B - D$	$-C + D$	?
$-C$	$A + B + C$	$-A$	?
?	?	?	?

Jasmine: Actually, we can fill in a lot of the missing entries by exploiting the fact that opposite corners of a 3-by-3 subgrid are the negatives of each other. That would get us to

A	B	C	$-A - B - C$
$-A + D$	$-B - D$	$-C + D$	?
$-C$	$A + B + C$	$-A$	-B
$C - D$	?	$A - D$	$B + D$

Emily: Wow … that "3-3 property" is a really useful observation. There are only 2 unknown entries left to figure out! This is working out much faster than I was expecting. Actually, before figuring out the missing entries, do the 4 corners of the grid add up to 0?

Jasmine: Oh, we better check that!

Emily: The sum is $A + (-A - B - C) + (C - D) + (B + D)$ … and that is 0!

Jasmine: Whew!

Emily: Since the 2 unknown entries are opposite corners of a 3-by-3 subgrid, they have to be opposites. So let's call them X and $-X$. I'll put X in the 2nd row, 4th column and $-X$ in the 4th row, 2nd column. Looking at the lower left 2-by-2 subgrid, we must have

$$-C + (A + B + C) + (C - D) - X = 0.$$

So, X must be $A + B + C - D$.

Jasmine: That $-X$ is a corner of another 2-by-2 subgrid, so it also must satisfy

$$(A + B + C) - A - X + (A - D) = 0,$$

and this equation says that $X = A + B + C - D$. Good, that's consistent!

Emily: We don't even have to check the 2-by-2 grids involving X, because those will follow by symmetry.

Jasmine: How do you mean?

Emily: Maybe "symmetry" isn't the right word. What I mean is this: if you look at the 2-by-2 subgrid in the upper right which has X in its lower right corner, all 4 entries can be regarded as the upper right corner of a 3-by-3 subgrid. Within each, the opposite corners constitute the

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Notes from the Club

These notes cover some of what happened at Girls' Angle meets. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 30 - Meet 5 March 3, 2022	Mentors:	Mandy Cheung, Cecilia Esterman, Bridget Li, Tharini Padmagirisan, Kate Pearce, Laura Pierson, Vievie Romanelli, Sakshi Suman, Jane Wang, Rebecca Whitman, Muskan Yadav, Angelina Zhang

It's a small miracle when a function is equal to its Taylor series. Computing Taylor series has become a bit of a fad at the club, with members computing the Taylor series of various functions such as arctan x and arcsin x. What's the most efficient way you can think of to compute the Maclaurin series for arcsin x? (The Maclaurin series is the Taylor series expanded around $x = 0$:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots,$$

where $f^{(n)}(0)$ is the nth derivative of $f(x)$, with respect to x, evaluated at $x = 0$.)

Session 30 - Meet 6 March 10, 2022	Mentors:	Amanda Burcroff, Mandy Cheung, Jenny Kaufmann, Bridget Li, Tharini Padmagirisan, Kate Pearce, AnaMaria Perez, Laura Pierson, Vievie Romanelli, Sakshi Suman, Rebecca Whitman, Muskan Yadav, Angelina Zhang

Some members rediscovered Fermat's little theorem. Now they are working on proving it, and they've come up with a really nifty strategy that's different from any proof I've seen. It's exciting. We'll see if they can pull it off...we're rooting for them!

In general, when you encounter a theorem, before reading a proof of it, try to prove it yourself. As mathematical thoughts occur to you, play around with them and see where they lead you. You might rediscover theorems without having to be told them. Maybe you'll discover something new! If you've not heard of Fermat's little theorem, try this: investigate geometric sequences in modular arithmetic. That is, for various integers $n > 1$ and a, look at the remainders when you divide each of the numbers 1, a, a^2, a^3, a^4, a^5, ... by n. What patterns do you notice? Don't look things up in books. Work out examples on blank sheets of scratch paper.

