Girls' Angle Bulletin

February/March 2017 • Volume 10 • Number 3

To Foster and Nurture Girls' Interest in Mathematics

From the Founder

People are so diverse. And so is mathematics. If you're a student who finds the math you're doing in school boring, take some time to explore math outside of class. There's bound to be something out there that you'll enjoy. If you can't find anything, consult someone who knows a lot of math, like our mentors. You might be surprised by what's out there.

- Ken Fan, President and Founder

Girls' Angle Bulletin

The official magazine of Girls' Angle: A Math Club for girls Electronic Version (ISSN 2151-5743)

Website: www.girlsangle.org
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This magazine is published six times a year by Girls' Angle to communicate with its members and to share ideas and information about mathematics.

Girls' Angle welcomes submissions that pertain to mathematics.

The print version of the Bulletin is printed by the American Mathematical Society.

Editor: Jennifer Silva
Executive Editor: C. Kenneth Fan

On the cover: Monic cubics with multiple roots. See page 20. Image created using MATLAB, a powerful suite of mathematical software produced by MathWorks.

An Interview with Sommer Gentry

Photo courtesy of Sommer Gentry

Sommer Gentry is Professor of Mathematics at the United States Naval Academy. She earned a Bachelor of Science degree in Mathematical and Computational Sciences and a Master of Science degree in Engineering-Economic Systems and Operations Research in 1998 from Stanford University and a PhD in Electrical Engineering and Computer Science in 2005 from the Massachusetts Institute of Technology.

The interviewer was Girls' Angle program assistant Long Nguyen.

Long: What got you interested in mathematics, computer science, and operations research?

Sommer: Before college, I never liked the repetitious, procedural sort of math that I saw at school, but I loved math puzzles and pop math books like those by Martin Gardner and Ivars Peterson. I craved certainty, which math seemed to offer. I was also interested in politics. I wanted to change the world for the better by understanding which was the best approach to solve some political or public policy problem. Operations research, which is the math of making better decisions, was an obvious fit for me.

However, I began my career believing that presenting facts, performing careful analysis, and using optimization to maximize a stated goal would result in great political decisions. Unfortunately, I must report that this is not at all how decisions get made! People make decisions out of emotion or tribal loyalty, and suffer many cognitive biases even when they attempt to examine facts. For instance, orders of magnitude more lives would be saved by investing public safety dollars in flood defenses, tornado shelters, and lead abatement rather than in "anti-terrorism" foolishness. More Americans are crushed to death by their furniture than are killed by terrorists – these are miniscule risks. I hope many more of us will come to value truth as the key to reducing human suffering.

Long: I watched "The Right Match," a short documentary you and your husband at Johns Hopkins made. I'd urge all readers of this interview to take a time out and watch it! In it, you describe the problem of matching kidney donors with those in need of kidney transplants. Could you please give us more details about how the algorithm that solves the problems you described in the documentary works?

Sommer: If one of your loved ones wants to donate a kidney to you, there's about a 30% chance they will be incompatible. There might be another pair out there with different blood types, and perhaps the other donor could donate to you while your donor donates to the other recipient - that's a kidney exchange. We create equations to

Dear Reader,

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For this issue, those who do not subscribe to the print version will be missing out on the remainder of this interview with Prof. Sommer Gentry and some other content. We hope that you consider the value of such content and decide that the efforts required to produce such content are worthy of your financial support.

We know that mathematical interest and talent is unrelated to economic status, which is why we provide so much content for free. But we hope that those of you who are comfortable financially will help us to continue in our efforts.

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Thank you and best wishes,
Ken Fan
President and Founder
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Quilt-doku!

Part 3

by Beth Malmskog | edited by Jennifer Silva

3. The Answer

Now that we're warmed up, let's work on the problem of 5 quilters:

> Arrange a passing scheme for 5 quilters so that no quilter passes to the same person twice, or prove that no such passing scheme exists.

When trying to answer Judy's question for 5 quilters, the first thing to notice is that there are essentially only two ways that the first trade could go. Either the group could pass the quilts in one big cycle, or two people could trade and the other three could pass in a cycle.

Case A. Say the first trade involves everyone passing the quilts in one cycle. By renumbering the people and reordering the rows, assume the first passing cycle is

$$0 \to 1 \to 2 \to 3 \to 4 \to 0.$$

The first two columns of the square are then determined.

$A =$

0	1	?	?	?
1	2			
2	3			
3	4			
4	0			

Let's introduce some notation. We have called our grid A, and we will let $A_{i,j}$ denote the number in the i-th row and j-th column of the grid. For example, $A_{1,1} = 0$ in the grid above. Following the 1 in the first row, we see that $A_{1,3}$ must be either 2, 3, or 4. It can't be that $A_{1,3} = 0$, because we want all 5 quilters to add a border to each quilt block and person 0 already handled the quilt block represented by row 1. Nor can it be that $A_{1,3} = 1$, because that would mean person 1 keeps the quilt block. It also can't be that $A_{1,3} = 2$, because the sequence (1, 2) already appears in the second row.

Say $A_{1,3} = 3$. Then $A_{1,4} \neq 4$ since (3, 4) already appears in the fourth row. So the first row must be (0, 1, 3, 2, 4). Now consider the second row. It must be that $A_{2,3} = 0$, 3, or 4. Given that the sequence (2, 3) appears in the third row and (2, 4) appears in the first row, $A_{2,3} = 0$. Since 4 already appears in the fifth column, we must have $A_{2,4} = 4$, which means the second row is (1, 2, 0, 4, 3).

	0	1	3	2	4
	1	2	0	4	3
A =	2	3			
	3	4			
	4	0			

We now see that it is impossible to fill the third row subject to the constraints because $A_{3,3}$ must be 4 (the third column is the only column that lacks a 4), but then the sequence (3, 4) would appear twice in the square. Thus, $A_{1,3} \neq 3$. By similar reasoning, we find that $A_{1,3} \neq 4$. Therefore, the first pass cannot work as a single cycle.

Case B. Assume that on the first quilt pass, two people swap and the other three pass in a cycle. If we renumber the people and reorder the rows, the first round of trades can give the following first two columns:

	0	1	?	?	?
	1	0			
A =	2	3			
	3	4			
	4	2			

We can rather quickly see that this will not work out. Notice that in each of the last three rows, we will need to place a 0 and a 1 in the last three columns. We can never put them next to each other, because (0, 1) appears in the first row and (1, 0) appears in the second row. So the 0 and 1 will always have to fall in the third and fifth columns. But we have three rows where we need to place 0's and only two columns to place them in; that would force us to place two 0's in some column, which is not allowed. So this whole case falls apart.

Judy was correct that her circle could not be squared. No RCLS(n) exists when n = 3 or 5, so these sizes of quilting circles will never work. By Williams's construction, an RCLS(n) always exists when n is even, so these exchanges can always be arranged for even-sized quilting circles. Can we completely classify the possible sizes of quilting circles for which these exchanges are possible – that is, can we determine every value of n for which an RCLS(n) exists?

It turns out that nobody knows the answer for sure! Recall that prime numbers are positive whole numbers that have exactly two positive whole number factors. Numbers that are not prime are called composite. J. Higham proved that it is possible to make an RCLS(n) for all composite n [1]. People have proven that there are no RCLS(n) for n = 3, 5, 7, and 11, but *nobody* has been able to prove that it is impossible to make an RCLS(13). It is conjectured that there are no row complete Latin squares of prime order, but proving this is still an open problem.

