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To Foster and Nurture Girls' Interest in Mathematics

Inside: Why Study Math? An interview with Melanie Matchett Wood Astronaut Rotations Number Sequences Origami Boxes and the Pythagorean Theorem Summer Fun National Pi Day otes from the Club

It Figures

# From the Director

I am very excited to report that the Mathematical Sciences Research Institute (MSRI) in Berkeley, California is giving a generous donation to Girls' Angle.

MSRI is one of the premier mathematics research institutes in the world and every year, mathematicians from all over gather there and engage in research. In fact, Girls' Angle advisor Lauren Williams is there right now because they are hosting a program in her specialty: Combinatorial Representation Theory.

Girls' Angle especially thanks Kathleen O'Hara, our contact at MSRI. She was instrumental in getting this donation to us.

Also, Girls' Angle welcomes Grace Lyo to the Advisory Board. Grace is a Moore Instructor in the math department at MIT and has a Ph.D. in mathematics from UC Berkeley.

Math does not stop when the school year stops! So in this issue, we have our first installment of Summer Fun. Summer Fun is a series of problems designed to introduce mathematics through doing instead of telling. All Girls' Angle members are urged to think about these problems and send in questions and solutions to girlsangle@gmail.com. We'll publish correct solutions in future issues of the Bulletin. See page 23 for details.

Ken Fan Founder and Director

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*Girls' Angle thanks the following for their generous contribution:* 

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The Mathematical Sciences Research Institute

### **Girls' Angle Bulletin**

The official magazine of Girls' Angle: A Math Club for girls

### girlsangle@gmail.com

This magazine is published about six times a year by Girls' Angle to communicate with its members and to share ideas and information about mathematics.

Girls' Angle welcomes submissions that pertain to mathematics. Subscriptions to the Girls' Angle Bulletin cost \$20 per year and support club activities.

Editor: C. Kenneth Fan

### **Girls' Angle:** A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and to empower girls to be able to tackle any field no matter the level of mathematical sophistication required.

FOUNDER AND DIRECTOR C. Kenneth Fan

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On the cover: "Labrynth" by origami artist Christine Edison. Photo courtesy of Christine Edison.

# Why Study Math?

The Cat often asks this important question: Why study mathematics?

People study math for very personal reasons, and there are all kinds of reasons why people study math! At Girls' Angle, we try to show you many reasons to study math. All the mentors enjoy studying math, so each can offer you their own reason, just ask! The women in the Girls' Angle support network each show ways that they use math in their work and each such application gives another reason to study math. Often, people study math for multiple reasons.

In the first issue of the Bulletin, I pointed out that studying mathematics is an ideal arena in which to improve one's ability to think. This is true because no other subject takes such care to define its objects and make evident the reasoning behind its arguments. Errors in thinking eventually reveal themselves through contradictions. In fact, the main aim of the mentors at Girls' Angle is to help members improve the way they think. The ability to think clearly and deeply is useful in all aspects of life. It saves you time and empowers you to figure out how to more effectively accomplish things.

For example, when I was a graduate student, we got a call from the police department. They were organizing some kind of drill. Unfortunately, I can't remember the specifics because this was some time ago, but the problem went something like this: Twenty police officers had to be divided into 4 teams with 5 people per team for a drill. The drill had to be repeated 5 times with different team compositions each time so that no two people were on the same team for more than one drill. They kept trying to find a way to do this before they finally decided to call the MIT mathematics department for help!

Being trained in mathematics, a first question that comes to mind in this situation is, is it even possible? If you are one of the police officers taking part in the drills and each time, you worked with a different group of 4 other police officers, then after 5 drills, you will have worked with  $4 \times 5$  or 20 different police officers. But, being one of twenty police officers, there are only 19 other police officers to work with in total. So what they hoped for was impossible!

Now, I am not saying that in order to see this sort of thing, you *have* to study math! I'm sure that if the thought occurred to the police to question their assumption of possibility, they would have realized the impossibility. But if you study math, you'll be more prepared to notice such things.

As society becomes more and more reliant on digital information, math's importance to society grows. The fact that I can write truthfully that if you are able to figure out a fast way to factor huge, three hundred digit numbers, you will be able to break into almost every bank account on the planet surely underscores how important mathematics has become!

With the right attitude, studying mathematics can be a lot of fun too. It's no fun if you feel that you must know everything! But if you are ok with not knowing all the answers, yet enjoy the *process* of figuring things out, then math will reward you with endless entertainment. Promise me one thing: If you ever wonder, "what's the point of this math?", before you push it aside, grab the nearest mentor and ask her!

I do not know what I may appear to the world; but to myself I seem to have been only like a boy, playing on the seashore and diverting myself and now and then finding a smoother pebble or a prettier seashell than ordinary, while the great ocean of truth lay all undiscovered before me.

-Sir Isaac Newton

# An Interview with Melanie Matchett Wood

Melanie Matchett Wood is a graduate student in mathematics at Princeton, one of the elite graduate programs in the world. Melanie has already made new contributions to mathematics. Her first paper appeared in the Journal of Number Theory in March of 2003.

**Ken**: Hi Melanie, thank you for agreeing to do this interview! What is it about being a mathematician that you enjoy?

**Melanie**: I enjoy finding structure and patterns. I enjoy finding reasons that explain those patterns. I enjoy trying to figure out if something that I know is true in one situation could be true in more general situations. This involves a lot of experimentation and playing around, nudging the situation this way or that just a little bit and seeing what happens. It also makes me focus on what is really essential in the reasoning I use to show something is true in the first situation.

I enjoy being a mathematician most when I get to do math with other people–this can mean working together, teaching someone else, or learning from someone else, but it is always a give-and-take two-way process.

**Ken**: Today, mathematics is a male-dominated field. Did this affect you as you were becoming a mathematician and does it affect you today?

**Melanie**: This does affect me in many ways, but yet I can still be a happy, successful mathematician. It affects me in ways that I realize and I'm sure also in ways I don't realize. For example, many people have close friends who have the same job, and many people are also closest friends with others of the same gender. This is much harder if there aren't so many women in my job, but there are certainly some, and I hold very dear the close women mathematician friends I have. This has been true since I was young. I think math would have been more enjoyable for me if I had been around lots of girls who were also doing math when I was young, and even today it is very exciting for me to get to be around a lot of other women mathematicians. I organize a group for the women mathematicians and math students at Princeton, and our activities are inspiring and stimulating for me.

There is also an effect called "stereotype threat". It turns out if you give two randomly selected groups of women a math test, and remind one group of women beforehand that women perform more poorly on the test than men, then the group who was told that will perform more poorly than the other group of women. Of course, the women don't necessarily feel like their performances were hurt by the remarks they heard. So this effect probably happens to me without me knowing it, but it has also been shown you can eliminate this effect by educating people about it, and so I try to be more conscious about my feelings about myself and my math performance to help avoid this stereotype threat.

Ken: How did you get interested in mathematics?

**Melanie**: I liked math in school at a young age, but I also liked everything else in school at that age. I first realized that math was particularly interesting to me when I did the Mathcounts competition in middle school. The problems in Mathcounts were much more interesting than the problems I saw in my math classes, and I had a lot of fun trying to figure them out.

Ken: How is contest mathematics related to research mathematics?

**Melanie**: Most students in middle and high school only have exposure to the math they learn in their math classes. Contest mathematics is much more like research mathematics than the math in those classes. The most important difference is that students have been taught how to solve the problems they are assigned in math class. The idea of (at least good) contest math problems is that students haven't been taught how to solve them ahead of time, and they apply their skills and creativity to a kind of problem they haven't seen before. Research mathematics is solving problems that no one knows how to solve, and so you certainly won't be taught ahead of time how to solve research problems. Of course, this has to be introduced gradually, because for students who have only done math in class before, it can be scary to see a problem that looks totally new.

Also, contest math gives students longer periods of time to work on problems than math classes. This brings it a step towards the way research math works. With contest problems, students progress from spending two minutes on a problem, to spending two hours, and with some contests like the math modeling contest, students can spend days on problems. Eventually, when you are doing research mathematics, you'll spend years working on problems!

A lot of the mathematics content that is part of the "contest curriculum" are basic fundamentals of research mathematics. In preparing for contests, I learned linear algebra, classical number theory, graph theory, generating functions, and a lot of other topics that aren't in traditional math classes but are part of research mathematics.

Of course, none of these features involve the competitive aspect of contests, so one could introduce all of these features of research mathematics without competition, but currently that doesn't happen on a large scale in this country.

**Ken**: In your interview with Math Horizons<sup>1</sup> you mentioned that by taking your time studying mathematics and not rushing to go to graduate school, you gained increased depth of understanding. What do you mean when you talk about a "deeper" understanding?

**Melanie**: A deeper understanding means knowing more things about the same thing as opposed to knowing less but about more things. When you want to solve a problem, you need lots of information about the situation that you are considering, and so without this kind of deep understanding you can't hope to solve many problems.

Ken: What kinds of things do you do when you are trying to prove something?

