Girls' Angle Bulletin

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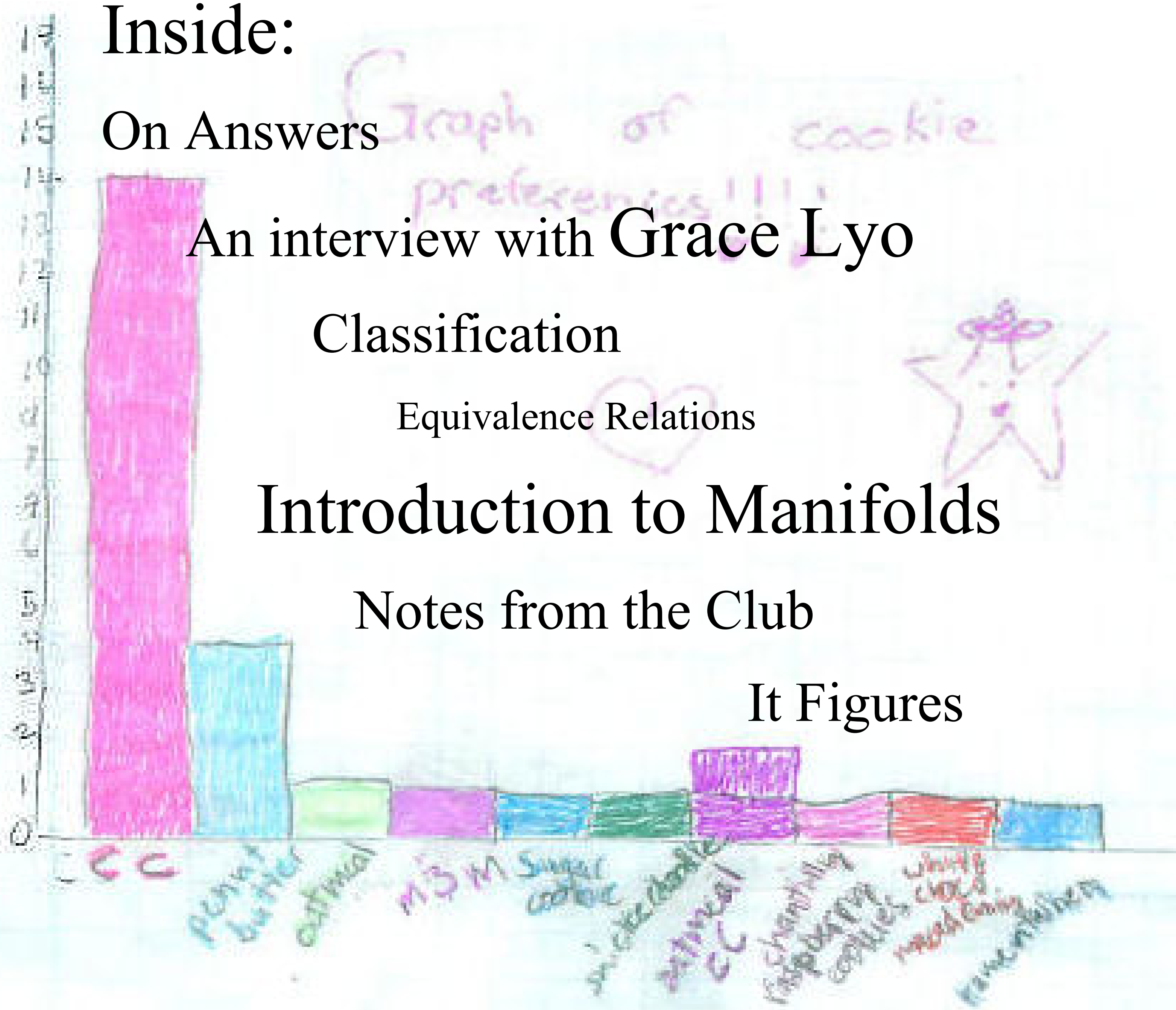
To Foster and Nurture Girls' Interest in Mathematics



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From the Director

It's a new year and a new session at Girls' Angle!

We're extending sessions by two meets to twelve, so we can have 20% more fun! Please see the calendar on page 18.

We've also welcomed two new members to the Girls' Angle Advisory Board: Dr. Elissa Ozanne and Dr. Elisenda Grigsby.

Dr. Ozanne earned her Ph.D. from Stanford and is a senior research scientist at Harvard Medical School. Many of you may be familiar with her from her visits during the first session. She has shown an interest in Girls' Angle from the beginning. Her work involves making decision models to help women make the best choices with respect to breast cancer treatment and prevention.

Dr. Grigsby earned her Ph.D. from the University of California at Berkeley and is a National Science Foundation postdoctoral fellow at Columbia University. She specializes in low-dimensional topology. Check out her article on manifolds in this Bulletin (see page 11)!

With Dr. Ozanne and Dr. Grigsby, Girls' Angle gains valuable new insight and advice that will keep Girls' Angle headed in the right direction!

Ken Fan
Founder and Director

Girls' Angle Donors

Girls' Angle thanks the following for their generous contribution:

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Girls' Angle Bulletin

The official magazine of
Girls' Angle: A Math Club for girls

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This magazine is published about six times a year by Girls' Angle to communicate with its members and to share ideas and information about mathematics.

Girls' Angle welcomes submissions that pertain to mathematics. Subscriptions to the Girls' Angle Bulletin cost $20 per year and support club activities.

Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and to empower girls to be able to tackle any field no matter the level of mathematical sophistication required.

On the cover: **sports car** made a bar chart of people's favorite type of cookie. Can you guess what "CC" stands for?

On Answers

At one of the Girls' Angle meets, **Honda** asked the mentors, "do *you* know the answers?"

She added, "because if you don't know the answers, how will we know if we're right or not?"

The world is filled with wonder, and there are far, far, far more things that are unknown than known! This imbues the world with mystery and the promise of discovering something new gives science and mathematics its vitality.

What are mathematicians? They are people who try to find the answers to questions that nobody knows the answers to. (Of course, they aren't interested in every unanswered question! They only think about those that are mathematical in nature.)

Because nobody knows the answers to the questions that these mathematicians ask, they have no mentors to seek answers from. So let's ask mathematicians **Honda**'s question. How do they know if they are right or not?

Now, there's a question that we will answer! Mathematics is a journey taken individually by each mathematician and collectively by all mathematicians. It is a journey of ever-increasing understanding. In order to make progress on this journey, mathematicians must develop a sense of confidence and certainty in their mathematical arguments (or proofs). The way they develop this confidence is to start with simple observations and conclusions that they can prove and keep thinking about and playing with things. Gradually, they make more intricate observations and conclusions which they prove using the less intricate statements that they proved in the past.

It's similar to learning about text messaging. A beginner might start with simple sequences, like "CU@10". Bt teen pros cn EzalE handL tngz lIk "Math iz :) & XcitN! kEp asking :-Qz & finding Ansz! Someday, we mentors hOp 2 Lern math frm U!"

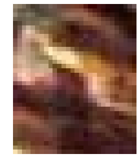
At Girls' Angle, we want to empower girls to be able to become researchers who are capable of finding the answers to unsolved questions. We want to help you develop your ability to verify your own observations and conclusions. And for that reason, we're less interested in giving you answers as we are in helping you verify your answers, and so in a profound sense, it does not matter if the mentors know the answers!