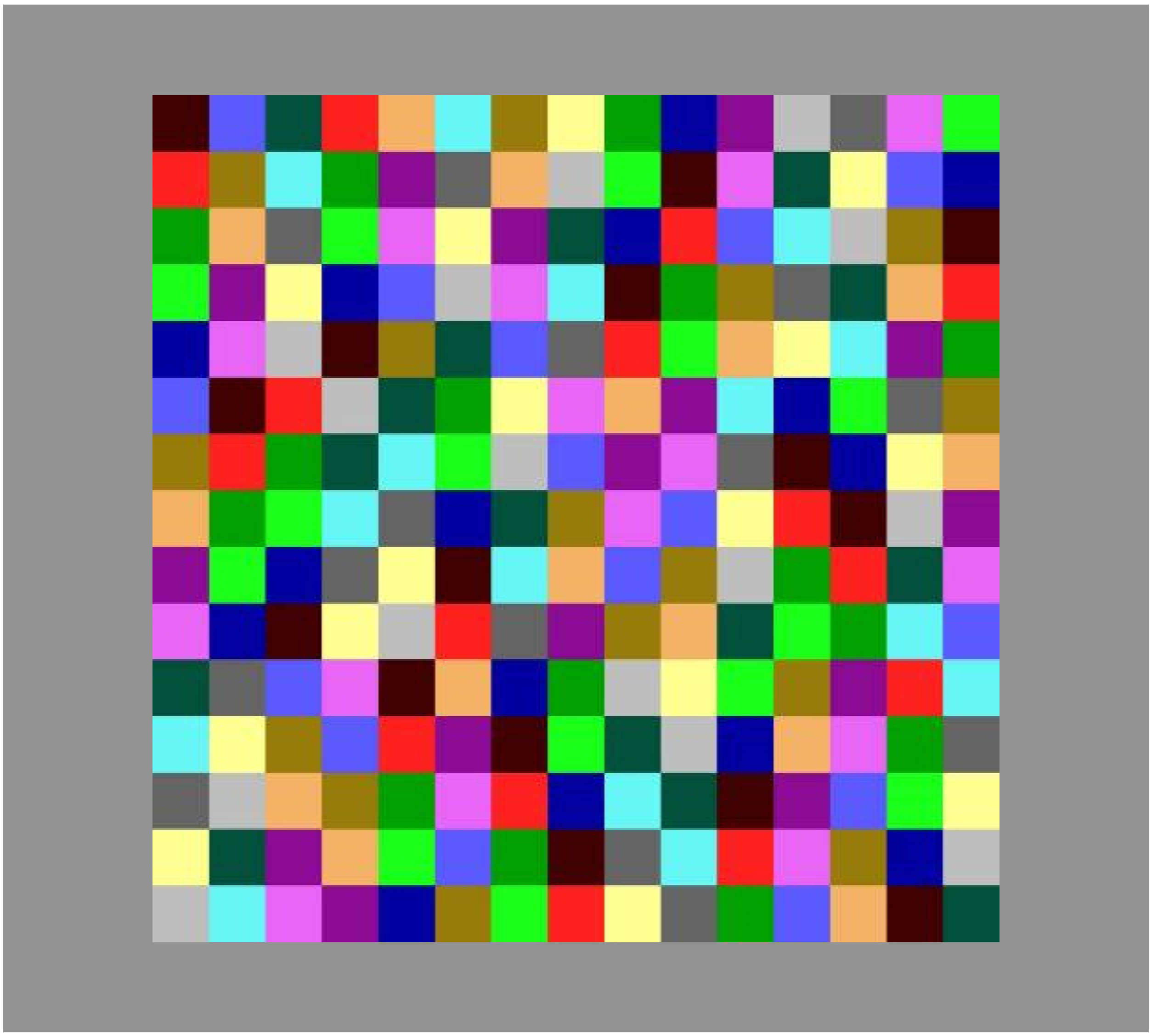
Acknowledgements: I would like to thank Judy Gilmore for the great question, and Katie Haymaker for working on this problem with me. If you'd like to know more about these squares, please email beth.malmskog@villanova.edu for a draft of our paper.

References

[1] J. Higham, Row-complete Latin squares of every composite order exist, *Journal of Combinatorial Designs*, **6**, no. 1 (1998).

[2] B. Malmskog and K. Haymaker, Quilting squares (2017). Under review for publication; available by email request at beth.malmskog@villanova.edu.

This work is supported in part by a grant from MathWorks.

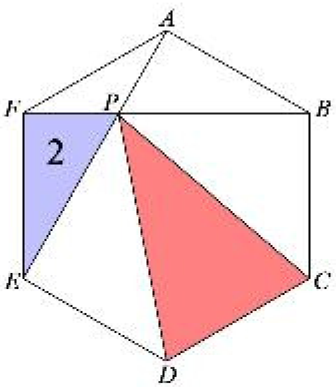


A colored version of a 15 by 15 row complete Latin square from "Some New Row-Complete Latin Squares" by Archdeacon, Dinitz, Stinson, and Tilson in the Journal of Combinatorial Theory, Series A 29, pp. 395-398 (1980). Note that every pair of colors appears side-by-side (both ways) somewhere in the pattern.

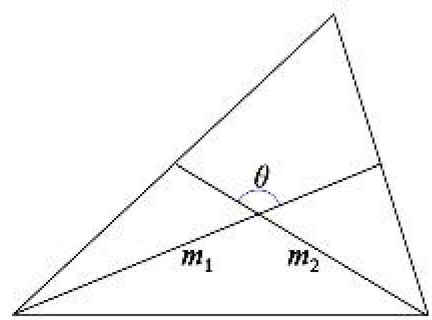
Area Area

by Ken Fan

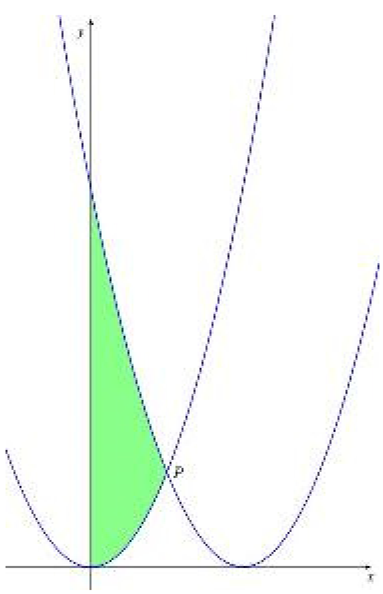
Here are 4 area problems. If you're just learning about the concept of area, these will be fairly challenging, but you're encouraged to give them a try. If they're too hard, don't worry, just put them aside and return to them in the future. If you're an advanced problem solver, see how many of these you can do entirely in your head. For hints and solutions, see page 26.



1. Inside regular hexagon *ABCDEF*, line segments *AE* and *BF* intersect at *P*. Given that the area of triangle *EFP* is 2 square units, what is the area of triangle *CDP*?

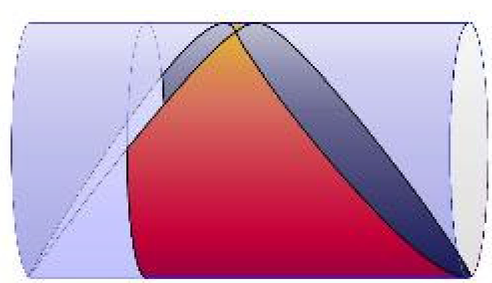


2. The lengths of two medians of a triangle are m_1 and m_2. The measure of the angle formed by these two medians is θ. What is the area of the triangle in terms of m_1, m_2, and θ?



3. Consider the graphs of the parabolas $y = x^2$ and $y = (x - d)^2$ in the *xy*-coordinate plane, where *d* is a positive constant. These two parabolas intersect at point *P*. In terms of *d*, what is the area of the green region? (The green region is the area between the parabolas and between the *y*-axis and the point *P*.)

4. Consider the cylinder given by $y^2 + z^2 = 1$, $-\pi/2 < x < \pi$ in *xyz*-coordinate space. This surface is sliced by the surface $z = \cos x$. What is the area of the part of the cylinder whose coordinates satisfy $z < \cos x$? (The illustration at right is only suggestive. It is not accurate.)



Anna's Math Journal

By Anna B.

Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.

Anna continues counting special *N*-tilings of a 1 by $\sqrt{2}$ rectangle.