**Melanie**: I write a lot. I use tons of paper and record all my ideas, thoughts, and attempts. I think about how I have proven similar things and try to use those ideas, and then I read how others have proven similar things and try to use their ideas. I work out small cases to see what is going on, first by hand, and then as necessary by computer. When doing examples by computer, I can do larger examples and thousands of them. I try to think about what makes the problem hard, and what the fundamental jump is going to be to construct a proof. Maybe there are two fundamental jumps (or more–oh no!). Recently, I have been working on trying to prove something, but I

<sup>&</sup>lt;sup>1</sup> See http://www.american.edu/academic.depts/cas/mathstat/Events/wood\_visit/MWoodInterview.pdf.

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don't even know the statement of what I am trying to prove! You see, I have found an algebraic phenomenon that strongly suggests it is a shadow of some nice geometrical fact. [Editor's Note: For an example of an algebraic fact that has a geometric interpretation, see the subsection on perfect squares on page 9.] So I am trying to find and prove the geometrical fact. But in this case, I still don't have a guess at the statement of the geometrical fact. I feel like once I find the statement, it won't be so hard to prove, and so here the hardest part is figuring out what to prove! Actually, it seems like a lot of my work happens this way (this isn't true of other mathematicians necessarily, just for me because of the area I work in). The largest part of my work is figuring out what is true, what the statement I am trying to prove is, and usually once I know it, the proof is much easier than figuring out the statement.

**Ken**: Many of the girls at Girls' Angle have not heard of graduate school. Could you please describe what it is like to be a graduate student at Princeton?

**Melanie**: Princeton has a quite unique graduate program, so I don't think it is very representative of the math graduate school experience. But I'll share some of the things that I think are sufficiently general. Graduate school is a strange blend of being a student and having a job. Unlike college students, graduate students do not pay tuition, and in fact are paid. Graduate students have offices, and I spend the hours of around 9 to 5 weekdays in my office working on my research. During the day, I also go to talks in the department. There are usually around 4 talks a week that are interesting to me here. I sometimes find other students or professors in their offices to ask them questions to talk to them about something I am working on. I spend some of my time learning new areas of math that I think are either interesting or relevant to my research (I hope both!). I spend some time just trying to prove my results, or as I mentioned, state the results I want to prove. I spend some time writing up my results, for myself and eventually for others to read.

Ken: Do you have any advice for girls who aspire to become mathematicians or scientists?

Melanie: I really only know about being a mathematician, so I'll answer about that.

It is important to know that to be a good mathematician, you don't have to love all math, you don't have to love it all the time, and you don't have to love it to the exclusion of other things. For example, I have never liked calculus, which is a subject most advanced students learn at the end of high school. I didn't like it then, and I still don't like it now, and I am on my way to becoming a professional mathematician. There are lots of different types of math, and they are really very different from one another. So if you find that you like algebra and hate geometry (or vice versa), it doesn't mean math isn't for you–it just might mean you'll become an algebraist instead of a geometer.

Also, even in the kind of math you do love, sometimes things can be difficult to learn and sometimes particular problems can be frustrating. In fact, if you never come across difficult material or annoyingly frustrating problems, you maybe should start searching for some more challenging stuff! But when the challenge gets to the point that makes the math less fun, I find the best solution is to talk to other people, especially nice people who know more than you, like the mentors at Girls' Angle!

Finally, there is sometimes a stereotype of a math nerd that is obsessed with math, and doesn't have any other interests or social life (except perhaps for some stereotypically nerdy pursuits).

Mathematics is all about creativity, so you actually have an advantage if you have lots of interests and can think in many different ways. I have always had a lot of hobbies and interests. In college I did at least as much theater as I did math, and many great mathematicians I know are curious and passionate about lots of aspects of life outside of mathematics.

Ken: Thanks Melanie! I hope you'll be able to visit Girls' Angle someday!



Header adapted from photo by Rochus Hess obtained from http://commons.wikimedia.org/wiki/Image:Pleiades-comet-Machholz.jpeg

# Number Sequences

When Tanya Khovanova visited Girls' Angle, she introduced many number sequences.

Where do these sequences come from? What use do they have?

In this article, we'll explore some of these sequences in more detail.

### **The Counting Numbers**

Who discovered them? Or, were they invented?

Though this is a primitive representation of counting numbers, *grouping by five* already exhibits a degree of sophistication!

Counting numbers also provide a convenient way to give each object in a large collection of objects a unique name. For example, when you open a book, typically, each page is given a different page number. Then, if you want to locate a quote, you can refer to the page number of the page that quote is on. How convenient! It helps that counting numbers are ordered in a natural way, but people use number names even when the order doesn't matter. Just look at your social security number!

The notion of counting is so powerful that people seem to want to count just about everything! They count how many times around the sun you've traveled, how many pennies you control, how many houses down the street you live, how many pounds you weigh, how many times your heart beats every minute, how many inches from the ground to the top of your head. There are people that obsess about how many seconds it takes them to run the length of a football field, how many points they got on a chemistry test, how many inches around their biceps measure, and how many miles they get per gallon of gasoline. There are even people who try to count how smart you are! Surely, that is taking counting too far!

Counting can be as easy as one, two, three...or as difficult as counting the number of Sudoku puzzles that have a unique solution, which, according to mathematician Timothy Chow, is unknown.

The notion that counting is so natural and reliable has given rise to idiomatic expressions that relate counting to trustworthiness and accuracy.

"You can count on me!" "Count me in!" "Who's counting?"

If you combine the impulse to count with an insatiable appetite for finding patterns, you will end up with mathematics!

### The Triangular Numbers

The triangular numbers count the number of dots in a triangular arrangement of dots.

Here are the first five terms:



August was quick to see that the differences between consecutive triangular numbers (if we start at zero), yield the counting numbers.

This means that the Nth triangular number is equal to 1 + 2 + 3 + 4 + ... + N, the sum of the first N counting numbers. What is a formula for this sum?

On July 10, 1796, the German mathematician Carl Friedrich Gauss wrote this in his diary:

E Y P H K A num = 
$$\Delta + \Delta + \Delta$$
.

Gauss realized that every positive integer is a sum of at most 3 triangular numbers.

### **The Square Numbers**

• • • • Also known as "perfect squares", the first few terms are: 1, 4, 9, 16, 25, 36, 49, 64, 81,

- ••••• 100, ...
  - • •
    - • They can be defined in a way analogous to the way triangular numbers are defined in
    - terms of counting the number of dots in square arrangements as shown at left for 49 dots.

**Sylvia** and **Ilana** noticed that the perfect squares also count the number of triangles that a triangle can be partitioned into using a number of lines drawn parallel to its sides as illustrated to the right. The big triangle is partitioned into 36 little triangles. The difference between this situation and what was done to get triangular numbers is that here, we are counting the triangles, whereas when we were getting triangular numbers, we were counting the points of intersection. What do you get if you count the number of line segments?



**Sylvia** also noticed that if you take the differences between consecutive perfect squares, you will get the sequence of odd numbers. In other words,  $N^2 = 1 + 3 + 5 + ... + (2N - 1)$ . The pictures show graphic interpretations of this formula. Count the number of triangles or squares in connected bands of the same color.

Now that you've seen triangular numbers and square numbers, do you think you can invent pentagonal and hexagonal numbers?





### A Note on Differences of Consecutive Terms

Notice that for triangular numbers and square numbers, it was fruitful to look at the differences of consecutive terms. This idea is just like the idea we saw when Tamara Awerbuch visited (see Volume 1, Number 2 of this Bulletin). To understand populations, she examined how a population changes in size from one moment to the next.

### The Fibonacci Numbers

The Fibonacci Numbers are named after an Italian mathematician Leonardo Fibonacci who lived eight centuries ago. In 1202, he wrote a book called *Liber Abaci*, which advocated the use of our modern decimal number system. In fact, **Sylvia** often multiplies numbers using the lattice multiplication method introduced to Europe in this very book!

Here are the first few terms: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...

Sometimes, people start the sequence with a zero: 0, 1, 1, 2, 3, 5, ...

Whether you take the first two terms to be 1 and 1 or 0 and 1, successive terms are found by adding the previous two terms.

At the club, we saw how the Fibonacci numbers give the number of paths to various intersections in the following map of one-way streets (see page 13 for a detailed proof):



We also saw how you can use the Fibonacci numbers to make a beautiful spiral of squares:



Amazingly, Fibonacci numbers appear in fascinating places in nature. It is conceivable that a flower lover could discover the Fibonacci sequence!

For example, the flower shown to the right is an *Echinacea purpurea* flower, sometimes called a purple coneflower. Below is a close up looking straight down on the flower. The central part of a coneflower has many tiny yellow florets with purple tips. Look carefully and you will see that the florets are arranged in spirals.





Photos by C. Kenneth Fan

You can see counterclockwise and clockwise spirals. To help you see these spirals, two of them are indicated in the image below, one going clockwise and the other going counterclockwise.



Photo by C. Kenneth Fan

The number of clockwise spirals and the number of counterclockwise spirals are two consecutive Fibonacci numbers! Count them and see for yourself!

The next time you eat a whole artichoke, after you've finished with the leaves, try to remove the artichoke hair very, very carefully to reveal the pattern of pits in the artichoke heart. The number of counterclockwise and clockwise spirals formed by those pits will also yield two consecutive Fibonacci numbers.