Yet, despite mathematicians' best efforts to accurately verify their answers, mistakes are still made. But not to worry: mathematicians help each other in verifying answers. How they do that is a discussion for another time.

Here's a more direct answer to **Honda**'s question: Girls' Angle mentors do know an awful lot of math and they will know the answers to all the questions you will be asked to solve at school, and many more questions beyond that. But they'll use this knowledge not so much to supply you with all the answers, but rather to guide themselves in judging best how to help you develop your own ability to verify your own answers. What's more, some mentors have proven original theorems, and so they know first-hand ways of finding answers to unanswered questions. Finding such mentors is common at major universities, but extremely rare in grade school. It is one of the hallmarks of Girls' Angle.

An Interview with Grace Lyo

Grace Lyo is a Moore Instructor at the Massachusetts Institute of Technology. She earned her Ph.D. in mathematics from the University of California at Berkeley.



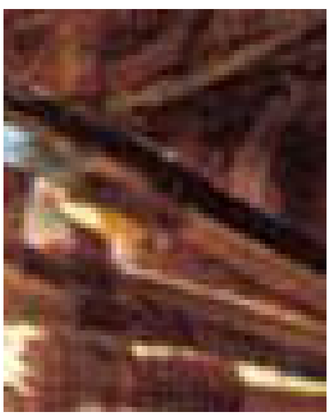
Ken: Hi Grace, thank you for agreeing to do this interview! My first question is: what got you interested in mathematics?

Grace: My interest developed gradually. When I went to college, I was thinking about majoring in physics or computer science, but eventually realized that the courses I liked best were the math/theory courses.

Ken: So you didn't realize your interest in math until college?

Grace: The way I was taught math, it didn't seem like something that one could do. It was easy for me to understand if someone said they did research in physics or biology, but if someone said they did research in math, I wouldn't have understood what they meant. What is there to research in math? In school, math seemed like just a tool to do other things, like physics or engineering. So I didn't really know what math was all about until I went to college.

Ken: What in college awakened you to mathematics?



Grace: When I took algebra, mathematics became really exciting. For the first time, math wasn't just a collection of facts that one had to learn. Instead, it was an arena where you could experiment and create. For instance, you can ask yourself, "what would happen if I set five equal to two?" In high school and before, such questions weren't allowed. But in real math, this is a perfectly valid question, and it leads you to create a number system where three equals zero. (Incidentally, in my research, I usually work with number systems like this, in which some prime [number] gets set equal to zero.) I found it exciting to learn that math is about building up structures from axioms and that you can change those axioms around and get different structures.

Ken: You make mathematics sound like a very creative endeavor!

Grace: It is! In school, math often seems like something where there is a definitive answer. But really, mathematical facts do not represent the end of something, but the beginning of something. There is a lot to explore and that's what makes math interesting. There is so much that is unknown—it's not like math homework where you know that there's an answer and that the problem is designed so that one can find that answer in a reasonable amount of time. In math, you don't always even know if there is an answer, or, if there is, how hard it will be to find.

Actually, this makes me think of something that I wish I had known much earlier. Maybe it might help the girls at Girls' Angle to know this. Mathematical research is about looking at something and seeing little things that may not seem directly related to the problem and playing with these things until you understand them. If you do that, gradually your understanding grows and you develop an intuition. It's kind of like doing a jigsaw puzzle. You start with lots of little pieces and, if you don't look at the cover, it's very hard to tell what the big picture is going to be. But you look at little details on individual pieces and gradually start to notice things. You keep going and suddenly, the big picture emerges.

Ken: Can you give a specific mathematical example of this kind of thing?

Grace: Well, when I was young, I used to take violin lessons with my brother, and so while he was having his lesson, I would have to sit and wait for my turn. During that half hour, I'd often think about little math questions. For example, I thought about how multiplication relates to even and odd numbers…how an even times an odd is even but an odd times an odd is odd…

Ken: We have some girls at Girls' Angle who like thinking about exactly that!

Grace: It may not seem like much, but if you keep thinking about such things, over time, you develop a lot of intuition and understanding. Then when you encounter a math problem many years later, you may find it's not so hard to solve because you've already solved half of it! You might not even realize that the reason the problem is so much easier for you than for everyone else is that you've thought about it already.

Ken: By the way, as a mathematician, when you say "understand", what exactly do you mean by that?

Grace: That's a hard question. Maybe it's like if you imagine yourself to be a little bug wandering around some surface. You see tiny things all around you and you don't understand what all these tiny things are or how they fit together. But if you keep thinking about these tiny little things you see and wonder how they fit together, a big picture might suddenly emerge, as if you've been lifted up to a higher vantage point and suddenly can see how it all fits together. So when you figure something out or understand something, it's kind of like zooming out and seeing how everything fits together.

Ken: Do you have any advice for Girls' Angle members?

Grace: Tinker with math. Keep asking questions and keep trying to find their answers.

Ken: Thanks Grace! I hope you'll be able to visit Girls' Angle someday!

The illustrations accompanying the interview with Dr. Lyo gradually reveal more and more of this self-portrait by the French painter Élisabeth-Louise Vigée-Le Brun. When did you sense what the images were? If you want to see one of her paintings in person, you don't have to travel to France! There's a portrait by her in the Museum of Fine Arts here in Boston.

All images in this interview are adapted from en.wikipedia.org/wiki/Image:Vigee-Lebrun1782.jpg.

Classification

At Girls' Angle, **Sylvia**, **Trisscar**, **Honda** and **sports car** worked out some classification systems. They classified people, birds, sports and cookies.

What are the ingredients of a classification system? Why are they useful in mathematics?

Suppose you have a set of objects, like birds or cookies. In order to classify the objects, you have to be able to note similarities and differences between the objects in your set. Once you observe and are able to articulate a difference, you can define a new class for your classification. The more you are able to see differences, the more extensive will be your classification.

In mathematics, people define many kinds of objects. There are numbers, polygons, mathematical operations like addition and multiplication, functions…math is a huge universe of objects of all sorts. One of the first things you can do when trying to understand some set of objects is to try to see similarities and differences among them, and the moment you start doing this, you are classifying the objects.

For example, think about triangles. You might notice that different triangles have different degrees of symmetry. Some have no symmetry at all. Some have a bilateral symmetry, meaning that you can draw a line through the triangle and see the triangle split into two mirror symmetric pieces. And some triangles show bilateral symmetry with respect to three different lines! At once, you have a classification of triangles according to how many lines of bilateral symmetry they have. Understanding this classification means increased knowledge about triangles.

When **Sylvia**, **Trisscar**, **Honda** and **sports car** worked out a classification system for people, the number of classes kept growing and growing. Before long, there were so many different classes of people that it became very difficult to draw a clear diagram of the classification scheme! There were smart, short brown-haired, hazel-eyed girls who lived in Europe and were about four feet tall. And among these girls, there were those that liked certain sports, certain cookies, certain kinds of music. And among those who liked tennis, chocolate chips, and rock music, there were those with various combinations of personality traits.