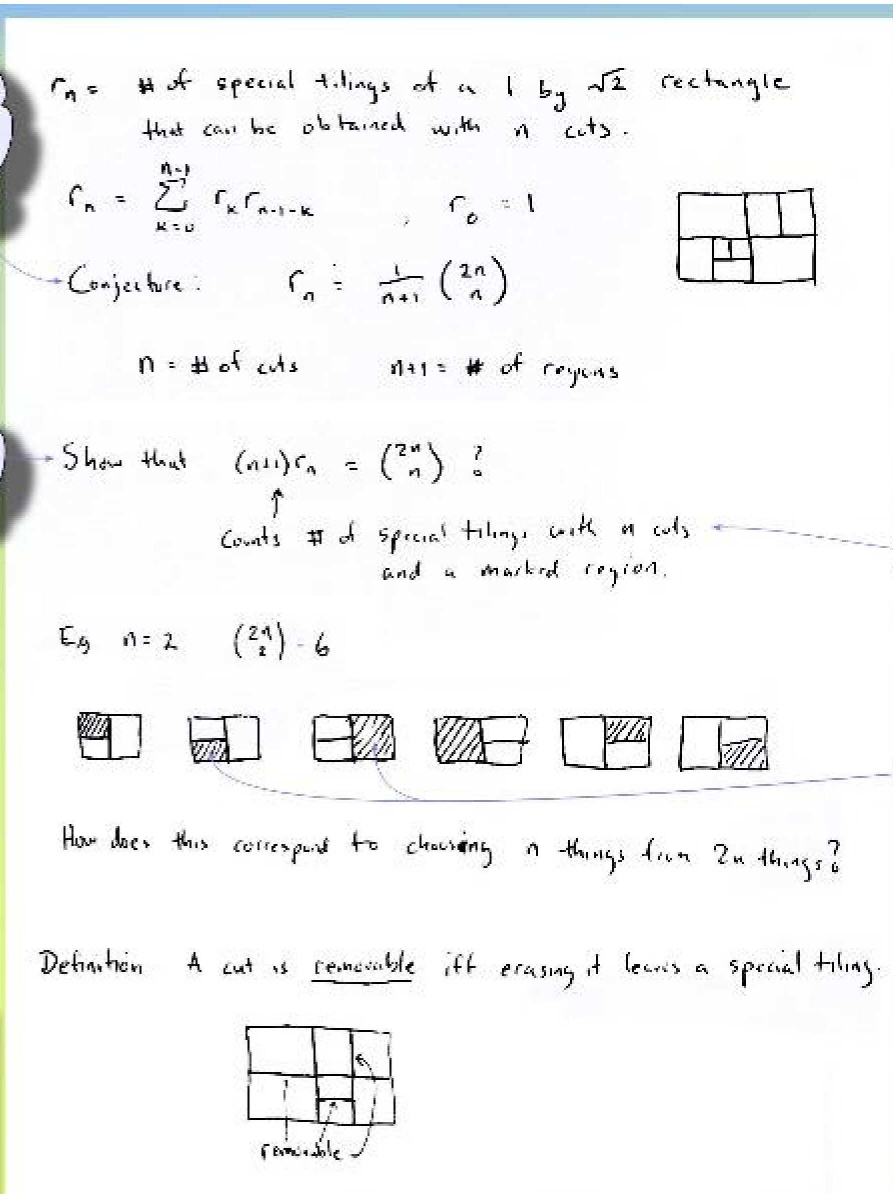
I'd like to show that the number of special tilings of a 1 by $\sqrt{2}$ rectangle that can be obtained with n cuts is one over $n + 1$ times $2n$ choose n.

Since $2n$ choose n has combinatorial meaning, I wonder if I can show that $(n + 1)r_n$ is $2n$ choose n.

With n cuts, the rectangle is split into $n + 1$ regions. So maybe I should try to count special tiles with a marked region.

Each region is a place where I could create a new cut.

Maybe I can count the total number of "removable" cuts found in all special tilings with $n + 1$ cuts and try to show it is $2n$ choose n.



Key:

Anna's thoughts

Anna's afterthoughts

Editor's comments

If I take all special tilings made with $n+1$ cuts and count the total number of removable cuts in all of them, I should/hope to get $\binom{2n}{n}$.

If T is a special tiling, define $R(T)$ to be the number of removable cuts from T.

Hey, $R(S+T)$ is equal to $R(S)+R(T)$, unless both S and T are uncut rectangles.

$$R(S+T) = R(S) + R(T)$$

unless $S = T = \square$, in which case $R(S+T) = 1$.

I'm using the notation "S + T" to mean rotating the special tilings S and T counterclockwise 90 degrees and joining them side by side, with S to the left of T.

Want to show that

$$\sum_{\substack{T \text{ is a} \\ \text{special tiling} \\ \text{with } n+1 \text{ cuts}}} R(T) = \binom{2n}{n}$$

$$\sum_{\substack{\text{Special tilings} \\ T \text{ with} \\ n+1 \text{ cuts}}} R(T) = \sum_{k=0}^{n} \sum_{\substack{\text{Special} \\ \text{tilings } X \\ \text{with } k \text{ cuts}}} \sum_{\substack{\text{Special} \\ \text{tilings } Y \\ \text{with } n-k \text{ cuts}}} R(X+Y)$$

$$= \sum_{k=0}^{n} \sum_{\substack{\text{Special} \\ \text{tilings } X \\ \text{with } k \text{ cuts}}} \sum_{\substack{\text{Special} \\ \text{tilings } Y \\ \text{with } n-k \text{ cuts}}} \left(R(X)+R(Y)\right)$$

$$= \sum_{k=0}^{n} \sum_{\substack{\text{Special} \\ \text{tilings } X \\ \text{with } k \text{ cuts}}} \left(r_{n-k} R(X) + \binom{2(n-k-1)}{n-k-1}\right)$$

$$= \sum_{k=0}^{n} \left(r_{n-k}\binom{2(k-1)}{k-1} + r_k\binom{2(n-k-1)}{n-k-1}\right)$$

$$= r_0\binom{2(n-1)}{n-1} + \left(r_{n-1} + r_1\binom{2(n-2)}{n-2}\right) + \left(r_{n-2}\binom{2\cdot 1}{1} + r_2\binom{2(n-3)}{n-3}\right)$$
$$+ \cdots + \left(r_1\binom{2(n-2)}{n-2} + r_{n-1}\right) + r_0\binom{2(n-1)}{n-1}$$

$$= 2\sum_{k=0}^{n} r_{n-k}\binom{2(k-1)}{k-1}$$

Hm. This feels too close to trying to show that the conjectured formula for r_n satisfies the recurrence relation. I think the problem is that the total number of removable cuts is too closely related to the number of special tilings. I think I'm going to have to try some other approach. I don't really want to try generating functions since such an approach probably won't give much insight, but I'm not sure what else to try. Maybe next time I'll do that.

Can you help Anna prove or disprove the formula for r_n?

ABB 2/25/17

Key:
- Anna's thoughts
- Anna's afterthoughts
- *Editor's comments*

In Search of Nice Triangles, Part 9

by Ken Fan | edited by Jennifer Silva | *special thanks to Alison Miller*

Emily and Jasmine continue their investigation into nice triangles. They've been using "nice" to denote angles that measure a rational multiple of π radians.

They are now looking for triangles with 3 nice angles and 2 sides of integer length.

Last time, they decided to embark on a study of the minimum polynomials of the cosines of rational multiples of π. They defined the polynomials $p_d(x)$, for $d > 1$, to be the product of all linear factors of the form $x - \cos(2\pi k/d)$, where $1 \le k \le d/2$ and $(k, d) = 1$. They defined $p_1(x) = x - 1$.

They observed that, for n odd,

$$T_n(x) - 1 = 2^{n-1} p_1(x) \left(\prod_{d|n, d>1} p_d(x) \right)^2,$$

and for n even,

$$T_n(x) - 1 = 2^{n-1} p_1(x) p_2(x) \left(\prod_{d|n, d>2} p_d(x) \right)^2,$$

where $T_n(x)$ is the nth Chebyshev polynomial of the first kind.