The explanation for these appearances of Fibonacci numbers in nature has to do with rational approximations to the golden mean, which is an irrational number. The golden mean is the positive solution to the quadratic equation  $x^2 - x - 1 = 0$ . To fully understand these ideas, you will need to learn about *continued fractions*. Some day, we will explore such things at the club. If you cannot wait to find out about it, you can read about it on your own in books such as *The Higher Arithmetic* by Harold Davenport, which is written for college math majors. To see how all this relates to the spiral patterns in flowers, look up *phyllotaxis*. (There's a beautiful phyllotaxis gallery located at Smith College, the nation's largest liberal arts college for women.)

For now, here's an interesting activity for you. Complete the following table of greatest common factors of pairs of Fibonacci numbers.

Table of Greatest Common Factors of Pairs of Fibonacci Numbers														
$F_m$ $F_n$	1	1	2	3	5	8	13	21	34	55	89	144		
1	1	1	1	1	1	1	1	1	1	1	1	1		
1	1	1	1	1	1	1	1	1	1	1	1	1		
2	1	1	2	1	1	2	1	1	2	1	1	2		
3	1													
5	1													
8	1													
13	1													
21	1											3		
34	1													
55	1													

Do you see any patterns?

Some of the patterns you find may be very difficult to prove, so do not be discouraged if you cannot prove them. In fact, it's good advice not to allow yourself to get discouraged if you cannot solve something. Just put it aside and work on something else. You can always come back to a problem later, because problems do have a nasty habit of not going away on their own! If you want to share your discoveries, even if you haven't proven them, please send them to girlsangle@gmail.com.

# A Proof Illustration: More Fibonacci Numbers

In mathematics, proofs are used to establish the truth of claims. A proof is simply a very clearly spelled out argument. In principle, each step in the argument should be something very easy to verify. We gave an example of a proof in the first issue of this Bulletin. Here's another example.

We're going to *prove* that the numbers of paths to the various intersections in the one-way street map below are really, truly the Fibonacci numbers...no ifs, ands or buts about it!



Many of you were able to assert this fact by directly counting and seeing that the first few numbers were part of the Fibonacci sequence, but stopped short of *proving* the fact. How can you know with *complete certainty* that you get the Fibonacci numbers? If this map were extended so that there were a million vertical streets, are you absolutely, 100% sure that the number of paths to intersections way over there would still be part of the Fibonacci sequence?

By counting the number of paths by hand, many found the answers for a few of the leftmost intersections:



Beyond that, it starts getting pretty tricky to keep track of paths! Even establishing that the 8 was correct wasn't easy! But we could recognize the emergence of the first few Fibonacci numbers and this might inspire us to *conjecture* that we would get the Fibonacci sequence if we continued. But how do we *prove* this conjecture?

At the seventh meet, **Tree** and I worked out a *proof* that enables us to state with absolute confidence that "Yes! They are the Fibonacci numbers!"

Here's how the proof went.

Recall that the Fibonacci numbers go like this: 1, 1, 2, 3, 5, 8, .... That is, they begin with two ones, and then each successive term is the sum of the previous two.

So what we believe is that the number of paths to the various intersections yields the Fibonacci numbers in this manner:



where we are writing  $F_k$  for the *k*th Fibonacci number.

It is here that many stopped working. But in counting the number of paths, say, to the intersection corresponding to  $F_9$ , it took a lot of tries to find 34 paths, and there were so many false starts and different answers too! Sometimes it would seem like there were only 30 paths, other times, 35, other times 25. When there is so much inconsistency in getting the answer, it doesn't give a lot of confidence that the number is really, truly 34.

But if we can *prove* the conjecture, then we can know for sure that  $F_9 = 34$ .

Here, we are trying to show that some sequence of numbers really is the Fibonacci sequence. To do that, the first thing we really must know is, what, exactly, are the Fibonacci numbers? In other words, what is the *definition* of the Fibonacci numbers?

Well, we know that. The Fibonacci numbers are the unique numbers obtained by starting with two ones and then getting each successive number by adding the previous two.

So, here's the idea: To show that the path counting also yields the Fibonacci numbers, we are going to show that the numbers that tell how many paths there are *also obey the defining properties of the Fibonacci numbers*. That is, we are going to show that the number of paths to the first two intersections is one in both cases. Then, we are going to show that for the other intersections, the number of paths *is equal to the sum of the number of paths to the previous two intersections* (using the zigzag order indicated in the diagram above). If we can show these things, that would *prove* that the number of paths does give us the Fibonacci sequence.

Right now, we *strongly suspect* that the number of paths gives the Fibonacci sequence, but we do not know this *for sure*. So let's label the number of paths to the various intersections N<sub>1</sub>, N<sub>2</sub>, N<sub>3</sub>, etc. By using a different symbol, we are emphasizing that we do not yet know with certainty that  $N_k = F_k$  for all positive integers *k*.

Let's show that the number of paths to the first two intersections is 1.

The first intersection is that starting intersection. Once you move from there, you can never get back! So there's only 1 way to stay there, and that's to just stay put. So  $N_1$  does equal  $F_1$ . A good start! (Remember, if you have any questions, please ask at the club or send us email!)

The second intersection is connected to the starting intersection by a single vertical road. Notice that there are no roads that let you move to the left. All the arrows point either straight down or have some movement to the right. So, if you start out by walking to the right from the starting intersection, you will never be able to get to the second intersection. The only way to get to the second intersection is to go straight down at the start. And, once you leave the second intersection following one of the two paths that leave it, you can never return. So there is only one path to the second intersection: go straight down. So  $N_2$  equals  $F_2$ . Progress!

If you're thinking, this is ridiculously easy, that's good! Mathematics is all about trying to make the obscure transparent and easy.

All that remains for us is to show that for the other intersections, the number of paths is equal to the sum of the number of paths to the previous two intersections. So let's go to one of these intersections. Pick any one.

Imagine that you have a list of all the paths to this intersection you've picked. We want to show that the number of paths in this list is equal to the sum of the number of paths to the previous two intersections. You might want to stop and think about how to show this before reading on.

Have you thought about it?

OK...no really...have you thought about it??? Please think about it before reading on!

When I was growing up, people sometimes told me that there's a difference between *active* reading and *passive* reading. Reading this article passively would mean reading it straight through without stopping and thinking along the way. It takes more energy to think actively, but think actively you must!

Wait! Don't tell me that the reason you don't want to think here is because you don't have scratch paper on which to work out your thoughts! Go get some! Try to get in the habit of reading math with a hefty stack of scratch paper and something to write with on hand at all times. No excuses. This time, I'll even give you space below to write down some thoughts.

# Your Thoughts Here.

I hope you just did some good thinking!

If you did, you are likely to find the next passage much easier to read, even if you didn't succeed in figuring out what to do.

OK, we want to show that the number of paths in this list is equal to the sum of the number of paths to the previous two intersections.

Here's an idea: We split the paths in the list into two types and show that the number of one type is the number of paths to the intersection two back and the number of the other type is the number of paths to the intersection one back.

Notice that at the intersection you picked (which shouldn't be one of the first two intersections...we've already dealt with those), there are exactly *two* incoming streets. Every path to your intersection must traverse *one* of these two streets, and no path can traverse both. So if we temporarily label these two incoming streets  $1^{st}$  street and  $2^{nd}$  street, we see that the paths to the intersection you picked can be split into two subsets:

Subset 1: Those paths that traverse  $1^{st}$  street. Subset 2: Those paths that traverse  $2^{nd}$  street.

The total number of paths is the sum of the number of paths in subset 1 plus the number of paths in subset 2.

Any path in subset 1 corresponds to a path to the intersection at the front of  $1^{st}$  street by erasing the part of the path along  $1^{st}$  street. Also, every path to the intersection at the front of  $1^{st}$  street can be converted to a path in subset 1 in only one way: by tacking on the walk along  $1^{st}$  street. *Therefore, the number of paths in subset 1 is equal to the number of paths to the intersection at the front of 1^{st} street.* 

Similarly, the number of paths in subset 2 is equal to the number of paths to the intersection at the front of  $2^{nd}$  street.

Combining the information in the previous three paragraphs, we see that the number of paths to the intersection you picked is equal to the sum of the number of paths to the previous two intersections!

This shows that the path sequence  $N_k$  satisfies the same defining properties as the Fibonacci sequence  $F_k$ , and so they must be one and the same.  $\Box$ 

To summarize, we had a problem where we wanted to show that one thing was the same as another. The way we did it is to show that the one thing has the *same defining properties* as the other thing.

Now, here's a question for you that may help you understand why proofs can be so important.

Consider the polynomial  $n^2 + n + 41$ . Compute the first few values of this polynomial, for n = 1, 2, 3, ... Is  $n^2 + n + 41$  prime for all positive integers n?

# Origami Boxes and the Pythagorean Theorem

by Christine Edison

As a math teacher I have taught students the Pythagorean theorem for years. I always knew it had practical value and even saw it used by carpenters when I volunteered to rebuild homes during a spring break in college, but until I started designing origami boxes I did not use it in my personal life.

The Pythagorean theorem has an amazing history, but that is for someone else to write. Basically it is a relationship that has been known, in some form or another, since at least 2,500 BC. Pythagoras was not the first to know about this relationship, but he wrote it down in a form we know today and gave us a proof that wasn't lost. This theorem is an equation that tells us the



A Rose Box designed and folded by the author.