A person couldn't be both male and female, but a person could like both peanut butter cookies and chocolate chip cookies. In other words, some classes were **mutually exclusive** while other classes **overlapped**.

And while a person couldn't have both type A and type B blood, everybody had to have either type A, type B, type AB, or type O blood. In other words, some classes together formed a **partition** of the set, meaning that every person belonged to some class of the partition, but no person belonged to more than one class of the partition.

All four girls showed considerable ease dealing with this enormously complex classification of people. Now here's a bit of welcome news: all the classification systems encountered in K-12 level mathematics are much simpler than the people classification. So if you feel comfortable with the people classification, you should have no trouble with any of the classifications you encounter in math for many years to come! If you do find yourself having trouble in math with a classification, it just means that you have to spend a little time getting more acquainted with the objects being classified. Talk to the objects, work with them! Find out how they are alike and how they differ.

Subjective versus Objective

The classification systems we played with can, themselves, be classified!

One such classification is to classify the classifications according to whether they are *subjective* or *objective*. In a subjective classification, people can disagree on which class an object belongs. For example, **Sylvia**'s classification of people was based on personality traits, such as whether a person was friendly, polite, boring or mean. One can imagine that different people could easily place the same person into different classes. With an objective classification, however, everyone will (eventually) agree how to place the objects into the various classes.

In mathematics, the best classifications are objective classifications whose classes are based on existing properties of the objects. It may be difficult to realize that a property exists, and some very important classifications in mathematics came about when a mathematician pointed out some previously unknown, exotic properties.

Triangles

For example, to classify triangles, you could classify them alphabetically by their name. However, if you did that, the resulting classifications may differ depending on what language you spoke. Why, there are even languages where it does not make sense to classify alphabetically, such as in Chinese.

Mathematicians do not like arbitrary choices, and the sound of the words that are assigned to various kinds of triangles are arbitrary. Even the word "triangle" differs from language to language. The *concept* of triangle is universal, but the *word* chosen to name that concept is not. In some other languages, the triangle concept is referred to as: triangolo, τριγωνο, Dreieck, triângulo, 三角, and さんかく. The words are important for communication, but don't confuse the word with the concept! If you didn't know anything about triangles, but saw all six of those words, you still wouldn't know a thing about triangles!

A triangle is a planar figure with no holes and whose boundary consists of three straight line segments. When you have three sides, there are three possibilities that automatically result when comparing their lengths. Either they are all equal, or, just two of them are equal, or, no two are equal. Notice that these distinctions have only to do with the nature of triangles. One almost has the feeling that once the notion of triangle is defined, this classification by comparing side lengths was already there!

As it turns out, the standard classification of triangles does not have a name for those triangles where just two sides have the same length. Instead, a name was devoted to a slightly larger set of triangles: those triangles where at least two sides have the same length. That's just another arbitrary accident of naming conventions!

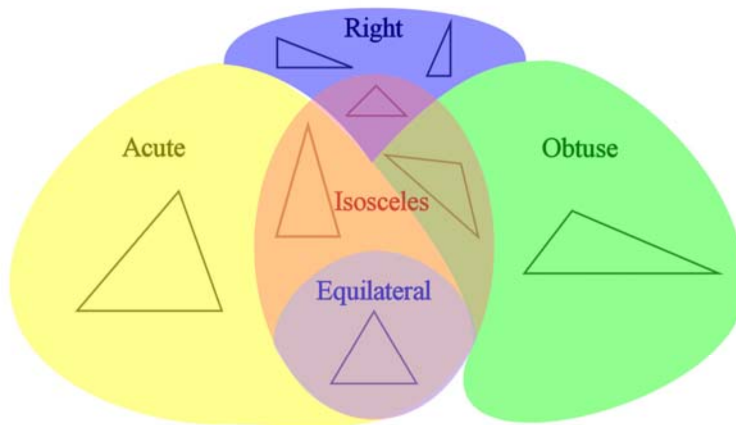
Here's a table of some commonly identified classes of triangles:

Equilateral	All three sides have the same length.
Isosceles	At least two sides have the same length.
Acute	All three angles are less than ninety degrees.
Right	There is a ninety degree angle.
Obtuse	There is an angle whose measure exceeds ninety degrees.
Scalene	No two edges have the same length.

Note that these classes often have alternative definitions. For example, the equilateral triangles can also be described as those triangles where all three angles are equal. (If you know about word roots, you will know that "equilateral" is formed from roots that mean "equal" and "side", but don't let that confuse you into thinking that a definition based on equal angles is therefore incorrect!)

By the way, in the definition of some of those classes, the angle whose measure is ninety degrees plays a key role. Do you think that the ninety degree angle is special enough to warrant a classification system that uses such angles?

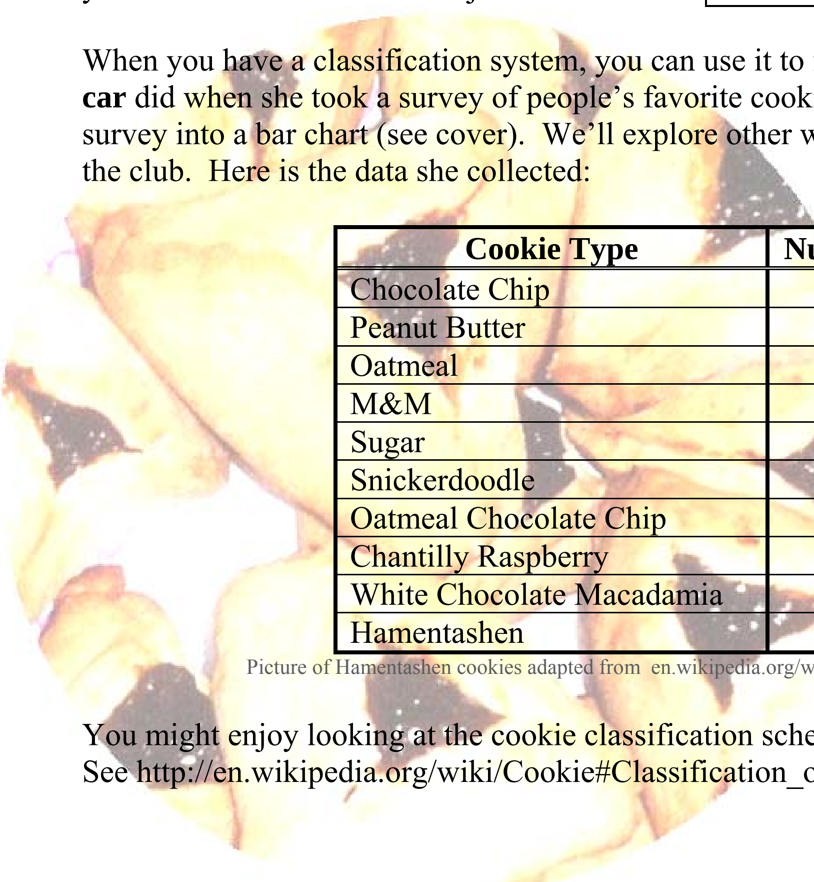
Try classifying quadrilaterals, numbers, or your favorite mathematical objects!



Triangles

We can show the relationship between classes with a *Venn diagram*. How do scalene triangles and the classes defined in terms of mirror symmetry fit into this picture?