They conjectured that the minimum polynomial of $\cos(2\pi k/n)$, where k and n are relatively prime, is $p_n(x)$.

Emily and Jasmine arrive at Cake Country.

Emily: It's crowded in here!

Jasmine: But the line's moving quickly.

They get in line to order. Sure enough, they arrive at the front in just a few short minutes.

Mr. ChemCake: What can I get for you mathophiles?

Emily: Hi, Mr. ChemCake! We'd love to share a slice of your Rainbow Royale ice cream cake.

Mr. ChemCake: Coming right up!

Mr. ChemCake produces a wedge of the 7-flavored marvel.

Jasmine: Thanks! Say, why's it so busy here?

Mr. ChemCake: A conference – at the university.

Emily pays and looks at Jasmine.

Emily: My treat!

The two friends grab the only empty table and dig in.

Jasmine: Thanks, Emily. Mmm, this Rainbow Royale hits the spot! Great idea to come here. It does make me feel a little better about the math.

Emily: We've really hit a wall. It would be so wonderful if the minimum polynomials of cosines of rational angles turned out to be what we think they are, but I'm at a loss for how to show it.

Jasmine: I actually can't decide whether I believe that $p_n(x)$ is the minimum polynomial of $\cos(2\pi k/n)$, where k and n are relatively prime, or not. Maybe $p_n(x)$ *does* factor. After all, we've only shown that $p_n(x)$ can't be factored for a few small values of n.

Emily: If these are the minimum polynomials, it would mean that if k and j are relatively prime to n, then $\cos(2\pi k/n)$ and $\cos(2\pi j/n)$ always come together as roots of polynomials with rational coefficients, which seems amazing.

Jasmine: That's true, because if $p_n(x)$ is their minimum polynomial, then we've seen that $p_n(x)$ must divide into any polynomial $f(x)$ with rational coefficients that either number is a root of.

And since both are roots of $p_n(x)$, both would then have to be roots of $f(x)$. It makes me think of complex conjugation.

Emily: It does?

Jasmine: Yes, because complex conjugation shows that complex conjugates $a + bi$ and $a - bi$ always come together as roots of polynomials with real coefficients.

Emily: Interesting. Hmm. Could it be that …

Stranger: Excuse me …

Emily and Jasmine look up from their table and see a woman standing before them.

Jasmine: Yes? May we help you with something?

Stranger: I'm sorry to interrupt, but I heard the two of you talking about polynomials, and I was curious to know what exactly you were discussing.

Jasmine: Oh, we were discussing a math problem. Do you know trigonometry?

Stranger: I know something about it. I should introduce myself! My name's Alison – Alison Miller. I'm a mathematician.

Emily: You're a mathematician?

Alison: Yes – I'm here for the math conference.

Jasmine: Neat! Mr. ChemCake didn't tell us that the conference is a *math* conference.

Alison: You're not part of the conference?

Jasmine: Oh no, we're not even in college.

Alison: That's funny, I just assumed you were since you were talking math. May I join you?

Emily: Of course! I'm Emily, and this is my friend Jasmine. We love math!

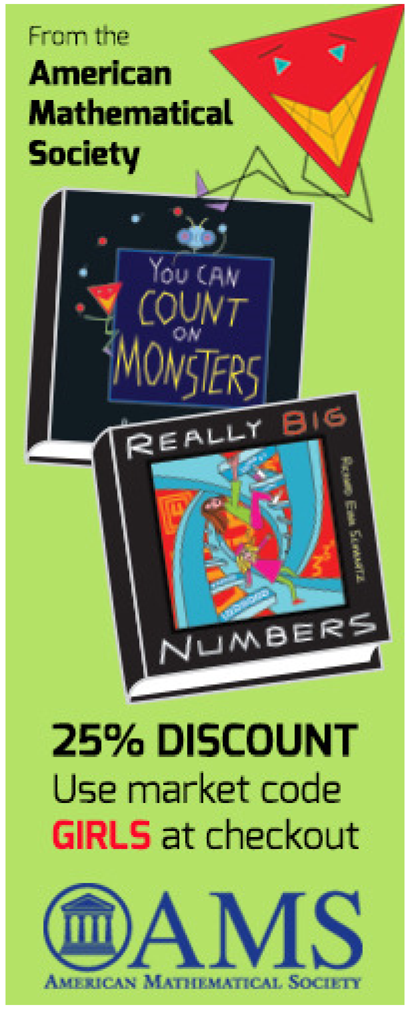
Alison: I can tell! So what math were you discussing?

Jasmine: It's kind of a long story.

Alison: I've got time.

Emily: Well, it all began when Jasmine saw an impossible triangle at our school. It was a triangle with sides of length 10, 17, and 20, but angles measuring 30, 60, and 90 degrees.

Alison: Oh dear!



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Learn by Doing

Gauss's Lemma
by Girls' Angle Staff

In this Learn by Doing, we'll explore Gauss's Lemma and some of its implications.

First, let's review polynomial multiplication. Let

$$f(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$$

and

$$g(x) = b_0 + b_1x + b_2x^2 + \ldots + b_mx^m,$$

be two polynomials in the variable x. The a_k are the coefficients of $f(x)$. For the sake of convenience, define $a_k = 0$ for $k > n$ and define $b_k = 0$ for $k > m$. Let $h(x) = f(x)g(x)$.

1. Show that the coefficient of x^d in $h(x)$ is given by

$$a_db_0 + a_{d-1}b_1 + a_{d-2}b_2 + \ldots + a_1b_{d-1} + a_0b_d.$$

Now suppose further that the a_k and b_k are integers and that the a_k have a common factor z. For example, if $f(x) = 6 + 9x + 3x^2$, we could take $z = 3$ since 3 divides evenly into all three coefficients.

2. Show that z is a common factor of every coefficient of $h(x)$ as well.

Let $f(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$ be a polynomial with integer coefficients. We define the **content** of $f(x)$ to be the greatest common divisor of the coefficients a_k. Let d be the content of $f(x)$. By definition, we can then write $a_k = d\alpha_k$, where α_k is an integer for all $0 \le k \le n$.

3. Show that $f(x) = dg(x)$, where $g(x) = \alpha_0 + \alpha_1x + \alpha_2x^2 + \ldots + \alpha_nx^n$ and the content of $g(x)$ is 1.

A polynomial with integer coefficients and content 1 is called **primitive**.

4. In the setup of Problem 3, suppose $f(x) = eh(x)$ where $h(x)$ is primitive. Show that $e = \pm d$.

5. Can you find primitive polynomials $f(x)$ and $g(x)$ whose product is *not* primitive?

If you made a good effort doing Problem 5, you might have begun to suspect that it is impossible. *That* is what Gauss's lemma says.

6. (Gauss's lemma.) Show that the product of primitive polynomials is primitive.

If you're having trouble with Problem 6, let's think about polynomial multiplication in a special case. Let $f(x) = a_0 + a_1x + a_2x^2$ and $g(x) = b_0 + b_1x + b_2x^2 + b_3x^3$. When we compute $f(x)g(x)$, we add up the products of pairs of terms where each pair consists of a term from $f(x)$ and a term from $g(x)$. This fact can be seen by systematically applying the distributive law. These products have been organized into the following table.

	b_0	b_1x	b_2x^2	b_3x^3
a_0	a_0b_0	a_0b_1x	$a_0b_2x^2$	$a_0b_3x^3$
a_1x	a_1b_0x	$a_1b_1x^2$	$a_1b_2x^3$	$a_1b_3x^4$
a_2x^2	$a_2b_0x^2$	$a_2b_1x^3$	$a_2b_2x^4$	$a_2b_3x^5$

The left column consists of the terms of $f(x)$ and the top row consists of the terms of $g(x)$. Each entry of the colored section is the product of the corresponding terms from $f(x)$ and $g(x)$. Note the pattern created by products of the same degree (indicated by color). Suppose the product $f(x)g(x)$ is not primitive. Let p be a prime number that divides the content of $f(x)g(x)$. Think about the divisibility of the coefficients of each entry in the table by p. Not every coefficient in the first column can be divisible by p since $f(x)$ is primitive. Similarly, not every coefficient in the first row can be divisible by p. What does this imply about the coefficients in the colored section of the table? Can you find like terms in the colored section whose coefficients add up to something that is *not* a multiple of p? Can you generalize your observations to arbitrary primitive polynomials? (In effect, we are considering the situation modulo p.)