Session 30 - Meet 7 March 17, 2022	Mentors:	Mandy Cheung, Cecilia Esterman, Jenny Kaufmann, Tharini Padmagirisan, Kate Pearce, AnaMaria Perez, Laura Pierson, Vievie Romanelli, Sakshi Suman, Jane Wang, Rebecca Whitman, Muskan Yadav

Some members are inventing mathematical definitions. In math, definitions are critical. Unlike definitions for everyday words, mathematical definitions are absolutely precise. This precision enables mathematicians to settle debates rigorously and move on, aiding progress. Because of this importance, the act of defining something in mathematics demands great care. A good definition can catalyze tremendous progress while a bad one can stymie it.

The impetus for a definition often comes from wondering about whether something can exist. For example, "can a space with 1.5 dimensions exist?" What's cool about math is that when such a question occurs, instead of hemming and hawing about whether such a thing exists or not, or preparing some sort of expedition or building some fancy machine to try to discover such a thing, *you can just go ahead and define it!* If your definition leads to interesting mathematics, it will be picked up by others and, eventually, become a *thing*–a generally recognized idea that's part of the overall mathematical landscape.

Session 30 - Meet 8 March 31, 2022	Mentors:	Amanda Burcroff, Mandy Cheung, Cecilia Esterman, Bridget Li, Tharini Padmagirisan, Kate Pearce, AnaMaria Perez, Laura Pierson, Vievie Romanelli, Sakshi Suman, Rebecca Whitman, Muskan Yadav, Rachel Zheng

Some members unscrambled the scrambled proof on page 17.

Session 30 - Meet 9 April 7, 2022	Mentors:	Mandy Cheung, Cecilia Esterman, Bridget Li, Tharini Padmagirisan, Kate Pearce, AnaMaria Perez, Laura Pierson, Vievie Romanelli, Sakshi Suman, Jane Wang, Rebecca Whitman, Muskan Yadav, Angelina Zhang, Rachel Zheng

There are a number of tiling projects underway. Some are concerned with tiles made by joining congruent squares edge-to-edge. Others are concerned with tiles made by joining congruent equilateral triangles edge-to-edge. Which such tiles can be used to tile the plane, or to tile various shapes?

Session 30 - Meet 10 April 14, 2022	Mentors:	Mandy Chueng, Cecilia Esterman, Bridget Li, Tharini Padmagirisan, Kate Pearce, Laura Pierson, Vievie Romanelli, Sakshi Suman, Rebecca Whitman, Muskan Yadav, Angelina Zhang

Here's member Sadie Piazza's object that casts shadows in the shapes of her initials. Special thanks to Prof. Silviana Amethyst of the University of Wisconsin Eau Claire for 3D printing this object exactly to Sadie's specifications and for the photo. For more about Prof. Amethyst, read this article in the Notices of the American Mathematical Society:

https://www.ams.org/journals/notices/202205/rnoti-p861.pdf

Session 30 - Meet 11 April 28, 2022	Mentors:	Amanda Burcroff, Mandy Cheung, Cecilia Esterman, Bridget Li, Tharini Padmagirisan, Kate Pearce, AnaMaria Perez, Laura Pierson, Vievie Romanelli, Sakshi Suman, Jane Wang, Rebecca Whitman, Muskan Yadav, Rachel Zheng

Four of our members have been hard at work creating, from scratch, a massive, collaborative math puzzle. We're looking forward to solving it at our last meet this spring!

Calendar

Session 29: (all dates in 2021)

September	9	Start of the twenty-ninth session!
	16	
	23	
	30	
October	7	
	14	
	21	
	28	
November	4	
	11	Karia Dibert, University of Chicago
	18	
	25	Thanksgiving - No meet
December	2	

Session 30: (all dates in 2022)

January	27	Start of the thirtieth session!
February	3	
	10	
	17	
	24	No meet
March	3	
	10	
	17	
	24	No meet
	31	
April	7	
	14	
	21	No meet
	28	
May	5	

Girls' Angle has been hosting Math Collaborations at schools and libraries. Math Collaborations are fun math events that can be adapted to a variety of group sizes and skill levels. We will soon have versions available that are designed for remote participation. For more information and testimonials, please visit www.girlsangle.org/page/math_collaborations.html.