When you have a classification system, you can use it to find out things about people, as **sports car** did when she took a survey of people's favorite cookie. She organized the results of her survey into a bar chart (see cover). We'll explore other ways of representing such information at the club. Here is the data she collected:



Cookie Type	Number of People
Chocolate Chip	14
Peanut Butter	4
Oatmeal	1
M&M	1
Sugar	1
Snickerdoodle	1
Oatmeal Chocolate Chip	2
Chantilly Raspberry	1
White Chocolate Macadamia	1
Hamentashen	1

Picture of Hamentashen cookies adapted from en.wikipedia.org/wiki/Image:Homemade_hamantaschen.jpg

You might enjoy looking at the cookie classification scheme on Wikipedia. See http://en.wikipedia.org/wiki/Cookie#Classification_of_cookies.

Equivalence Relations

At the club, we briefly talked about equivalence relations. An equivalence relation is used to describe "sameness". Often, when you have a collection of objects, there are qualities which some objects share. An equivalence relation is a formal way of stating that two objects share something in common or are similar in some way without having to be identical.

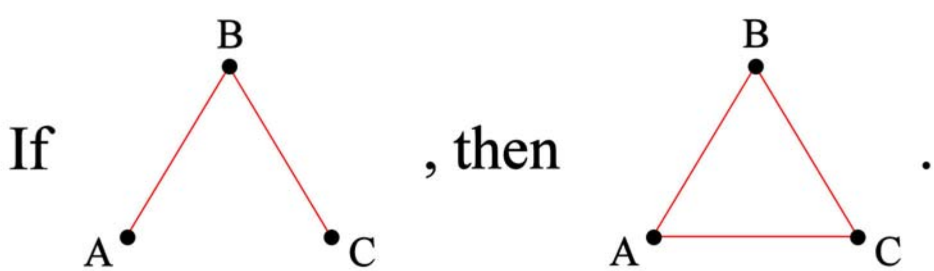
For example, we could say that two items at a store are equivalent if they cost the same. Or, we could say that two people are equivalent if they have the same color hair. **Honda** pointed out that with this latter equivalence relation, being equivalent would be the same as being equal because she believes that no two people have exactly the same color hair!

The reason we talked about equivalence relations is because if you have an equivalence relation, you automatically get a classification into classes that form a partition! All you have to do is look at the subsets consisting of all objects that are equivalent to each other. For the pricing equivalence relation then, there would be the class of all items that sell for a penny, the class of all items that sell for two cents, the class of all items that sell for three cents, etc. And if **honda** is correct about hair color, the equivalence classes for the hair color equivalence relation would number as many people as there are and each equivalence class would contain a single person.

The notation to express that some object A is equivalent to another object B is to write $A \sim B$ and the properties that define an equivalence relation are:

1. $A \sim A$ (reflexivity).
2. If $A \sim B$, then $B \sim A$ (symmetry).
3. If $A \sim B$ and $B \sim C$, then $A \sim C$ (transitivity).

Trisscar gave a very nice visual interpretation of transitivity. Suppose you use dots to represent objects and draw a line between dots that represent equivalent objects. **Trisscar** saw that transitivity means that whenever you have two sides of a triangle, you can draw in the third.



Trisscar's visual interpretation of transitivity

The second property, symmetry, is the property that allows one to represent the equivalence of two objects by connecting the two dots with a line segment. If the relation were not symmetric, then instead of connecting dots with line segments, you could connect them with an arrow to indicate a one-sided nature. The relationship of being "less than" is an example of a relationship that is neither symmetric nor reflexive, though it is transitive!

Introduction to Manifolds

by Elisenda Grigsby

Not all mathematics involves numbers. "What on Earth is math without numbers?" you may ask. Well, remember that mathematics is just a systematic way of tackling abstract questions–so mathematical techniques can be brought to bear on almost any question of interest.

For example, if we had been alive 2,500 years ago, we would probably have been very interested in the following question:

What is the general shape of the surface of the Earth? Is it a plane? Is it a disk that you would fall off if you sailed too far?

These days, you can find a toy globe in any hobby shop, and we have all seen satellite images of the Earth from space. We know the Earth is round–its surface is a sphere. Two and a half centuries ago, however, you can probably imagine how difficult it would have been to figure something like that out. The Earth is big, and without the aid of airplanes and spaceships, we are confined to a small section of it. When our distant ancestors looked at the surface of the Earth–the ground beneath their feet–it looked pretty flat, albeit with some hills and mountains thrown in.

When we speak of the "general" or "overall" shape of the Earth, we are ignoring hills, mountains, canyons and other bumpy findings. If you wish, think of the Earth as covered entirely by a perfectly calm ocean. In fact, even when this is done, the surface is not really a sphere! The Earth bulges at the equator and is more like an ellipsoid. However, to a mathematician who works in topology, a topologist, a sphere and an ellipsoid are considered equivalent. One can be deformed into the other without having to damage the surfaces by cutting them; one just has to stretch and bend a bit.

Conceptualizing the surface of the Earth as a sphere helped us build the global transportation and communication systems that we enjoy today. Study of the sphere and similar objects led mathematicians to define a category of geometric objects known as manifolds. The sphere is an example of a manifold, more specifically, a 2-dimensional manifold. It is a manifold because it looks "locally flat". In other words, around any point of the sphere, there is a small neighborhood around that point that looks like a neighborhood of the origin in the Cartesian plane (see the previous issue of this Bulletin). It is 2-dimensional because any coordinate system used to map positions on the Earth's surface requires two numbers (for example: latitude and longitude).

Let me pause for just a moment to clear up a point that can be very confusing the first time you encounter a manifold: why is a sphere considered to be 2-dimensional? Isn't it sitting inside of a 3-dimensional space?

The best way to approach this question is to think of a sphere as a completely abstract object. In other words, imagine what would happen if someone were to squash you absolutely flat, so you looked like a paper doll. Then imagine that someone were to stick you onto the sphere so that you could only move north, south, east or west, but no longer up or down off of it.

Confined in this manner, you only know the 2 dimensions that comprise the sphere's surface: you have no idea that the sphere is sitting inside a higher-dimensional space. In fact, it's fun to imagine that our universe, which looks 3-dimensional to us, is actually embedded in a higher-dimensional space that we can't see!!! Some physicists believe that this is the case, but we'll leave discussion of this to a future Bulletin.

Instead, let's return to our original question: suppose we are living 2,500 years in the past. How can we tell what the overall shape of the Earth is? Unfortunately, the fact that the general shape of the Earth's surface is a manifold makes this question very hard, for we've already noted that every point on a manifold looks nearly the same as every other: locally like a little disk in a plane. A larger perspective on how these points fit together to form the surface of the Earth would help a lot! But, remember, 2,500 years ago, the technology to get a photo of the Earth was not available

Let's do what every mathematician does when confronted with a puzzle: study more examples! So, let's gather some examples of 2-dimensional manifolds and explore their properties.