Photo courtesy of Christine Edison

relationship between the lengths of the legs and hypotenuse of a right triangle. The hypotenuse is the side across from the right angle and is also the longest side. In its simplest form it is "The sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse."



I use this theorem for two things when I fold: to find out the finished width and length for my box given a specific origami square and to find out how big an origami square to use if I want the finished box to have a specific length and width. It gets more complex depending on the box, but the math today is based on a traditional box design called the Masu Box. We start with an origami square and name the side length *s*. Since it is a square the length and width will be the same. If you look at the darker lines in the crease pattern on the next page you can see the right triangle I am looking at to establish a relationship between the size of the original paper and the

width of the finished box. The line segment called g is the length of the finished box. The hypotenuse of the finished box is half the distance of the original side.



This is an origami square with the crease pattern of a Masu Box drawn in. The right isosceles triangle indicated can be used to relate the side length of the square s to the side length of the Masu Box g.

Since we are dealing with a square the right triangle we are dealing with is special. It has two congruent legs and is called an isosceles triangle. Now I substitute the variables for the lengths of the legs and hypotenuse into the Pythagorean theorem:

$$g^2 + g^2 = \left(\frac{s}{2}\right)^2,$$

which can be simplified to

$$8g^2 = s^2$$
.

Now we can go in two separate directions. I can solve for g in terms of s or for s in terms of g.

At the club, each girl will receive a Masu box folded by the author. Christine used origami squares 6

inches on a side. Can you figure

out how big your boxes will be?

Solving for g in terms of s, we find that 
$$g = \sqrt{\frac{s^2}{8}} = \sqrt{\frac{1}{8}s} = \frac{\sqrt{8}}{8}s = \frac{\sqrt{2}}{4}s$$
.

With this equation I can find the length and width of the finished box given the dimensions of the original square. For example if I gave you a 12 inch by 12 inch square you would substitute s = 12 into the equation and find that  $g \approx 4.24$  inches, which means that the length and width of the final box would be about 4.24 inches. Notice that g represents the length of something real, and so it cannot be negative. This means we only need to use the positive, or principal, square root.

Solving for *s* in terms of *g*, we find that  $s = \sqrt{8g}$ .

If I want a box of a specific length and width I substitute in this desired length for g into this equation and can find out how big my origami square needs to be before I start folding.

The more I designed boxes the more complex they got, but the basic principal never changed. The trick always is finding the relationship of the side length to the hypotenuse. In the Masu box it is a simple relationship as the hypotenuse is half of the side length. The rose box is an example of how the crease pattern starts to get more complex. Without knowing the Pythagorean theorem, I would be stuck with finding dimensions by trial and error. With the Pythagorean theorem, I know my dimensions every time.

### Rose Box - Christine Edison, November, 2006



variation: crease various twentieths. The height of the box will increase.



Use a Gauge Sheet for the creases at 2/5=4/10=8/20, then fold the edges in for 3/10=6/20. The center line is at 5/10=10/20, and now the creases at 7/20 and 9/20 are easy.

Go to http://www.flickr.com/photos/christine42/306063819/in/set-72157594252973513/ for more detailed instructions and to see more of Christine's origami art.

### Iore Astronaut Rotations by Leia Stirling Rotation about the y-axis 1. From the desired neutral position, move the limbs such that the arms are equally inclined from y. The legs can be outstretched or tucked. 2. The arms are rotated about y until the desired net rotation is obtained. 3. The limbs are returned to the chosen neutral position. 1. From the desired neutral position, move the limbs such that the arms are next to the body and the legs are bent such as when on a bicycle. 2. The legs are then pedaled about the offset y until the desired net rotation is obtained. 3. The limbs are returned to the chosen neutral position. 1. From the desired neutral position, move the limbs such that the arms inclined from z. The legs can be outstretched or tucked. 2. The arms are rotated about the offset z in the same direction until the desired net rotation is obtained. 3. The limbs are returned to the chosen neutral position. 1. From the desired neutral position, move the limbs so that the arms are Rotation about the z-axis next to the body and the legs are at an angle from the body's z-axis as shown. 2. The legs are then rotated about the offset z, such that one leg is 180 degrees out of phase with the other, until the desired net rotation is obtained. 3. The limbs are returned to the chosen neutral position. ▼× (2)(5) (3 (1)1. In the initial stage, the body is straight, arms down, and legs spread to the side. 2. The torso is twisted about the axis of intended rotation. 3. The moment of inertia is increased at the top of the body by spreading the arms and decreased in the lower part of the body by closing the legs. 4. In the final stage, the body is untwisted at the waist and the arms are lowered.

Header adapted from NASA photo obtained from http://commons.wikimedia.org/wiki/Image:Ssc2007-10a1.jpg

# National Pi Day

March 14 is National Pi Day. In mathematics, the Greek letter pi, written  $\pi$ , has come to be reserved for the unique constant that is the ratio of the circumference of a circle to its diameter. Implicit in this definition is the observation that *all* circles are similar. Recall that in similar figures, the ratios of corresponding lengths are all given by a single number called the *similarity constant* or the *dilation factor*.

One way to understand similarity is to imagine that you are watching your friend walk away from you. From your point of view, her apparent size will get smaller and smaller. If you take several photos of her as she walks into the distance using the same camera settings (no automatic zoom, please!), she will appear smaller and smaller in each successive image.

But even though she appears smaller and smaller, all of her proportions will be the same in every picture. The ratio of her height to the length of one of her forearms will be the same no matter in which photo you make the measurements. Perhaps in one photo, she will be 3.5 inches tall and her forearm 0.5 inches long. The ratio is 3.5:0.5 or 7:1. In another photo, if she is 1 inch tall, then her forearm will measure one seventh of an inch in that photo. (If you actually do this, you will have to take care to account for foreshortening.)

You can probably imagine that any two circles could be thought of as images of the same circle viewed from different distances. That is because any two circles are similar to each other, and so the ratio of any two corresponding lengths will be the same. This implies that the ratio of the circumference to the diameter of any circle will be the same for all circles, and this number is what is known as  $\pi$ .

To celebrate National Pi Day, one demonstration went like this: Line up three cups with circular mouths in a row so that each is touching the next. Stretch a string across all three cups, end to end, and cut. Question: will the string be able to wrap all the way around the mouth of one of the cups?



At first blush, many people think the string is long enough. But it isn't, because  $\pi > 3$ .

Computing the value of  $\pi$  ever more accurately is important in engineering applications that demand higher and higher precision. The Greek mathematician Archimedes determined that  $\pi < 22/7$ . Sir Isaac Newton dashed off a computation of  $\pi$  accurate to 15 decimal places. Today, there's some kind of horse race among some computer programmers and mathematicians who want to hold the record for the most digits of  $\pi$  computed. Currently, the record stands at some one trillion decimal digits of  $\pi$ .

For another demonstration, cut a circle into 32 identical sectors. Arrange these sectors to form an almost rectangular shape whose area can be estimated using the formula for the area of a rectangle and estimates for the rectangle's length and width. As the number of sectors used increases, this estimate increases in accuracy and makes it seem quite plausible that the area of a circle is given by  $\pi r^2$ , where r is the radius of the circle.



Because the figure to the right is just a rearrangement of pieces that make a circle, the two shapes have the same area. The figure to the right is not exactly a rectangle. It's not even a parallelogram because two of its sides consist of lots of little arcs. But, it is pretty close to rectangular, and it is pretty close in size to a rectangle with width r and length half the circumference of the circle. Because the circumference has length  $2\pi r$  (by definition!), the figure to the right has area approximately given by  $r \cdot \pi r$  or  $\pi r^2$ .

To make this argument rigorous, one must increase the number of sectors systematically (say, by doubling their number each time) and find explicit formulas for upper and lower bounds of the area. Alternatively, one can employ the powerful techniques of calculus. Perhaps on National Pi Day a couple of years from now, we will talk about this.

Notice that the fact that the circumference of a circle has length given by pi times the diameter is the *definition* of pi. This definition exploits the fact that all circles are similar to each other. However, the fact that the area is given by  $\pi r^2$  is not a definition. It is a theorem that must be deduced and proven.

We'll end this article with a few facts that point to a mysterious connection...

Draw a line segment. We're going to make this line segment the base of a triangle. If you draw a point that represents the apex of the triangle, the triangle consists of all the line segments from this apex to the points on the base. If the length of the base is *b*, the area of the triangle is  $\frac{1}{2}bh$ , where *h* is the height of the triangle relative to that base.

Now draw a circle. The full circular disk consists of all the line segments from the center of the circle to the points on the circumference. If the circumference has length *C*, the area of the circle is  $\frac{1}{2}Cr$ , where *r* is the radius of the circle.

Analogously, the volume of a pyramid with height *h* and base area *B* is  $\frac{1}{3}Bh$ , whereas the volume of a sphere with radius *r* and surface area *S* is ..., well, what do *you* think it is?

For more information, check out Petr Beckmann's book A History of Pi.

# Summer Funl

The best way to learn math is to do math!

We've made a bunch of fun problem sets for you to work on over the summer.

We invite Girls' Angle members and subscribers to the Bulletin to send any questions and solutions to girlsangle@gmail.com. We'll give you feedback and put your solutions in the Bulletin!

Take a quick look at all the problems soon so that if you have any questions you can ask the mentors on May 1 or May 8. Remember, May 8 is the last meet before we break for the summer!