7. Let $f(x)$ and $g(x)$ be polynomials with integer coefficients. Show that the content of $f(x)g(x)$ is the product of the content of $f(x)$ and the content of $g(x)$.

8. Let $f(x)$ be a nonzero polynomial with rational coefficients. Show that there exists a rational number r such that $rf(x)$ is a primitive polynomial.

9. Let $f(x)$ be a nonzero polynomial with rational coefficients. Suppose r and s are rational numbers such that both $rf(x)$ and $sf(x)$ are primitive polynomials. Show that $r = \pm s$.

10. Let $f(x)$ be a non-constant primitive polynomial. Assume that $f(x)$ cannot be factored as a product of non-constant polynomials with integer coefficients. Show that $f(x)$ cannot be factored as a product of non-constant polynomials with *rational* coefficients.

Gauss's Lemma can be used in the proof of a famous result of Eisenstein:

11. (Eisenstein's irreducibility criterion.) Let $f(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$ be a polynomial with integer coefficients. Suppose that there exists a prime number p such that p does not divide a_n, p^2 does not divide a_0, but p does divide a_k for $0 \le k < n$. Show that $f(x)$ cannot be factored as a product of non-constant polynomials with rational coefficients.

The following problem is an inseparable companion to Eisenstein's irreducibility criterion. Every book I have seen contains both or neither. Eisenstein's proof of the irreducibility of prime-indexed cyclotomic polynomials is much simpler than Gauss's.

12. Let p be a prime number. Show that $1 + x + x^2 + \ldots + x^{p-1} = (x^p - 1)/(x - 1)$ cannot be written as a product of non-constant polynomials with rational coefficients. Hint: replace x by $x + 1$. (These polynomials are cyclotomic polynomials. See this issue's installment of Emily and Jasmine on page 12.)

For further reading: Cox, D. A. "Why Eisenstein Proved the Eisenstein Criterion and Why Schönemann Discovered It First," American Mathematical Monthly 118 (2011), 3-21.

More on Quadratics

by Addie Summer | edited by Jennifer Silva

I love a good picture. That's why, after explaining (in the previous *Bulletin* issue) how I found the quadratic formula, I decided to make a map of the sea of quadratic functions.

To review, the quadratic formula gives the solutions to a quadratic equation in terms of its coefficients: If a, b, and c are constants, then the solutions to the equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

For $ax^2 + bx + c = 0$ to be a *quadratic* equation, we must have $a \neq 0$; otherwise, the equation reduces to the linear equation $bx + c = 0$. Since $a \neq 0$, we can divide the quadratic equation by a to arrive at the equation $x^2 + (b/a)x + (c/a) = 0$, an equation with lead coefficient 1. We therefore don't lose much generality by focusing on quadratic equations of the form

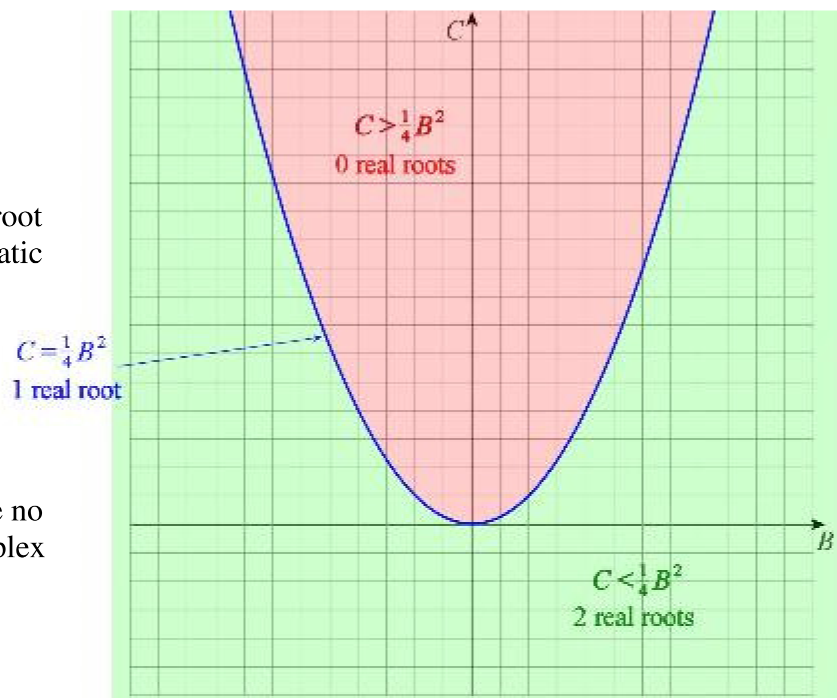
$$x^2 + Bx + C = 0,$$

where B and C are real numbers. In this case, the quadratic formula tells us that the solutions are

$$x = \frac{-B \pm \sqrt{B^2 - 4C}}{2}.$$

Since the quadratics $x^2 + Bx + C$ are determined by the two numbers B and C, we can represent each by a point in a Cartesian plane with horizontal coordinate labeled by B and vertical coordinate labeled by C. For example, in this "plane of quadratics," the point (5, 3) represents the quadratic $x^2 + 5x + 3$.

In the plane of quadratics, the points (B, C) whose coordinates satisfy $B^2 - 4C = 0$ represent precisely those quadratics $x^2 + Bx + C$ that have only one root (namely, $x = -B/2$), as the quadratic formula informs us. The points (B, C) with $B^2 - 4C > 0$ represent the quadratics $x^2 + Bx + C$ that have two roots, and the points (B, C) with $B^2 - 4C < 0$ represent the quadratics $x^2 + Bx + C$ that have no real roots (or two non-real complex roots). I decided to map these regions (see figure at right).



Roots

Next, I pondered the following: In the plane of quadratics, do the roots of the quadratic $x^2 + Bx + C$ have a graphical interpretation?

The roots of $x^2 + Bx + C$ are the solutions to the quadratic equation

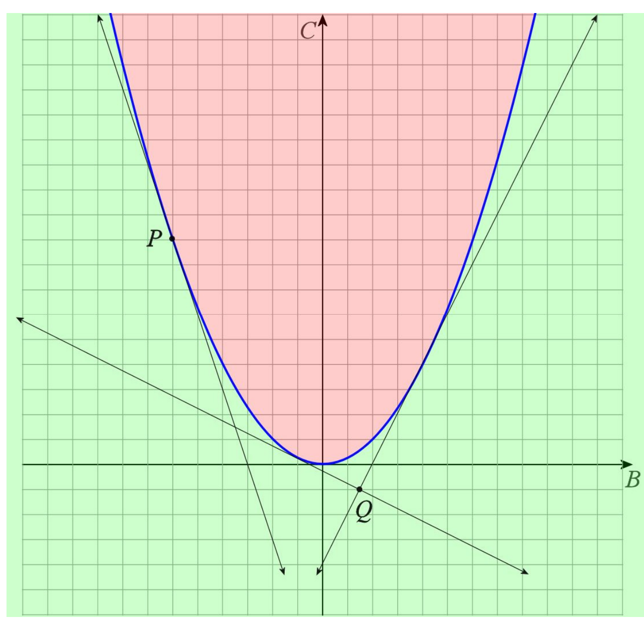
$$x^2 + Bx + C = 0.$$

All along, I had been thinking of this as a "quadratic equation in the variable x." But if we flip our perspective and view x as a constant and B and C as variables, we see that the equation describes a line in the BC-coordinate plane!