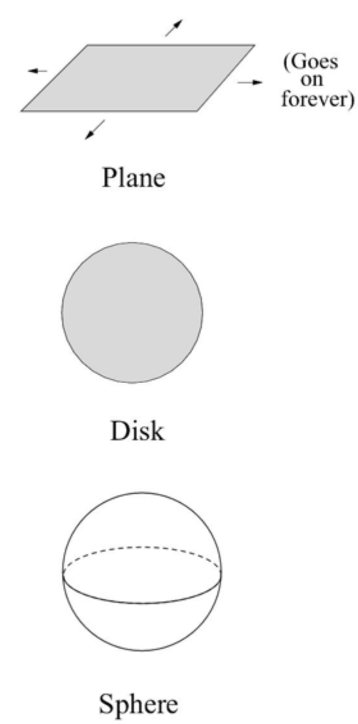
There are three 2-dimensional manifolds that spring to my mind immediately: the plane, the sphere, and the disk. These are all objects that look locally flat, and if you were a tiny paper doll confined to the object, your movement would be confined to the reach of two perpendicular directions. Let's list some characteristic properties of each.

The plane's most obvious characteristic is that it is infinite. In other words, if you and a friend meet for lunch on the coordinate plane, and then after lunch you say goodbye and head off in opposite directions, you will never meet again, no matter how many days, months, years, centuries, or even millennia you walk.

The sphere, on the other hand, is not infinite. If you and a friend meet for lunch on a sphere, then walk off in opposite directions, eventually you will run into each other again (though it may be many, many years later–depending on the size of the sphere and how fast you walk).

The disk shares some properties with the plane and with the sphere, but is actually quite different from both. Like the plane, if you and a friend meet for lunch on a disk and then start walking away from each other in opposite directions, you would never meet again. However, a disk is finite, like the sphere. In fact, the disk has a special feature that neither the plane nor the sphere shares: it has a boundary, an edge. In other words, when you start walking away from each other, you would do well to watch where you're going, because you may fall off!

The Plane, Disk, and Sphere

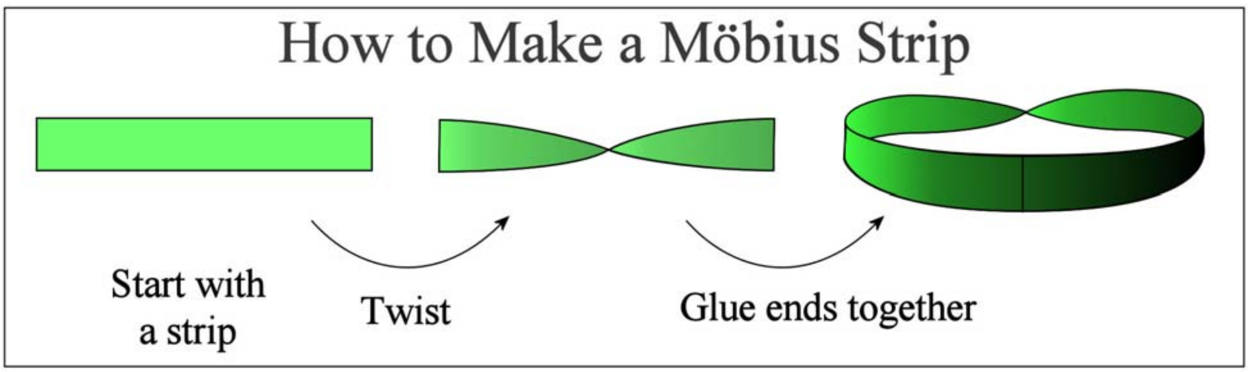


The plane, disk, and sphere are really quite different from each other! Unlike the sphere and the ellipsoid, they cannot be deformed one into the other without some kind of cutting.

Actually, this is a good time to mention that, technically speaking, the disk is not a 2-dimensional manifold–it is a 2-dimensional manifold *with boundary*. Notice that the points on the edge of the disk (on the boundary) have neighborhoods that look fundamentally different from points in the interior of the disk (not on the boundary). What I mean by this is that if you're on the boundary of the disk, there are a whole slew of directions in which you cannot travel without falling off of the disk. On the other hand, if you're in the interior of the disk, you can travel (for at least a little while, anyway) in any (planar) direction you like without falling off.

Is this all? Are there other 2-dimensional manifolds? Well...yes, of course there are! Actually, this isn't obvious, but if we think carefully, we can figure out how to construct some others. Now, when I say "construct," I really mean it. The point is that the easiest way to construct new manifolds from old manifolds is to cut and paste them together along parts of their boundary.

Try the following (You've probably seen this before, and if you haven't, you're in for a treat!). Cut a strip off of a piece of paper. Hold the two short edges of the strip, twist one side once, then glue the two short edges together along their common edge.

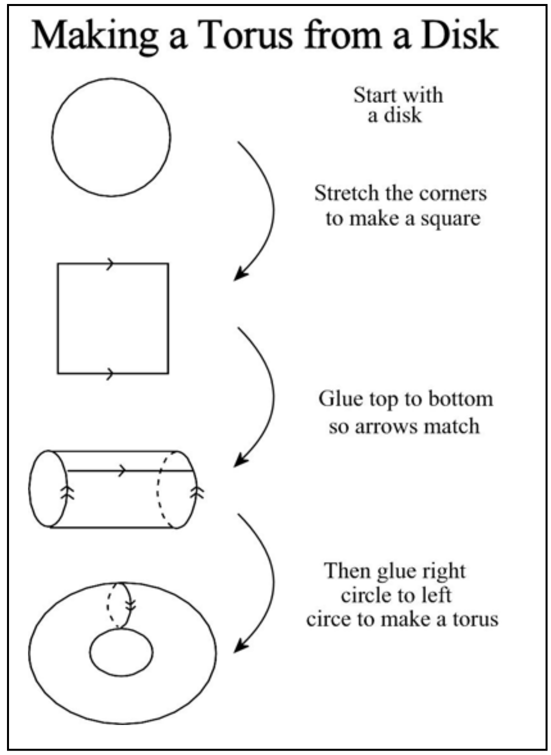


Now, starting anywhere on the strip, start drawing a line lengthwise along the strip, continuing until your line connects back to where it started. Notice anything strange? If you glued properly, you'll see that the 2-dimensional manifold you constructed has only one side! Unlike the plane, the sphere, and the disk, which all have tops and bottoms (in the case of the sphere, we call it the outside and the inside), the Möbius strip (as this 2-dimensional manifold is called) has only one side!

In mathematical terms, we say that the plane, the sphere, and the disk are all orientable, while the Möbius strip is non-orientable. In other words, if you and your friend meet for lunch on the Möbius strip, then start walking away from each other in opposite directions, when you run into each other again, your friend will be upside-down (and to her, *you* will look upside-down).

Let's close with one more example (my favorite): the surface of a doughnut. What's cool about this example (besides the fact that it's tasty) is that it can be constructed, just like we constructed the Möbius strip, by taking a 2-dimensional manifold we understand–the disk–and gluing together parts of the boundary in a prescribed way. Of course, it's not always that easy to glue things together (particularly when they're non-elastic, like a sheet of paper), but let's try to imagine the following.

Start with a disk made from some stretchable material like spandex. First, stretch it into the shape of a square. Now glue the top straight to the bottom to form a cylinder. This cylinder now has two ends, both of which are circles. We can now glue these circles together (this is where we run into difficulty if we're using paper), and voila! We have a doughnut (or rather, the surface of a doughnut, which mathematicians like to call a torus).