Girls' Angle can offer custom math classes over the internet for small groups on a wide range of topics. Please inquire for pricing and possibilities. Email: girlsangle@gmail.com.

Girls' Angle: A Math Club for Girls

Membership Application

Note: If you plan to attend the club, you only need to fill out the Club Enrollment Form because all the information here is also on that form.

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address (the Bulletin will be sent to this address):

Email:

Home Phone: ____________________________ Cell Phone: ____________________________

Personal Statement (optional, but strongly encouraged!): Please tell us about your relationship to mathematics. If you don't like math, what don't you like? If you love math, what do you love? What would you like to get out of a Girls' Angle Membership?

The $50 rate is for US postal addresses only. **For international rates, contact us before applying.**

Please check all that apply:

- ☐ Enclosed is a check for $50 for a 1-year Girls' Angle Membership.
- ☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com.

A Math Club for Girls

Girls' Angle
Club Enrollment

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

The club is where our in-person mentoring takes place. At the club, girls work directly with our mentors and members of our Support Network. To join, please fill out and return the Club Enrollment form. Girls' Angle Members receive a significant discount on club attendance fees.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-12. We welcome *all girls* (in grades 5-12) regardless of perceived mathematical ability. There is no entrance test. Whether you love math or suffer from math anxiety, math is worth studying.

How do I enroll? You can enroll by filling out and returning the Club Enrollment form.

How do I pay? The cost is $20/meet for members and $30/meet for nonmembers. Members get an additional 10% discount if they pay in advance for all 12 meets in a session. Girls are welcome to join at any time. The program is individually focused, so the concept of "catching up with the group" doesn't apply.

Where is Girls' Angle located? Girls' Angle is based in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org/page/calendar.html or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Peirce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science, worked in the mathematics educational publishing industry, and taught at HCSSiM. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, founder and director of the Exploratory
Yaim Cooper, Institute for Advanced Study
Julia Elisenda Grigsby, professor of mathematics, Boston College
Kay Kirkpatrick, associate professor of mathematics, University of Illinois at Urbana-Champaign
Grace Lyo, assistant dean and director teaching & learning, Stanford University
Lauren McGough, postdoctoral fellow, University of Chicago
Mia Minnes, SEW assistant professor of mathematics, UC San Diego
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, associate professor, University of Utah School of Medicine
Kathy Paur, Kiva Systems
Bjorn Poonen, professor of mathematics, MIT
Liz Simon, graduate student, MIT
Gigliola Staffilani, professor of mathematics, MIT
Bianca Viray, associate professor, University of Washington
Karen Willcox, Director, Oden Institute for Computational Engineering and Sciences, UT Austin
Lauren Williams, professor of mathematics, Harvard University

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. In this way, we hope to help girls become solvers of the yet unsolved.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: Club Enrollment Form

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Please fill out the information in this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. Names:

____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: Girls roughly in grades 5-12 are welcome. Although we will work hard to include every girl and to communicate with you any issues that may arise, Girls' Angle reserves the discretion to dismiss any girl whose actions are disruptive to club activities.

Personal Statement (optional, but strongly encouraged!): We encourage the participant to fill out the optional personal statement on the next page.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: _______________________

(Parent/Guardian Signature)

Participant Signature: ____

Members: Please choose one.	Nonmembers: Please choose one.
☐ Enclosed is \$216 for one session (12 meets)	☐ I will pay on a per meet basis at \$30/meet.
☐ I will pay on a per meet basis at \$20/meet.	☐ I'm including \$50 to become a member, and I have selected an item from the left.

☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Personal Statement (optional, but strongly encouraged!): This is for the club participant only. How would you describe your relationship to mathematics? What would you like to get out of your Girls' Angle club experience? If you don't like math, please tell us why. If you love math, please tell us what you love about it. If you need more space, please attach another sheet.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____ Date: _______________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: __________________________________