In the June issue of the Bulletin, we will publish solutions to the first problem of each set and give hints for the second problem. In the August issue, we will give complete solutions. You could wait until the August issue to see the answers, but you will learn a lot more if you try to solve these problems before seeing solutions.

By the way, some of these problems are going to be very unlike those you will find at school. Usually, problems that you get at school are readily solvable. However, some of these problems are not meant to be solved immediately. In her interview, Melanie pointed out that mathematicians sometimes take years to solve problems! They wouldn't take years to solve a problem if they could solve it in a few hours.

If you are used to solving problems quickly, it can feel frustrating at first to work on problems that take years to solve. I've felt this frustration. But now, I long for problems that take years to solve! There's something about the journey that is very enjoyable. It's like hiking up a mountain or rock climbing. Getting to the top rewards one with a spectacular view, but during the journey, there's a lot to see and experience. So there's a meta-problem for those of you who feel frustrated at times doing these problems: see if you can dissolve that frustration and replace it with a relaxed, optimistic sense of adventure!

This is Summer Fun, not Summer Torture!



# Parity: Are you Even or Odd?

by Ken Fan

In this problem set, when I say "number", I mean an integer, that is, a counting number, the negative of a counting number, or zero.

1. We've encountered parity many times at the club. Just knowing whether a number is even or odd can often be powerful information! Recall that a number is even if and only if it is divisible by 2. Otherwise, it is odd.

- a. Write down the first 10 (positive) even numbers and the first 10 (positive) odd numbers.
- b. Is zero even or odd?
- c. Which prime numbers are even? Which prime numbers are odd?

2. When you have a concept that pertains to numbers, it is often a good idea to see how that concept relates to basic number operations, such as addition and multiplication. Let N and M be two numbers.

a. Complete this table:

Table of Parities								
N	М	N + M	N-M	$N \times M$	$N^2$	$N^3$		
even	even	even						
even	odd							
odd	even							
odd	odd							

- b. Is it possible for 2N = 2M + 1?
- c. Can you show that  $N^2 + N$  is always even?
- 3. How does parity relate to the Fibonacci numbers? To the triangular numbers?

4. Here's another classic problem that involves parity. One hundred lockers are lined up in a row. They are numbered one through one hundred and are all closed. One hundred people pass by this row of lockers. As the first person passes by, she opens any closed locker and closes any open locker. As the second person passes by, she only pays attention to the even numbered lockers, again opening closed lockers and closing open lockers. The third person does the same, only paying attention to those lockers whose number is a multiple of three. In general, the Nth person goes to the lockers numbered by a multiple of N, closing open lockers and opening closed lockers. After all 100 people have passed by the lockers, which lockers end up open?



# Pascal's Triangle and Binomial Coefficients

by Elisenda Grigsby

Suppose that x and y are variables. We can manipulate the symbols without having to know what numbers they represent. For example, we can write "x + y", and it means "add x and y". If x represents 2, and y represents 3, then "x + y" represents "5". The point is that the expression "x + y" itself doesn't depend on what numbers x and y actually represent, and we can think about what we can say about these expressions in general, without worrying about some particular choice of numbers for x and y.

As an example, let's consider the following question: Let x and y be numbers. What can we say about the expression  $(x + y)^n$ , when n = 0, 1, 2, 3, ...?

So,  $(x + y)^0 = 1$ ; (the result of multiplying a number by itself 0 times is usually defined to be 1)  $(x + y)^1 = x + y$ ;  $(x + y)^2 = (x + y)(x + y)$ ;  $(x + y)^3 = (x + y)(x + y)(x + y)$ ; etc....

1. Show that  $(x + y)^2 = x^2 + 2xy + y^2$  and  $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ . What is  $(x + y)^4$ ? (Hint: Use the distributive property of numbers: (x + y)(z) = xz + yz, for all numbers x, y, and z. Also, remember that multiplying two numbers doesn't depend on the order in which we multiply them. So, if x and y are two numbers, then xy = yx.)

2. Consider the collection of numbers, arranged into rows, shown at right. Can you see a pattern? Can you fill in the next row? Can you state precisely how each row is obtained from the row above it? (Assume that the top row is given.)
1 1 1 2 1 1 3 3 1 1 4 6 4 1 1 5 10 10 5 1

3. This collection of numbers is called *Pascal's triangle* after the French mathematician Blaise Pascal. Do you see a relationship between the rows of Pascal's triangle and the expressions you were writing down in Problem 1? Can you explain why this relationship exists?

4. Why is Pascal's triangle symmetric across the vertical line through the center of the triangle? In other words, notice that the numbers on a row to the right of the vertical line are the same as the numbers to the left. Can you explain why this is the case?

5. (Bonus challenge!) It's standard to call the top row of Pascal's triangle row zero and the leftmost number in each row the zero-eth number (instead of the first). Can you show that the *k*th entry in the *n*th row of Pascal's triangle is the number of different *k*-element subsets that can be formed from a set with *n* elements? As an example, suppose we want to see how many different 3-element subsets we can form from the set  $\{1, 2, 3, 4\}$ . We see that there are exactly four:  $\{1, 2, 3\}$ ,  $\{1, 2, 4\}$ ,  $\{1, 3, 4\}$  and  $\{2, 3, 4\}$ . And if we look at the third number in row four, it is indeed four! (Remember to start your count from zero!)



# Slope Problem Set I

by Lauren McGough

For this problem set, we're going to be dealing a lot with lines. If you think about a straight line on a coordinate plane where the horizontal and the vertical directions are defined, it has a certain steepness associated with it. Maybe it is completely parallel to the horizontal, in this direction: – —. Or maybe it is parallel to the vertical, in this direction: |. Or maybe it makes some angle with the horizontal, like this: /, or this: \. Each of these lines has a different "steepness" associated with it. There is a number we use to measure this property of steepness: it's called *slope*. The slope of a line is just the ratio of the amount the line goes up for every unit it goes over. We can measure slope just by taking two points on a line, and calculating the change in the vertical direction divided by the change in the horizontal direction. First, let's make sense of this definition!

1. The first question here is: draw some lines, and calculate their slopes using a few sets of different points. Is the slope always the same no matter what points you use? Why or why not? (We hope it is, because otherwise, the definition of the "slope of a line" doesn't make sense– the line could have a different slope at every point!)

2. Now that we've hopefully found that the definition of slope makes sense, let's see why this quantity actually measures the "steepness" mentioned before:

- a. What does a line of slope 1 look like?
- b. How about a line of slope 0? What does this look like?
- c. What is the slope of a vertical line?
- d. What does a line with negative slope look like?
- e. Draw lines with slopes of 0.5, 1, 5, 0, -0.5, -1, and -5. What does it look like when one line has a more positive slope than another line? How about a line with a more negative slope than another line? What do lines with slopes that are less than 1 and greater than -1 look like?

3. Let's assume that we're working with a specific coordinate plane. For now, let's specify points on this coordinate plane as (a, b), where a is the horizontal distance of the point from a specific vertical line, and b is the vertical distance of the point from a specific horizontal line (these specific lines are called our *axes*). How many lines of a specific slope s, where s is any real number, is it possible to draw on this coordinate plane? Do you notice anything special about all lines of a specific slope?

3. Try drawing pairs of lines that make right angles with each other, and measuring their slopes. Lines that make right angles like this are called "perpendicular." Do you notice anything special about the relationship between the slopes of lines that are perpendicular?

# Slope Problem Set II

by Lauren McGough

Let's continue using the same set up of Slope Problem Set I.

1. Sometimes, people like to express all of the points on a line using an equation that relates a and b for all points (a, b) on the line. Consider a line of slope 5 that goes through the intersection of the two axes on the plane– that is, through the "origin". Can you think of a relationship that all of the points (a, b) satisfy– that is, can you write an equation using a, b and the slope of the line such that if a and b satisfy the equation, then (a, b) is a point on the line and vice versa?

2. Can you generalize the equation you found in problem 1? So, say that we have a line of slope m that also passes through the origin. Can you write an equation involving a, b and m such that if a and b satisfy the equation, then (a, b) is a point on the line and vice versa?

3. What does an equation for a horizontal line (slope 0) look like, using the generalization you found in problem 2? What happens in the case of a vertical line?

4. So far, we've been using a coordinate system that relies on distances of points from specific horizontal and vertical axes. However, we've talked about other coordinate systems before! Do you remember the coordinate system where points were specified by  $(r, \theta)$ , where *r* is the point's distance from a special point called the "origin", and  $\theta$  is the angle the line connecting the point to the origin makes with the horizontal ray pointing from the origin to the right? Consider all of the lines that go through the origin in this type of coordinate system. Say that you have a specific line that goes through the origin in mind. How could you use an equation that uses r and/or  $\theta$  in order to tell me exactly what line you have in mind?



5. So far, we've been dealing with straight lines, which go off to infinity in the same direction



forever. The "slope" of such a straight line defines how "quickly" it rises or falls, in a sense. What about for a curve? Curves rise and fall, too, though they don't necessarily always rise or fall at the same rate as you move along it. How would you express the steepness, or slope, of a curve at a point? Would your definition result in the same slope at all points of the curve? How could you calculate the slope of a curve at a point using your definition?



# Origami Math

by Ken Fan

roughly as shown.