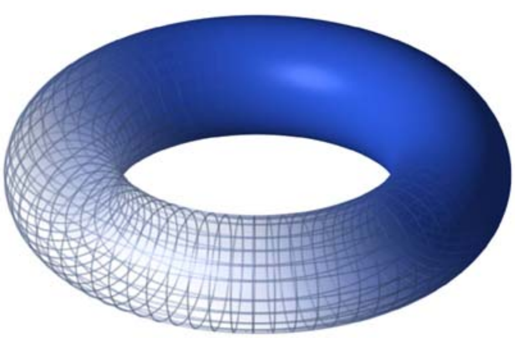
Questions:

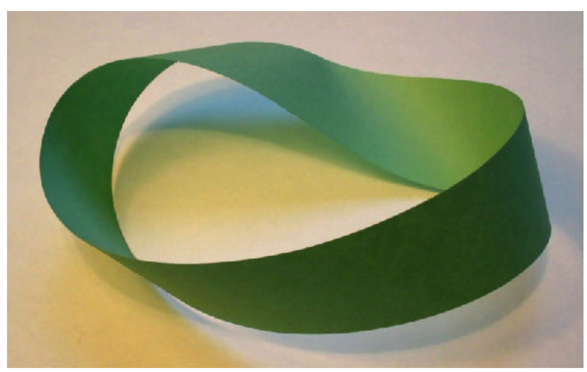
1. Is the torus orientable? Does it have a boundary? Is it finite or infinite?

2. Name some ways in which the torus is different from the sphere.

3. Can you figure out how to glue two disks together to get a sphere?

4. Can you think of other examples of 2-dimensional manifolds? Play around on this one: try to think of other ways that you can glue the 2-dimensional manifolds we've already constructed together to form other 2-dimensional manifolds.





A Torus and a Möbius strip.

Möbius strip: en.wikipedia.org/wiki/Image:M%C3%B6bius_strip.jpg

Notes from the Club

These notes cover some of what happened at Girls' Angle meets. They are not meant to be complete.

Session 2 – Meet 1 – January 31, 2008

Mentors: Hilary Finucane, Alison Miller, Ken Fan, Lauren McGough

After introductions and a quick round of the Introduction problem from the first session, we got to work on a problem from network theory. A group of people are trying to share information as quickly as possible. They start with each member knowing a unique piece of information. In each time step, the members can pair up, and the two members of each pair can fully exchange the information they know between each other. Note that if there are an odd number of members, there must be at least one person left out of the communication for each time step. What is the minimum number of time steps necessary for the members to share all their information so that everybody knows everything?

The girls quickly figured out the answer when the group has fewer than five people. However, with five girls, it starts getting tricky.

Here's a table for a few group sizes:

Group Size	1	2	3	4	5	6	7	8
Minimum Number of Time Steps	0	1	3	2	?	?	?	3

This is a challenging problem!

The Cat observed that it is possible to use a pairing scheme for groups sizes of 2, 4, and 8 so that the pairings correspond in a natural way with the edges of a line segment, square, and cube, respectively. She also saw a connection with powers of two.

August observed that if the group size is even, the group can be split into two equal-sized groups. If, in the first step, the members in one half each pair up with a different member in the second half, and then the two halves proceed in parallel as they

The Logarithm

In thinking about the network problem, **The Cat** noticed that it would be useful to have notation that gave the value of the exponent, given a base and a power of that base.

In other words, if $p = b^n$, what notation is used to denote n when given p and b? Mathematicians use the logarithm for this purpose.

If $p = b^n$ then we write $n = \log_b p$.

You might enjoy trying to show the following properties of the logarithm:

$$\log_b xy = \log_b x + \log_b y$$

$$\log_b x^n = n \log_b x$$

Can you find other properties of the logarithm?

would for a group of half the size, then all the information will be successfully spread among all members of the group. This enables one to conclude that the minimum number of time steps required for a group of size 2N is no more than 1 more than the minimum number of time steps required to handle a group of size N.

Ilana observed that if a member wanted to increase its knowledge quickly, the member should try to pair up with another member who had the most *complementary* knowledge.

Tree used props to represent the unique pieces of information and in the last twenty minutes, all the girls used the props to carry out the exchange of information for eight people, testing their understanding of the problem at that point.

<u>Session 2 – Meet 2 – February 7, 2008</u>

Mentors: Ken Fan, Lauren McGough

We split into two groups. Those who wanted to work on the network problem worked with Lauren and those who didn't worked with Ken on classifications (see page 7).

After the break, Lauren led a Dream Time on the Hotel Infinity.

The Hotel Infinity is an imaginary hotel with infinitely many rooms, one room for each positive integer. Unlike a normal hotel with finitely many rooms, when the Hotel Infinity is fully occupied, it can still accommodate more guests!

Lauren first asked the girls how a fully occupied Hotel Infinity could accommodate one more guest. The girls figured out that if the occupant in room N is told to move to room N + 1, that would free up room 1 while still accommodating all of the current occupants, so the new guest can move into room 1.

Lauren then asked how a fully occupied Hotel Infinity could accommodate any finite number, say G, of new guests. The girls figured this out too: just tell the guest in room N to move to room N + G. Rooms 1 through G will then be left vacant and ready to be occupied by the new guests.

Lauren then said that a crisis occurred at a second Hotel Infinity. This second Hotel Infinity was fully occupied when it suddenly went out of business. Can the first Hotel Infinity accommodate all of the guests in the second? **August** had the idea that multiplication by two might be helpful. **The cat** gave a clear and complete argument that, indeed, you can ask the guest in room N to move to room 2N. This will leave all the odd numbered rooms vacant and ready for all of the former occupants of the second Hotel Infinity. (At the beginning of Meet 3, **August** even provided a formula, stating that the guest who once occupied room K in the second Hotel Infinity could be placed in room 2K – 1 of the first Hotel Infinity.)

> The Grand Hotel Infinity was first presented by the German mathematician David Hilbert. Hilbert gave a famous talk in 1900 where he discussed a number of problems which have influenced the development of mathematics throughout the twentieth century and beyond. His speech began:
>
> "Who of us would not be glad to lift the veil behind which the future lies hidden; to cast a glance at the next advances of our science and at the secrets of its development during future centuries? What particular goals will there be toward which the leading mathematical spirits of coming generations will strive? What new methods and new facts in the wide and rich field of mathematical thought will the new centuries disclose?"
>
> *Translated from German by Dr. Maby Winton Newson.*

Next, Lauren said that there were *infinitely many* Hotel Infinities, numbered 1, 2, 3, 4, 5, etc. In other words, not only did each Hotel Infinity have one room for each positive integer, there was also an entire Hotel Infinity for each positive integer! All of them were fully occupied, when suddenly, all but one of them closed. Now can all the guests be accommodated in a single Hotel Infinity?

Session 2 – Meet 3 – February 14, 2008

Mentors: Beth Schaffer, Ken Fan, Lauren McGough

We continued thinking about the Hotel Infinity.