That is, if we fix x, the points (B, C) in the plane of quadratics that satisfy $x^2 + Bx + C = 0$ comprise a line with slope $-x$ and vertical intercept $-x^2$. All of the points (B, C) on this line correspond to quadratic equations that have this fixed value of x as a root.

Since it is so customary to think of "x" as a variable, I'll relabel using r for x in the previous paragraph and restate it in order to avoid confusion: the equation $r^2 + Br + C = 0$ represents a line in the BC-coordinate plane with slope $-r$ and vertical intercept $-r^2$. Every point (B, C) on this line corresponds to a quadratic function $x^2 + Bx + C$ that has r as a root. (As a reminder, the "x" is no longer the fixed root of the previous paragraph. Here, "x" is once again regarded as a variable, and "r" denotes the fixed root.)

Of all the quadratics that have r as a root, there is the special quadratic $(x - r)^2 = x^2 - 2rx + r^2$, which has only the one root r. We saw earlier that quadratics that have just one root are exactly the ones represented by points on the blue parabola. That means the line of quadratics that have r as a root *intersects the blue parabola* at the point $(-2r, r^2)$. And since a quadratic cannot have both a real root and a non-real root, the line of quadratics that have r as a root must be situated entirely outside of the red zone. Thus, the line of quadratics that have r as a root is *tangent* to the blue parabola. In summary, the roots of the quadratic $x^2 + Bx + C$ are exactly the negatives of the slopes of the lines through (B, C) that are tangent to the blue parabola, and tangent lines to the blue parabola represent loci of quadratics that share a common root.



The negatives of the slopes of the lines through Q that are tangent to the blue parabola are the roots of the quadratic represented by Q. The quadratic represented by P only has one real root.

If the point (B, C) is in the green zone, there will be two different lines through it that are tangent to the blue parabola, each representing one of the two roots of $x^2 + Bx + C$. If (B, C) is on the blue parabola, then there is only one line through (B, C) tangent to the blue parabola, and the negative of its slope is the one root of $x^2 + Bx + C$. And if (B, C) is in the red zone, no line through (B, C) will be tangent to the blue parabola.

Vertical and Horizontal Translation

Note that vertically translating the parabola $x^2 + Bx + C$ corresponds to adding or subtracting a constant from it; in the plane of quadratics, this corresponds to moving up and down the vertical line through (B, C). The fact that precisely one of these vertical translates will have one root corresponds to the fact that every vertical line in the plane of quadratics intersects the blue parabola in one point.

However, translating the parabola $x^2 + Bx + c$ horizontally cannot, in general, correspond to moving horizontally through (B, C) in the plane of quadratics. After all, horizontal translation of a parabola does not change the number of its real roots. But in the plane of quadratics, horizontal lines above the B-axis pass through both the red and green regions and the blue parabola. So what curve of quadratics is traced out in the plane of quadratics by horizontal translation of a parabola?

Let's fix a parabola $x^2 + B_0x + C_0$. Horizontal translation by t units to the right corresponds to moving $x^2 + B_0x + C_0$ to

$$(x + t)^2 + B_0(x + t) + C_0.$$

If we expand and gather like terms, we get $x^2 + (B_0 + 2t)x + (t^2 + B_0t + C_0)$. Thus, in the plane of quadratics, horizontal translation by t units to the right corresponds to moving the point (B_0, C_0) to the point $(B_0 + 2t, t^2 + B_0t + C_0)$. As t varies, the points $(B_0 + 2t, t^2 + B_0t + C_0)$ trace out the curve that represents horizontal translates of the parabola $x^2 + B_0x + C_0$. When a curve is given by specifying the various points on the curve as a function of a variable t, the curve is said to be **parameterized** by t, and t is called the **parameter**.

We can exhibit the parameterization of the curve as the graph of an equation by setting

$$B = B_0 + 2t$$
$$C = t^2 + B_0t + C_0$$

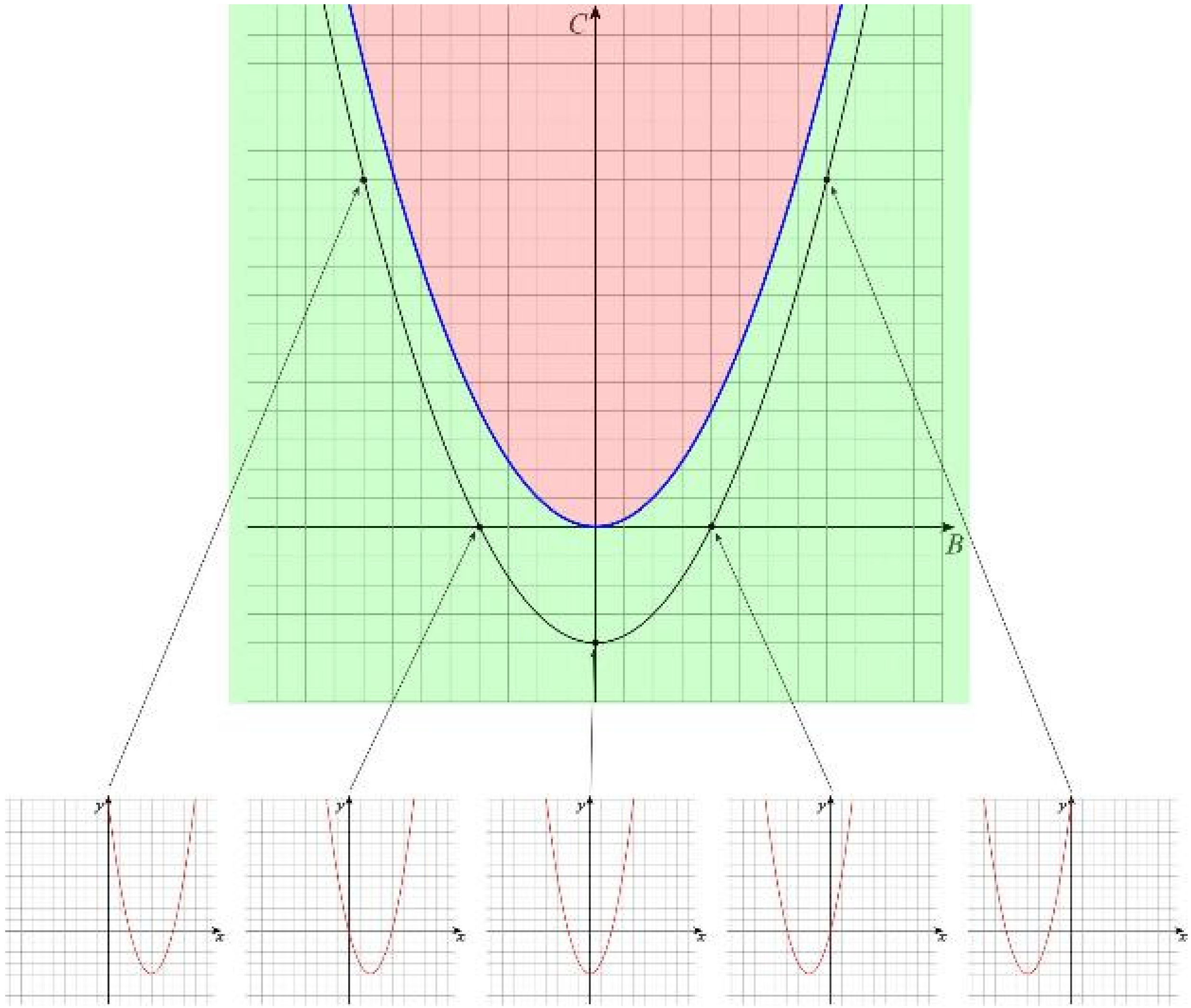
and eliminating t. To eliminate t, we can solve for t in the first equation, and substitute the result into the second equation. From the first equation, we find $t = (B - B_0)/2$. We substitute this into the second equation and, after some algebraic manipulation, we find

$$C = B^2/4 + (C_0 - B_0^2/4).$$

Do you recognize the graph of this equation? It's a vertical translate of the blue parabola!