Origami artists start with square pieces of paper and fold them into works of art. There is quite a bit of mathematics related to origami. Christine's article (page 17) gives an example and this problem set introduces some others.

- 1. Take an origami square.
  - a. If you fold the square in half along a crease parallel to a side, what is the resulting shape?
  - b. If you fold the square in half along a diagonal, what is the resulting shape?
  - c. If you fold an origami square in half repeatedly a total of *n* times, how many layers of paper will there be?

2. Because an origami square starts out as a square, you don't have to do anything to make an origami square! However, an origami equilateral triangle is a little bit harder.



The diagram to the right is identical to the diagram in step 3 with a few more lines added. Notice the right triangle drawn in red. Let s be the length of the side of the original square.

- a. What is the hypotenuse of this red right triangle?
- b. What is the length of the shorter leg of this red right triangle?
- c. What are the angles of this red right triangle? (Do you recognize the red right triangle as half of a special triangle?)
- d. Can you *prove* that this folding sequence produces an exact equilateral triangle?



3. Can you extend the folding sequence for the equilateral triangle to make a three dimensional regular tetrahedron? A tetrahedron is a 3D object with four triangular faces, six edges, and four vertices. See if you can make an origami regular tetrahedron that holds its shape.

# Getting a Balanced Diet

by Lauren Williams

An "average" human being consumes about 1940 calories per day. All food is comprised of carbohydrates, fat, and/or protein. One gram of protein or carbohydrates provides 4 calories, and one gram of fat provides 9 calories. Suppose a person decides to get the 1940 calories by eating 50 grams of protein, 300 grams of carbohydrates and 60 grams of fat. Let's see how this person could do that with different kinds of foods!

1. A pat of butter contains 4 grams of fat (and no significant protein or carbohydrates). A large pear contains 30 grams of carbohydrates (and no significant amount of protein or fat). A can of



You can see this painting by Spanish painter Luis Meléndez at the Museum of Fine Arts in Boston.

Photo courtesy of http://commons.wikimedia.org/wiki/Main\_Page

tuna canned in water contains 40 grams of protein (and no significant quantity of fat or carbohydrates). In order to consume 50 grams of protein, 300 grams of carbohydrates, and 60 grams of fat, while eating only butter, pears, and canned tuna, how much of each quantity of food should the person eat?

2. A large cantaloupe contains 65 grams of carbohydrates (and no significant protein or carbohydrates). A whole egg contains 5 grams of protein and 5 grams of fat. A chicken pot pie contains 25 grams of fat, 40 grams of carbohydrates, and 15 grams of protein. In order to consume 50 grams of protein, 300 grams of carbohydrates, and 60 grams of fat, while eating only cantaloupe, eggs, and chicken pot pie, how much of each quantity of food should the person eat?

3. One cup of mushrooms contains 2 grams of carbohydrates and 2 grams of protein. One whole avocado contains 20 grams of fat, 10 grams of carbohydrates, and 3 grams of protein. One cup of carrots contains 10 grams of carbohydrates and 1 gram of protein. In order to consume 50 grams of protein, 300 grams of carbohydrates, and 60 grams of fat, while eating only mushrooms, avocados, and carrots, how much of each quantity of food should the person eat?

4. A cookie contains 3 grams of protein, 35 grams of carbohydrates, and 10 grams of fat. A donut contains 6 grams of protein, 41 grams of carbohydrates, and 10 grams of fat. A croissant contains 4 grams of protein, 40 grams of carbohydrate, and 20 grams of fat. In order to consume 50 grams of protein, 300 grams of carbohydrates, and 60 grams of fat, while eating only cookies, donuts, and croissants, how much of each quantity of food should the person eat? What is wrong with your answer? Explain.

Different people have different caloric requirements. Using the same tools that you developed to solve the problems in this problem set, you could consult a nutritionist and design meals for yourself that are best suited to your own needs!

# Discovering Square Roots Via the Pythagorean Theorem

by Anda Degeratu



Triangle inequality:

For an arbitrary triangle with sides of length a, b and c, we have

a+b > c b+c > ac+a > b

...but if the triangle has a right angle between the sides of length a and b, more can be said:

a < c and b < c

and we also have the powerful Pythagorean theorem:

 $a^2 + b^2 = c^2$ 

Square roots:

Recall that the *principal square root* of a positive number *a* is the positive number *x* that satisfies  $x^2 = a$ . We denote it by  $\sqrt{a}$ .

For example, the square root of 1.69 is 1.3 because  $1.3^2 = 1.69$ .

In this problem set we are going to look at square roots using triangles, which we construct using only the following three tools: a ruler which is marked 1, 2, 3 and 4, a right angle, and a compass.

For example, if we construct a triangle with a right angle with short sides of length 1, the Pythagorean theorem tells us the length of the long side: it is  $\sqrt{2}$ .

Now, using the segment of length  $\sqrt{2}$  as a leg we can construct another right triangle with the other leg of length 1. In this new triangle, the long side has length  $\sqrt{3}$ .

The triangle inequality in this new triangle gives  $\sqrt{3} \, < 1 + \sqrt{2} \; . \label{eq:constraint}$ 

Moreover, since this is a right triangle, we also get  $\sqrt{2} < \sqrt{3}$ (which you already knew since 2 < 3 implies  $\sqrt{2} < \sqrt{3}$ ).

1. Find **two** ways to construct a segment of length  $\sqrt{5}$ . (For both ways, you should use only the three tools given to you. Note that the compass can be used to construct segments of a certain length, once you have constructed them somewhere else on your sheet of paper.)

2. Inferring (as much as possible) from the above constructions, try to rearrange the following numbers in increasing order:

 $3, \sqrt{2}, 1, \sqrt{5}, 1 + \sqrt{2}, \sqrt{2} + \sqrt{3}, \sqrt{3}, 1 + \sqrt{4}, 2.$ 

3. Using the three given tools, devise a way of constructing a segment of length  $\sqrt{147}$ .

4. With the same tools, can you devise a way of constructing a rectangle with area  $\sqrt{147}$ ? Would it be possible to construct a square with area  $\sqrt{147}$ ?

## Notes from the Club

These notes cover some of what happened at Girls' Angle meets. They are not meant to be complete.

Mentors: Beth Schaffer, Alison Miller, Ken Fan, Lauren McGough

Special Visitor: Tanya Khovanova, independent mathematician

The fourth meet began with a brief introduction to the concept of a function. Functions play an extremely important role in mathematics. The girls played a game where someone thought of a function and the others had to try to figure out which function. To gain information, the girls would provide a number and the person who thought of the function returned the function of the provided number.

Whenever you have two sets, say S and T, and a way of associating an element of T to each element of S, you have a function from S to T. If f is the name of the function, we write

$$f: S \to T$$

and the element in T that is associated with the element s in S is written like this: f(s). When S and T are sets of numbers, one way to make a picture of the function is to plot the points whose coordinates are (x, f(x)) in the Cartesian plane.

Tanya Khovanova introduced the concept of a number sequence. She gave several examples: the constant sequence, the counting numbers, the triangular numbers, the Fibonacci numbers, prime numbers, perfect numbers, pizza numbers and cake numbers. All of these number sequences arose as answers to questions and we explored some of these questions at the following meet. See page 8 for more on sequences.

Tanya also described various kinds of properties that numbers can have. Some of the properties reflect deep facts about numbers and others are rather whimsical! Check out her Number Gossip website at http://www.numbergossip.com/. One of the properties of numbers that has emerged as an extremely important property is that of being prime.



### Session 2 - Meet 5 - March 6, 2008

Mentors: Beth O'Sullivan, Cammie Smith Barnes, Ken Fan, Lauren McGough

The fifth meet began with demonstrations involving Möbius strips. For instructions on how to make a Möbius strip, read Eli Grigsby's article on manifolds in Volume 1, Number 3 of this Bulletin!

We then worked on a number of problems whose answers are various sequences.

**Sylvia** worked with Ken dividing triangles into a perfect square number of triangles (see page 9). **Sports car** and **Honda** worked on the pizza numbers that Tanya introduced from the prior meet. The Nth pizza number is the maximum number of pieces that a circle can be partitioned into using N straight cuts. Similarly, the Nth cake number is the maximum number of pieces that a cube (or a cylinder) can be partitioned into using N straight cuts.

Ν	0	1	2	3	4	5	6	7
Pizza Numbers	1	2	4	7	11	16	22	29
Cake Numbers	1	2	4	8	15	26	42	64

Other girls worked on counting paths in street maps. A street map that leads to not just a sequence of numbers, but a whole array of numbers is this one:



If you count the number of paths to each intersection, you will end up with a famous array of numbers. See page 25.

### <u>Session 2 – Meet 6 – March 13, 2008</u>

Mentors: Cammie Smith Barnes, Beth Schaffer, Ken Fan, Lauren McGough

We started the sixth meet working in small groups.

Cammie worked mostly with **Trisscar**, though also a bit with **The Cat**. They worked on the sequence problems handed out at the previous meet. They conjectured a formula for pizza numbers but found it challenging to imagine cuts in three dimensions to compute cake numbers. Try to develop your mind's eye by imagining geometric configurations in your head. Start with simple configurations, like two planes in space, and gradually increase the complexity. For example, try to visualize the intersections of objects such as a line and a torus or a plane and a cylinder. Can you picture the shape that results when two cylinders of equal radii intersect at right angles? Although somewhat outmoded in style, the book *Flatland* by Edwin Abbott is a good book to help one visualize geometry in two, three and four dimensions.