Ilana explained how to handle the last question from Meet 2. She and **August** worked with Beth to develop a precise formula. Beth reports, "My group was able to figure out the formula pretty quickly in imprecise English words. However, it then took awhile to convert that into a nice concise formula. We first had to learn how to sum $1 + 2 + 3 + \ldots + n$. Then they had to figure out the rest of the formula for Infinite Hotel Infinities which they did by looking at first 1 then 2 then 3, 4, 5, and then H hotels to find the pattern. They were slightly off several times but were able to recognize this by plugging in specific cases. At the end, we found a complete formula, but the girls weren't entirely certain on one of the parts and didn't get a chance to test-check it." Hopefully, we'll get to see their formula in the next issue of the Bulletin!

Some girls worked with Lauren on the network problem. Lauren reports, "I worked with **The Cat** and **Tree** on the problem introduced on the first meeting this session: how many time steps does it take to spread information to a group of N people if each person can communicate with one other person at each time-step. **The Cat** had determined numbers that she believed to be the least number of time-steps for N from one to ten; we spent some time looking for patterns in these numbers and predicting values for higher values of N. After not finding any consistent pattern, we spent time verifying the numbers that **The Cat** had determined, and realized that though her numbers were upper bounds, we were sometimes able to find ways of spreading the information in fewer time-steps than had previously been determined. We discussed ways of determining whether a given way of spreading information was optimal, and different ways of notating the spread of information to make optimal solutions clearer. Most interesting to us were odd N's, as in the case where N is odd, it is less clear whether a solution is optimal. While all three of us were working on verifying the numbers **The Cat** had previously determined, **The Cat** noted that our work might be more productive if we could just program a computer to go through all of the possibilities and determine the minimum number of time-steps it took to spread all of the information for a given N. She commented that we could then spend more time finding patterns in the minimum number of time-steps themselves. We then spent some time discussing advantages and disadvantages of doing these calculations by hand, and in finding an analytical method of proving whether a solution is optimal rather than testing every possible solution, and we discussed how we might program a computer to determine the least number of time-steps required for a given N."

The rest of the girls worked with Ken on more classifications. After exploring the complexities of people classifications, we discussed the concept of the equivalence relation and how an equivalence relation automatically gives you a partition. See page 10.

Special Announcements

On April 10, 2008, Girls' Angle will meet at MIT. Further details will be sent as the date approaches.

Calendar

Session 2: (all dates in 2008)

January	31	Start of second session!
February	7	
	14	
	21	No meet
	28	Visitor: Tanya Khovanova, mathematician
March	6	
	13	Visitor: Elissa Ozanne, Harvard Medical School
	20	
	27	No meet
April	3	
	10	Visitor: Karen Willcox, MIT Aeronautics Department
	17	Visitor: Leia Sterling, MIT Aeronautics Department
	24	No meet
May	1	
	8	

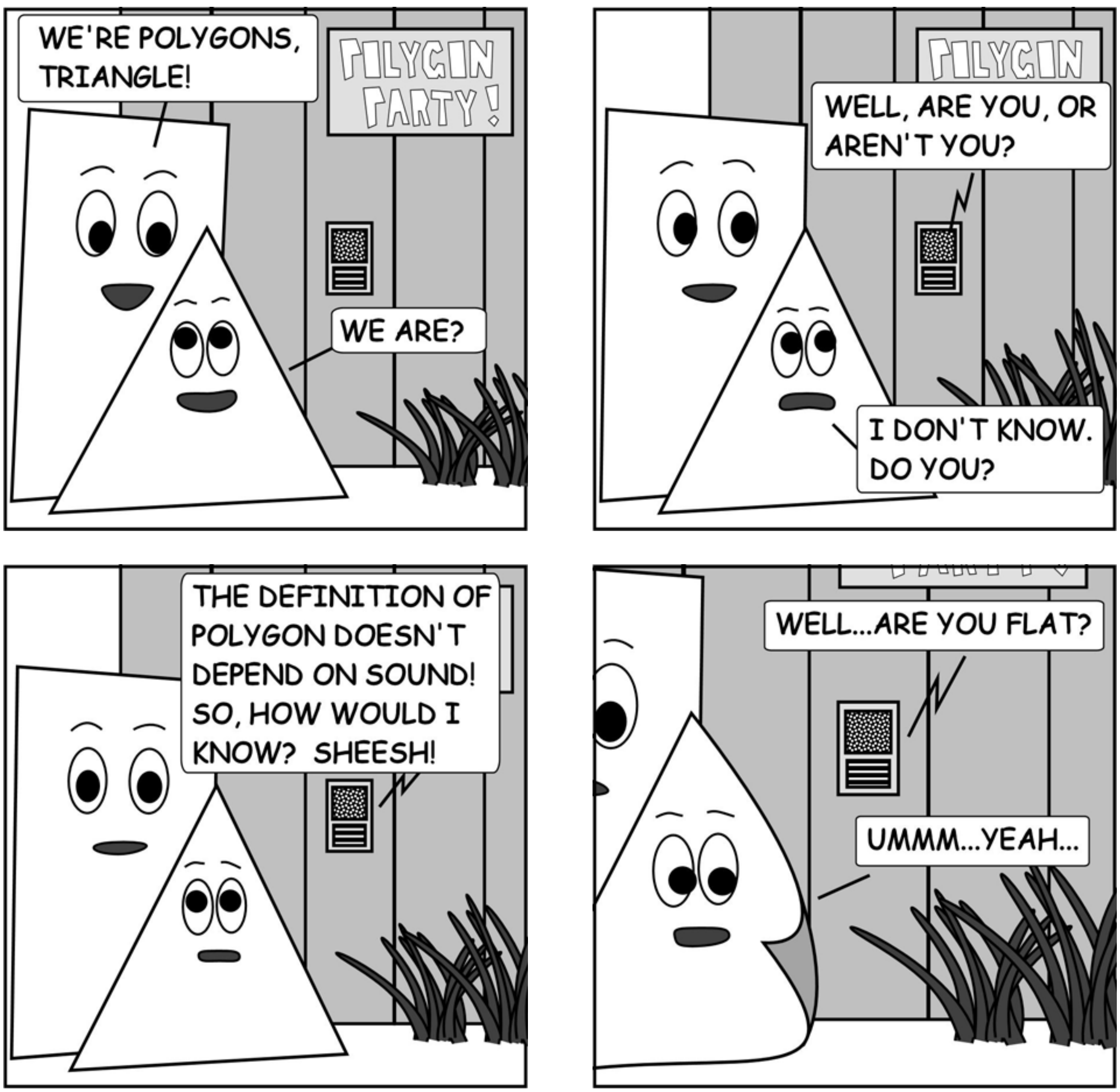
Session 3: (likely dates will be Thursdays in the months from September to December)