Thus, in the plane of quadratics, horizontal translation of a parabola corresponds to traveling along a vertical translate of the blue parabola. Isn't it satisfying how this picture neatly accommodates the fact that horizontal translation doesn't change the number of real roots?

The Plane of Quadratics



$(x - 4)^2 - 4$ $(x - 2)^2 - 4$ $x^2 - 4$ $(x + 2)^2 - 4$ $(x + 4)^2 - 4$

In the plane of quadratics, horizontal translation of a parabola corresponds to travel along a vertical translate of the blue parabola. Notice that horizontal translation of the parabola to the left corresponds to rightward motion in the plane of quadratics.

Recall that "completing the square" corresponds to horizontally translating a parabola so that it becomes symmetric about the vertical axis. In our plane of quadratics, completing the square corresponds to moving from (B_0, C_0) to the apex of the parabola $C = B^2/4 + (C_0 - B_0^2/4)$.

Further Questions

In the plane of quadratics, how are the quadratics that correspond to points on a horizontal line related?

Can you devise a picture analogous to the plane of quadratics for cubic polynomials? (For a small hint, take a look at the picture on the cover.)

A Self-Referential True/False Quiz

by Ken Fan

Can you get a perfect score on this True/False quiz? Good luck!

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Notation

by Ken Fan

Good notation facilitates communication. To learn notation, use it. Practice makes perfect!

Subscripts

At the club, some members have been exploring properties of Fibonacci numbers. The Fibonacci numbers are a sequence of numbers that begin with two 1s. After that, numbers can be found by adding up the previous two numbers in the sequence. One finds

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, \ldots$$

If you play with Fibonacci numbers a lot, you'll soon find that you'd like a way to refer to particular Fibonacci numbers in the sequence without having to write, "the fifth Fibonacci number" or "the twelfth Fibonacci number" or "the hundredth Fibonacci number". You could declare that F is the hundredth Fibonacci number, and then each time you had to refer to that Fibonacci number, you could then just write "F". You could introduce new labels each time you had to repeatedly refer to other Fibonacci numbers: let G be the two-hundredth Fibonacci number, let H be the three-hundredth Fibonacci number, and so on. But if you had to refer to many of them in this way, you would soon run out of symbols.

What's more, it is often useful to refer to consecutive terms in the sequence. For example, here's a beautiful fact about Fibonacci numbers: the product of consecutive terms is equal to the sum of the squares of all the terms up to and including the first term in the product. Wouldn't it be nice to have a succinct way to state this fact? How can that be done?

Subscripts are a solution!

Instead of introducing symbols for each term in the Fibonacci sequence as we need them, we think of one symbol for the whole sequence and decorate that symbol with a small number written below and to its right, like this: F_3. The little number is called a **subscript** and tells us which term of the sequence we are referring to. Thus, we might declare, "Let F_n, $n = 1, 2, 3, \ldots$, be the Fibonacci sequence." Then, F_{100} refers to the one-hundredth Fibonacci number and F_{200} refers to the two-hundredth. We can refer to any term in the Fibonacci sequence we wish without fear of running out of symbols! Notice that we can refer to a generic term in the sequence by replacing the subscripted number with a variable: F_n is the nth Fibonacci number.

With this notation, we can write the recurrence relation that defines the Fibonacci numbers like so: $F_{n+1} = F_n + F_{n-1}$, for $n > 1$. The sum of squares observation can be written:

$$F_n F_{n+1} = F_1^2 + F_2^2 + F_3^2 + \ldots + F_n^2.$$

Convenient, isn't it?

Get the hang of subscripts by expressing your thoughts concerning your favorite sequence using subscript notation.

Area Area, continued
Hints and Solutions

by Ken Fan

Here are some hints and solutions to the area problem on page 9.

First, some hints:

1. Regular hexagons have many symmetries and nice angles. What can you say about the angles and triangles in this figure?

2. Can you build a figure that relates nicely to the given information and whose area relates to the given triangle in a way that can be readily seen?

3. What's the difference between the two parabolas?

4. How might you find the area of regions on the lateral surface of a cylinder in general?

Next, some bigger hints:

1. The ratio of the areas of triangles with the same height but different bases is equal to the ratio of the lengths of their bases.

2. Two congruent triangles can be fit together to form a parallelogram.

3. Does the volume of a stack of coins change if the coins in the stack are shifted horizontally in a haphazard manner?

4. What is the shape of the desired surface if the cylinder is unrolled and made flat?

Finally, the answers:

1. The area of triangle CDP is 5 square units.

2. The area of the triangle is $\frac{3}{2}m_1m_2\sin\theta$.

3. The green region has area $d^3/4$. (The region has the same area as a right triangle with legs of length d^2 and $d/2$.)

4. The defined surface has area $7\pi^2/4$. (Unfurled, the surface becomes a square with a tip of area 1/8 the area of the square clipped off.)

Notes from the Club

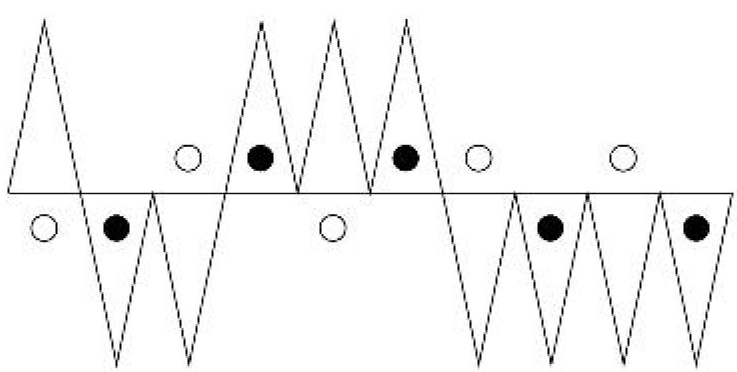
These notes cover some of what happened at Girls' Angle meets. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material. Less than 5% of what happens at the club is revealed here.

Session 20 - Meet 1
January 26, 2017

Mentors: Jessica Dyer, Anna Ellison, Neslly Estrada, Isabel Macenka, Jennifer Matthews, Suzanne O'Meara, Christine Soh, Isabel Vogt, Jane Wang

We began our first meet with a quick ice-breaker activity. We asked members to sort themselves according to various categories, such as by middle name. This forced members to learn more about each other. When asked to sort themselves by birthday, members asked if they could increase the challenge and accomplish the task without anybody speaking. Amazingly, they succeeded!

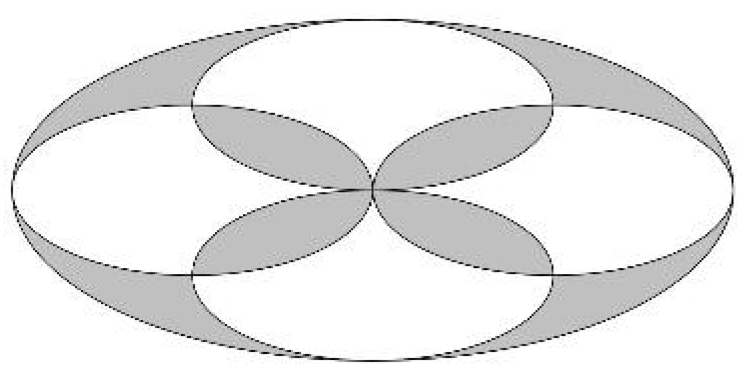
Later, a group of girls played "Describe This Drawing," which is a game we created some years ago and have been playing off and on ever since. In each round of this game, one of the participants serves as the "Describer" and the others serve as "Drawers". The Describer is given a drawing to study for a minute. After a minute, the drawing is taken away from the Describer and the Describer, facing away from the Drawers, has 2 minutes to verbally explain the drawing with the goal of getting at least one of the Drawers to recreate the drawing up to scale. The Drawers are not allowed to speak. They may only draw. The Describer may choose to cut short her 2 minutes of explanation time. If any Drawer has succeeded in replicating the drawing up to scale at the end of the explanation time, that is a win for the group.