Beth Schaffer worked mostly with **PowderPuff**. She was interested in algebra so Beth gave her a couple of classic algebra problems. First, if x + y = 10 and xy = 15, what is  $x^2 + y^2$ ? After figuring that out, Beth asked her to figure out  $x^3 + y^3$  as a follow up question. This led to a digression on the binomial theorem (see page 25). After the digression, she was able to work out  $x^3 + y^3$ . Finally, she was left with evaluating

$$\sqrt{3+\sqrt{3+\sqrt{3+\sqrt{3+\dots}}}}$$

These problems lead to a lot of interesting mathematics. When you solve math problems, try to generalize them. For instance, for the first two problems, after figuring out  $x^2 + y^2$  and  $x^3 + y^3$ , it would be natural to try to work out the value of  $x^n + y^n$  for all *n*. For the last problem, what happens if you replace all the threes with 1 or 2, or, more generally, with *n*?

Lauren McGough worked with **Sylvia** and **Honda**. **Sylvia** worked on the street map problems and she made a lot of progress on the infinite grid street problem. She also worked for a while on the pizza problem, trying to understand the differences between sequences that follow a pattern and sequences that match the phenomenon that we are trying to understand. **Honda** worked a bit on the problems from the handout, but Lauren ended up finding a different problem that interested her: they talked about averages, different definitions of averages, and worked on an algebra problem that involved using information about several different averages to find a value in a set of numbers.



Ken worked with **Ilana** and **The Cat**. **Ilana** thought about pizza and cake numbers. She gave an example of a comb-shaped region to show that there are shapes whose associated cutting sequence differs from that of a circle. She also came up with a beautiful conjecture: any polygon that does not have an interior angle



exceeding 180 degrees must have the same cutting sequence as a circle. Is there a criterion that will also address regions with curvy boundaries? **The Cat** was led to thinking about how to represent three-dimensional tables of

numbers during her investigations into the network problem from the first meet. She made a three-dimensional table of numbers using color and perspective to clarify the information. This

is really quite a challenging problem and many people earn their living thinking about such things!

The meet concluded with a presentation on pi by Lauren McGough in anticipation of National Pi Day. She presented an argument that suggests why the area of a circle is given by  $\pi r^2$ , where r is the radius of the circle.

### <u>Session 2 – Meet 7 – March 20, 2008</u>

Mentors: Hilary Finucane, Beth Schaffer, Ken Fan, Lauren McGough, Beth O'Sullivan

Special Visitor: Elissa Ozanne, Harvard Medical School and Massachusetts General Hospital

The seventh meet began with work in small groups. **Tree** helped create a proof that one of the street maps gave the Fibonacci numbers (see page 13).

Before Elissa arrived, two more demonstrations were shown involving pi (see page 21).

Elissa Ozanne talked about decision models in breast cancer treatment and prevention. She discussed many of the factors one must consider in order to help women make the best choices on this matter. She presented some graphs that illustrate the effects of certain choices on various health outcomes and she showed decision flow charts that she constructed to organize the decision making process.

Elissa's use of mathematics underscores one very important aspect of mathematics: its objectivity. Health issues can be very emotional. Having people anticipate the needs and questions that a person may have when they are ill and use mathematics to find the optimal answers is an important way to help people continue to make rational decisions during emotional times. In addition, Elissa's application of mathematics to actual life and death issues shows how vital mathematics can be.

### Session 2 - Meet 8 - April 3, 2008

Mentors: Beth Schaffer, Katrin Wehrheim, Beth O'Sullivan, Ken Fan



In preparation for our visit to Professor Willcox's lab at MIT, we discussed periodic functions and gave many examples of phenomena where periodic functions arise, such as with vibrating strings, tides, orbits, circular motion, pendulums and sound.

In music, we talked about how the sounds instruments make are not pure sine waves. Instead, they are described by very complex periodic functions. However, these complex periodic functions are sums of pure sine waves called the *Fourier components* of the wave. The ear has the remarkable ability to break up these complex periodic functions into its pure components. When you strike a note on a piano, you can hear the components as harmonics.

If you know how to read music, here are the pitches you can hear if you listen very closely after striking and holding down the C three octaves below middle C:



Try it!

If you play a note on another instrument, you can hear the same pattern of pitches transposed so that the low C in the staff becomes the note you are playing. However, the strength of the various Fourier components in the sound are characteristic of the instrument. On different instruments, you will hear the harmonics (or *overtones*) with different strengths. That is, an instrument's unique sound quality is determined by its pattern of how loud and soft the harmonics of its notes are.

By the way, this could be an example of how mathematics helps one improve one's senses, if it helps you hear something that you never knew was there before!

After the break, some girls worked with Katrin Wehrheim on pizza numbers and others played a card game called Cliffhanger.

In Cliffhanger, each person is dealt three cards. Then, a target card is flipped over. Using the numbers of the cards in one's hand, one creates a number sentence. The goal is to try to find a number sentence whose value is as close to the target as possible. All three cards must be used in the number sentence. The absolute value of the error represents how far the player moves down a number line toward a cliff. If you fall over the cliff, you lose. A special rule is that if one is dealt a King, one can either use the King as a 13, or trade the King in for two more cards. This may not be an advantage because if you trade in a King, you will then have four cards all of which must be used!

In fact, **sports car** had a situation where she had a King and traded it in and ended up with two 2s, a 3 and a 9 when the target was 4. That didn't prevent her, however, from hitting the target on the head! Can you see a way to do that? **Trisscar** was a bit unlucky with the hands she was dealt, but she did somersaults to stay very competitive! For the last hand, we modified the rules. This time, there was no target flipped over. Instead, the target would be the number produced by whoever could make the highest number. **Sylvia** got dealt a King which she traded in and got

another King, and ended up with five cards. She accurately multiplied all five numbers together tossing everyone else over the cliff!

Girls' Angle was very fortunate to have Prof. Wehrheim at this meet. She is an assistant professor of mathematics at MIT and a professional geometer.

### <u>Session 2 – Meet 9 – April 10, 2008</u>

Mentors: Anda Degeratu, Beth Schaffer, Ken Fan, Lauren McGough, Cammie Smith Barnes

Special Visitor: Karen Willcox, Department of Aeronautics and Astronautics, MIT

For the ninth meet, we traveled to MIT to meet Professor Willcox at her aeronautics labs.

We spent the first hour in small groups. **The Cat** worked on finding a formula for the area of a regular octagon in terms of the length of one of its sides. She seems to have come up with this problem for herself, and that's wonderful! It's already a nice observation to note that the area of a regular octagon should be expressible in terms of just the length of its side. It is excellent to get into the habit of asking and answering one's own math questions! Such a formula could be used to figure out how many cans of red paint one would need to paint a thousand stop signs. Also, if you take a unit circle and circumscribe and inscribe it with two regular octagons, compute the associated side lengths and use the formula to find their areas, you would be able to approximate  $\pi$ .





Beth Schaffer introduced some girls to a well-known math game. The game involves starting at zero and adding an integer between 1 and some fixed limit, like 5, inclusive. Players alternate adding numbers. The winner is the girl who is first to reach a target number. At Girls' Angle, we've played a number of games. In fact, there is a whole branch of mathematics devoted to games: Game Theory. Game theorists analyze games and try to find winning strategies.

**August** worked with Anda on a formula related to the Hotel Infinities. Anda was visiting from the Max Plank Institute in Berlin.

Karen Willcox explained how periodic functions are used to help analyze and design aircraft controls. She reinforced the idea that complex functions can be written as sums of simpler functions, explaining that analysis of how airplanes react to conditions described by the simpler functions can help to understand how they react to complex functions. One such application is analyzing how planes react to turbulence, and she showed a computer simulation of a glider hitting turbulence which showed just how much a wing can warp. Some girls were skeptical that a wing could bend so much. Karen suggested that the next time you fly on an airplane, watch the wing. It supposedly changes shape quite a bit!

Karen then took us to a room with helicopter models and the girls took turns flying it. It didn't look very easy to control, and external factors such as a breeze could send the helicopter reeling. For this reason, there is interest in developing computer algorithms that fly helicopters. As

another example, two of Karen's students demonstrated a peculiar, asymmetric helicopter that they designed and built. It had a main helicopter blade that had a long helicopter blade only on one side counterbalanced by a small propeller on the opposite side.

Girls' Angle thanks Professor Willcox for hosting us at her labs! It was a real treat!

### <u>Session 2 – Meet 10 – April 17, 2008</u>

Mentors: Katherine Körner, Amy Manson, Ken Fan, Lauren McGough, Alison Miller, Eli Grigsby, Grace Lyo

Special Visitor: Leia Stirling, Department of Aeronautics and Astronautics, MIT

At the tenth meet we had more mentors than students! Katherine is a math graduate student at Harvard and her friend Amy happened to be visiting from Bristol University in England where she is a graduate student in math. Grace is a Girls' Angle advisor and Eli is a Girls' Angle director and advisor. All four were visiting for the first time.