Describe This Drawing is about pattern recognition, articulation, and attention to detail.

For the drawing on the left, the Describer might begin by noting a sequence of congruent isosceles triangles that follow the pattern 1 above, 2 below, 3 above, 4 below, and that the open circles and filled circles alternate.

For the drawing on the right, the Describer might start by describing a large circle and 4 circles of half the diameter, each internally tangent to the larger circle and with their points of tangency at the four primary compass points, then direct the Drawers to squash the drawing vertically by a half.



We recommend rotating systematically through the group of participants to select the Describer for each round. Our mentors were responsible for creating drawings for a Describer of a complexity commensurate with the Describer's abilities. For a young Describer, it might be suitable to use a stick figure with a few unusual characteristics, such as a square and triangle for eyes. For seasoned Describers, an intricate "Proof Without Words" might be amusing.

The purpose of this game is to give participants practice in organizing information for precise retrieval as well as to develop more ways to be articulate about drawings, which generally includes developing geometric vocabulary. For example, depending on the drawing, participants can learn more about symmetry and coordinate geometry.

Session 20 - Meet 2 February 2, 2017	Mentors:	Bridget Bassi, Anna Ellison, Isabel Macenka, Suzanne O'Meara, Christine Soh, Isabel Vogt, Jane Wang

At the first meet, **Allie** asked how one could tell if a given number is a Fibonacci number. We created a station to explore this question.

We also created a station to develop the connection between algebra and geometry. At this station, the mentor began by announcing a geometric shape. To illustrate, we'll use a circle. The mentor then asked participants to describe properties of the shape. For a circle, participants might say, "it has rotational symmetry," "it has mirror symmetry," "it's the locus of points equidistant from a given point," and "lines can intersect it in 0, 1, or 2 points."

After collecting a number of properties, the mentor then writes down an equation whose zeroes correspond to an example of the geometric shape in the coordinate plane. For a circle, the equation could be $x^2 + 4x + y^2 - 21 = 0$.

The next task for the participants is to go through each of the properties they came up with and verify that each property is satisfied by the graph of the equation.

Later, we'll give participants an algebraic equation and ask them to determine the geometric properties of its graph.

Session 20 - Meet 3 February 9, 2017	Snowed out!

Meet 3 was cancelled due to a snowstorm.

Session 20 - Meet 4 February 16, 2017	Mentors:	Bridget Bassi, Sarah Coleman, Anna Ellison, Neslly Estrada, Jennifer Matthews, Suzanne O'Meara, Jane Wang

Learning how to handle the general case and not be confined to specifics is an underlying current that we find ourselves riding over and over again in different guises. It involves a basic kind of pattern recognition, together with learning tools and language skills to articulate these patterns, such as the notion of a variable or the ability to craft a definition. The payoff comes in understanding the more general truth versus seeing only the specific fact.

Learning language and notation suited to working with infinite sequences is a good place to start. See Notation Station on page 25 for a brief explanation of how subscripts are used to refer to terms of a sequence. Especially, members at this meet who were working on showing that the term by term sum of two sequences that satisfy the Fibonacci recurrence does so too will probably find this idea to be a breath of fresh air!

Calendar

Session 19: (all dates in 2016)

September	15	Start of the nineteenth session!
	22	Jane Kostick, woodworker
	29	
October	6	
	13	
	20	
	27	
November	3	
	10	
	17	
	24	Thanksgiving - No meet
December	1	
	8	

Session 20: (all dates in 2017)

January	26	Start of the twentieth session!
February	2	
	9	
	16	
	23	No meet
March	2	
	9	
	16	
	23	
	30	No meet
April	6	
	13	
	20	No meet
	27	
May	4	

Girls' Angle has been hosting Math Collaborations at schools and libraries. Math Collaborations are fun math events that can be adapted to a variety of group sizes and skill levels. For more information and testimonials, please visit www.girlsangle.org/page/math_collaborations.html.

Girls' Angle can offer custom math classes over the internet for small groups on a wide range of topics. Please inquire for pricing and possibilities. Email: girlsangle@gmail.com.

Girls' Angle: A Math Club for Girls

Membership Application

Note: If you plan to attend the club, you only need to fill out the Club Enrollment Form because all the information here is also on that form.

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address (the Bulletin will be sent to this address):

Email:

Home Phone: ____________________________ Cell Phone: ____________________________

Personal Statement (optional, but strongly encouraged!): Please tell us about your relationship to mathematics. If you don't like math, what don't you like? If you love math, what do you love? What would you like to get out of a Girls' Angle Membership?

The $50 rate is for US postal addresses only. **For international rates, contact us before applying.**

Please check all that apply:

- ☐ Enclosed is a check for $50 for a 1-year Girls' Angle Membership.
- ☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com.

Girls' Angle

A Math Club for Girls

Girls' Angle
Club Enrollment

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

The club is where our in-person mentoring takes place. At the club, girls work directly with our mentors and members of our Support Network. To join, please fill out and return the Club Enrollment form. Girls' Angle Members receive a significant discount on club attendance fees.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors strive to get members to do math through inspiration and not assignment. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-12. We welcome *all girls* (in grades 5-12) regardless of perceived mathematical ability. There is no entrance test. Whether you love math or suffer from math anxiety, math is worth studying.

How do I enroll? You can enroll by filling out and returning the Club Enrollment form.

How do I pay? The cost is $20/meet for members and $30/meet for nonmembers. Members get an additional 10% discount if they pay in advance for all 12 meets in a session. Girls are welcome to join at any time. The program is individually focused, so the concept of "catching up with the group" doesn't apply.

Where are Girls' Angle meets held? Girls' Angle meets take place near Kendall Square in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org/page/calendar.html or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Peirce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science, worked in the mathematics educational publishing industry, and taught at HCSSiM. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, lecturer, Harvard University
Julia Elisenda Grigsby, assistant professor of mathematics, Boston College
Kay Kirkpatrick, assistant professor of mathematics, University of Illinois at Urbana-Champaign
Grace Lyo, Instructional Designer, Stanford University
Lauren McGough, graduate student in physics, Princeton Univeresity
Mia Minnes, SEW assistant professor of mathematics, UC San Diego
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, associate professor, The Dartmouth Institute
Kathy Paur, Kiva Systems
Bjorn Poonen, professor of mathematics, MIT
Gigliola Staffilani, professor of mathematics, MIT
Bianca Viray, assistant professor, University of Washington
Karen Willcox, professor of aeronautics and astronautics, MIT
Lauren Williams, associate professor of mathematics, UC Berkeley

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. In this way, we hope to help girls become solvers of the yet unsolved.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: Club Enrollment Form

Applicant's Name: (last) ______________________________ (first) ______________________________

Parents/Guardians: ____

Address: ____ Zip Code: _________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Please fill out the information in this box.

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. Names:

____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about?

____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: Girls roughly in grades 5-12 are welcome. Although we will work hard to include every girl and to communicate with you any issues that may arise, Girls' Angle reserves the discretion to dismiss any girl whose actions are disruptive to club activities.

Personal Statement (optional, but strongly encouraged!): We encourage the participant to fill out the optional personal statement on the next page.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: _______________________

(Parent/Guardian Signature)

Participant Signature: ____

Members: Please choose one.	Nonmembers: Please choose one.
☐ Enclosed is $216 for one session (12 meets)	☐ I will pay on a per meet basis at $30/meet.
☐ I will pay on a per meet basis at $20/meet.	☐ I'm including $50 to become a member, and I have selected an item from the left.

☐ I am making a tax free donation.

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Also, please sign and return the Liability Waiver or bring it with you to the first meet.

Personal Statement (optional, but strongly encouraged!): This is for the club participant only. How would you describe your relationship to mathematics? What would you like to get out of your Girls' Angle club experience? If you don't like math, please tell us why. If you love math, please tell us what you love about it. If you need more space, please attach another sheet.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: ____________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____