After doing so much with exponentials and periodic functions, we asked the girls two questions:

- 1. What is the units digit of  $2^{100}$ ?
- 2. For what *n* is  $2^n + 3^n$  divisible by 5?

By the way, notice that the first question could have been put like this:

What is the remainder when  $2^{100}$  is divided by 10?

And the second question could have been put like this:

For what *n* is the units digit of  $2^n + 3^n$  equal to 0 or 5?

Here's a harder variant of the first problem:

What is the remainder when  $2^{1000}$  is divided by 100?

Leia Stirling introduced the concept of linear and angular momentum. Linear momentum is conserved in the absence of external forces. Angular momentum is conserved in the absence of external torques. The linear momentum of an object is equal to mv where m is its mass and v is its velocity. The magnitude of the angular momentum is given by  $I\omega$  where I is the *moment of inertia* and  $\omega$  is the angular velocity. The moment of inertia is a measure of how mass is distributed about an axis.

Leia then showed how understanding the conservation of angular momentum allows one to devise maneuvers which astronauts can use to rotate themselves in zero gravity environments, such as on the International Space Station. She showed us videos of astronauts performing some of those maneuvers. Then, the girls got to practice these maneuvers sitting on swivel chairs. For a sampling of her rotation maneuvers, see pages 7 and 20.

# Special Announcements

Remember that all members and subscribers are invited to send mathematical questions and discoveries that they make to girlsangle@gmail.com. This extends beyond the Summer Fun problem sets. We encourage members and subscribers to take advantage of this opportunity!

# Calendar

Session 2: (all dates in 2008)

January	31	Start of second session!
February	7	
-	14	
	21	No meet
	28	Visitor: Tanya Khovanova, mathematician
March	6	
	13	
	20	Visitor: Elissa Ozanne, Harvard Medical School*
	27	No meet
April	3	
	10	Visitor: Karen Willcox, MIT Aeronautics Department**
	17	Visitor: Leia Sterling, MIT Aeronautics Department
	24	No meet
May	1	Visitors: Sarah Ackley and Adele Schwab, MIT Physics
	8	

\*Dr. Ozanne's visit was postponed one week and took place on March 20. \*\*Girls' Angle went to MIT for Prof. Willcox's presentation.

Session 3: (likely dates will be Thursdays in the months from September to December)

# Feedback

Please send feedback to girlsangle@gmail.com. We'd love to hear from you!

















# Girls' Angle: A Math Club for Girls

### Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

What is Girls' Angle? Girls' Angle is a math club for girls that aims to foster and nurture girls' interest and ability in mathematics. Instead of modeling after the traditional classroom experience, Girls' Angle is inspired by the lively activity in math department common rooms. Our philosophy is that mathematical ability is best developed through interaction with people who have both a deep understanding of mathematics and a genuine interest in helping others learn. Rather than 'teach math' at the club, we'll have helpers who work on motivation, motivation, motivation! The helpers, who will mostly be women, will introduce the girls to all kinds of activities, objects, and ideas that are math related trying to hook their interest. Once hooked, we will encourage them to explore, to think, and to ask and seek the answers to questions. We will show them all kinds of techniques that help one find answers, and we will show them how to craft questions that promote progress. The goal is to empower girls to be able to tackle any level of mathematics in the future so that no field, no matter how technical, will be off limits. We aim to overcome math anxiety and build solid foundations, so we will be welcoming all girls, not just those deemed gifted in mathematics.

**Who can join?** Ultimately, we hope to open membership to all women. Initially, we will be opening the doors primarily to girls in grades 5-9. We welcome *all girls* regardless of perceived mathematical ability.

In what ways can a girl participate? There are 3 ways: membership, premium subscriber, and basic subscriber. Membership is granted for per session or per meet and includes access to the club, the math question email service, and a subscription to the Girls' Angle Bulletin. Premium subscriptions last one session and include the math question email service and subscription to the Girls' Angle Bulletin. Basic subscriptions are one-year subscriptions to the Girls' Angle Bulletin. We operate in 10-12 meet sessions, but girls are welcome to join at any time. The program is individually focused so the concept of "catching up with the group" doesn't apply.

**What is the Girls' Angle Bulletin?** The Girls' Angle Bulletin is a bimonthly (6 issues per year) electronic publication that will feature articles and information of mathematical interest as well as a comic strip that teaches mathematics.

What is the math question email service? The math question email service allows a subscriber to email math questions that will be answered by staff or addressed during club meetings. Please note that we will not do math problems that *appear to us* to be for homework.

What do members get? Members get a one-year subscription to the Bulletin and one session of access to the club and the math question email service. The club will be a friendly place staffed mainly by women who have been selected for their deep understanding of mathematics and their desire to truly help others learn math. Helpers will take a personal interest in each member, assessing her mathematical abilities and working with her to motivate an interest in mathematics and mathematical topics by encouraging questions and explaining strategies and techniques for finding answers. Helpers will also organize fun activities that serve to introduce, explain, and clarify mathematical topics.

**Where is Girls' Angle located?** Girls' Angle is located about 10 minutes walk from Central Square on Magazine Street in Cambridge. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

**When are the club hours?** Girls' Angle meets Thursdays from 3:45 to 5:45. Please inquire about the calendar. It is very important that you pick up your child promptly at 5:45.

**Can you describe what the activities at the club will be like?** Girls' Angle activities will be tailored to each girl's specific needs. We will assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

**Are donations to Girls' Angle tax deductible?** Yes. If you believe in our approach and goals and want to help support us, we appreciate any contribution you can make. Currently, Science Club for Girls, a 501(c)3 corporation, is holding our treasury. Please make donations out to **Girls' Angle c/o Science Club for Girls** and send checks to Ken Fan, P.O. Box 410038, Cambridge, MA 02141-0038. Also, please alert us to your donation by sending email to girlsangle@gmail.com. Thanks!

**Who is the Girls' Angle director?** Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was an assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science and worked in the mathematics educational publishing industry. Ken believes that mathematics education in this country can be improved significantly. Also, through the years, he has witnessed instances of gender bias in mathematics and in math education. The last two summers Ken volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were hung at Boston Children's Museum. The girls of Science Club for Girls showed a lot of creativity and ingenuity and were able to realize their ideas in the final project, something that may not have happened in a co-ed environment. These experiences have motivated him to create Girls' Angle.

Who ensures that Girls' Angle adheres to its mission? Girls' Angle has a stellar Board of Advisors:

Connie Chow, executive director of Science Club for Girls Yaim Cooper, graduate student in mathematics, UC Berkeley Julia Elisenda Grigsby, NSF postdoctoral fellow, Columbia University Grace Lyo, Moore Instructor, Massachusetts Institute of Technology Lauren McGough, advanced high school student who founded her school's math club Beth O'Sullivan, co-founder of Science Club for Girls. Elissa Ozanne, Senior Research Scientist, Harvard Medical School. Kathy Paur, graduate student in mathematics, Harvard Lauren Williams, Benjamin Pierce assistant professor of mathematics, Harvard

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematical competence required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known. Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle helpers can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

# **Girls' Angle: A Math Club for Girls**

Membership Application

Applicant's Name: (la	)(first)
Applying For:	Member (Access to club, Math question email service, Receive Bulletin) Premium Subscriber (Math question email service, Receive Bulletin) Basic Subscriber (Receive Bulletin)
Parents/Guardians:	
Address:	Zip Code:
Home Phone:	Cell Phone: Email:
Emergency contact	me and number:
<b>Pick Up Info</b> : For sa They will have to sign	ty reasons, only the following people will be allowed to pick up your daughter. her out. Names:
Medical Information know about?	Are there any medical issues or conditions, such as allergies, that you'd like us to
<b>Photography Releas</b> in all media forms. W use your daughter's in	Occasionally, photos and videos are taken to document and publicize our program will not print or use your daughter's name in any way. Do we have permission to age for these purposes? Yes No
<b>Eligibility:</b> For now, hard to include every Girls' Angle has the c	rls who are roughly in the grade 5-8 range are welcome. Although we will work rl no matter her needs and to communicate with you any issues that may arise, cretion to dismiss any girl whose actions are disruptive to club activities.
<b>Permission:</b> I give m everything on this reg	daughter permission to participate in Girls' Angle. I have read and understand tration form and the attached information sheets.
	Date:
(Parent/Guardian Sign	ture)
Membership-Applica	Signature:
<ul> <li>□ Enclosed is a</li> <li>□ \$216 for a</li> <li>□ \$20 for a c</li> </ul>	neck for (indicate one) (prorate as necessary)2 session membership□ \$100 for a 12 week premium subscription2 year basic subscription□ I am making a tax free charitable donation.
$\Box$ I will pay on	per session basis at \$20/session. (Note: You still must return this form.)
Please make check pa 410038, MA 02141-0 girlsangle@gmail.cor but not the math ques	able to: <b>Girls' Angle c/o Science Club for Girls</b> . Mail to: Ken Fan, P.O. Box 8. Please notify us of your application by sending email to Paying on a per session basis comes with a one year subscription to the Bulletin, on email service. Also, please sign and return the Liability Waiver.

### Girls' Angle: A Math Club for Girls Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s participation in the Program.

Signature of applicant/parent:	Date:
Print name of applicant/parent:	
Print name(s) of child(ren) in program:	
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	<b>J</b> Angle
A Mat	h Club for Girls