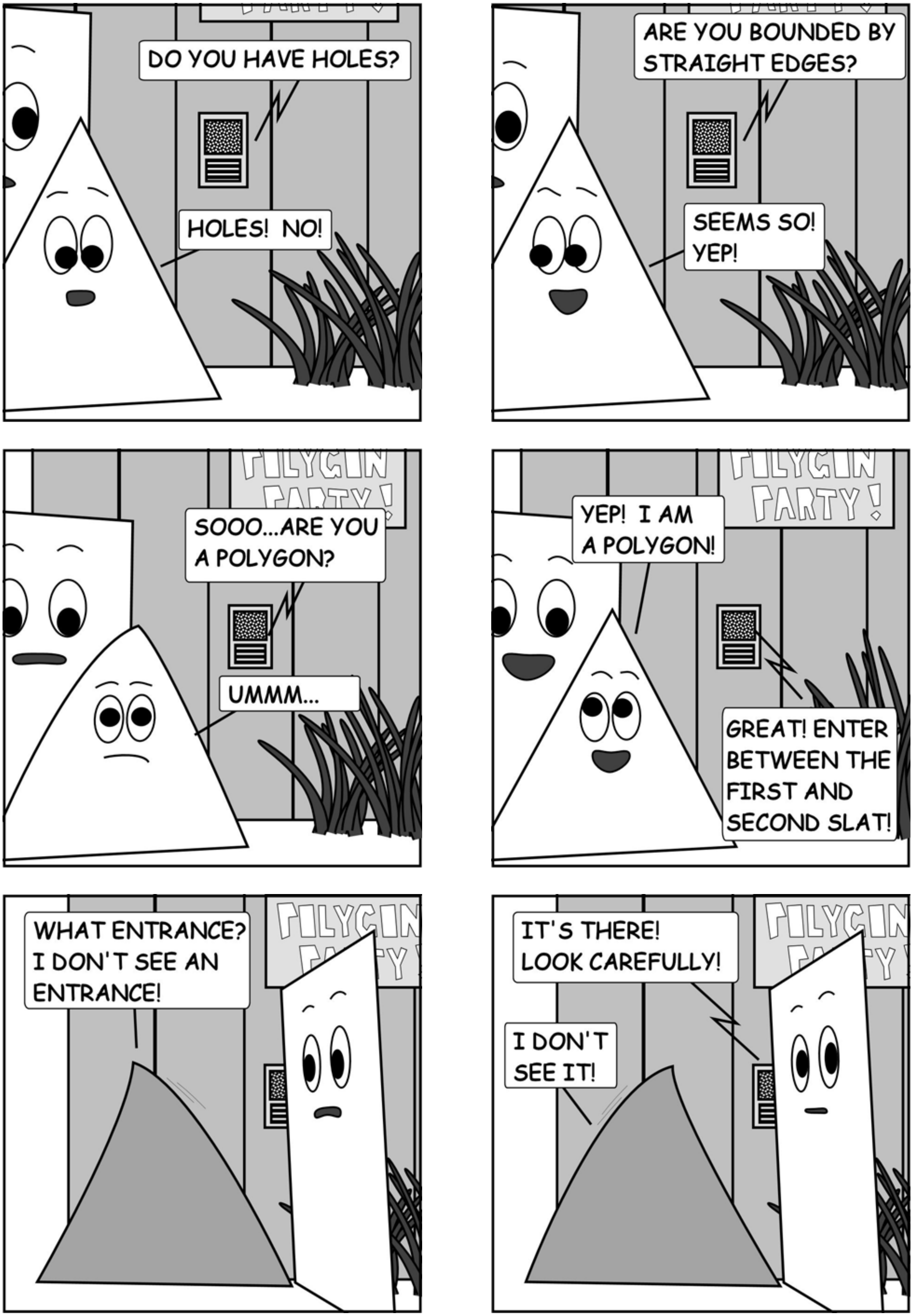
Feedback

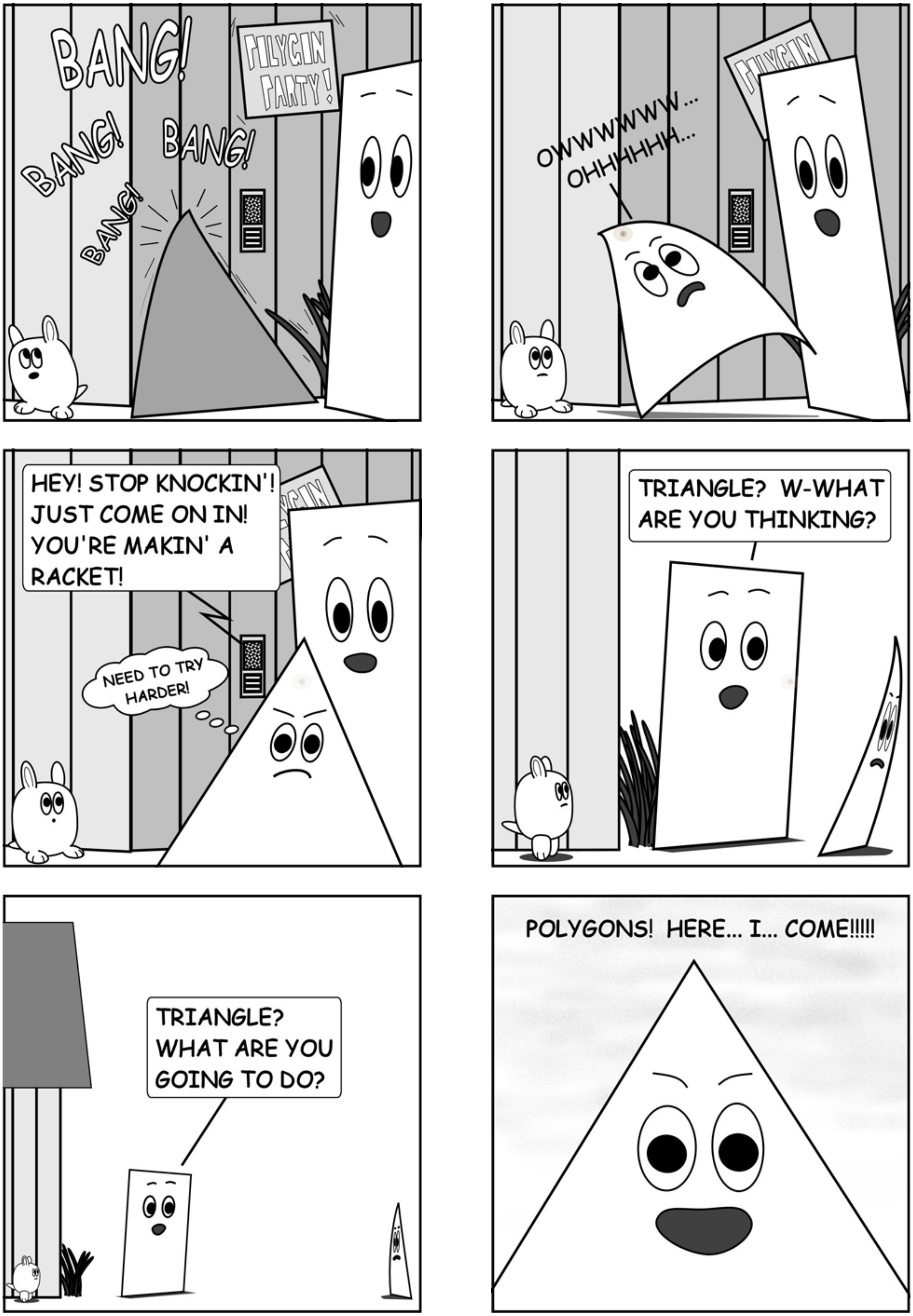
Please send feedback to girlsangle@gmail.com. We'd love to hear from you!

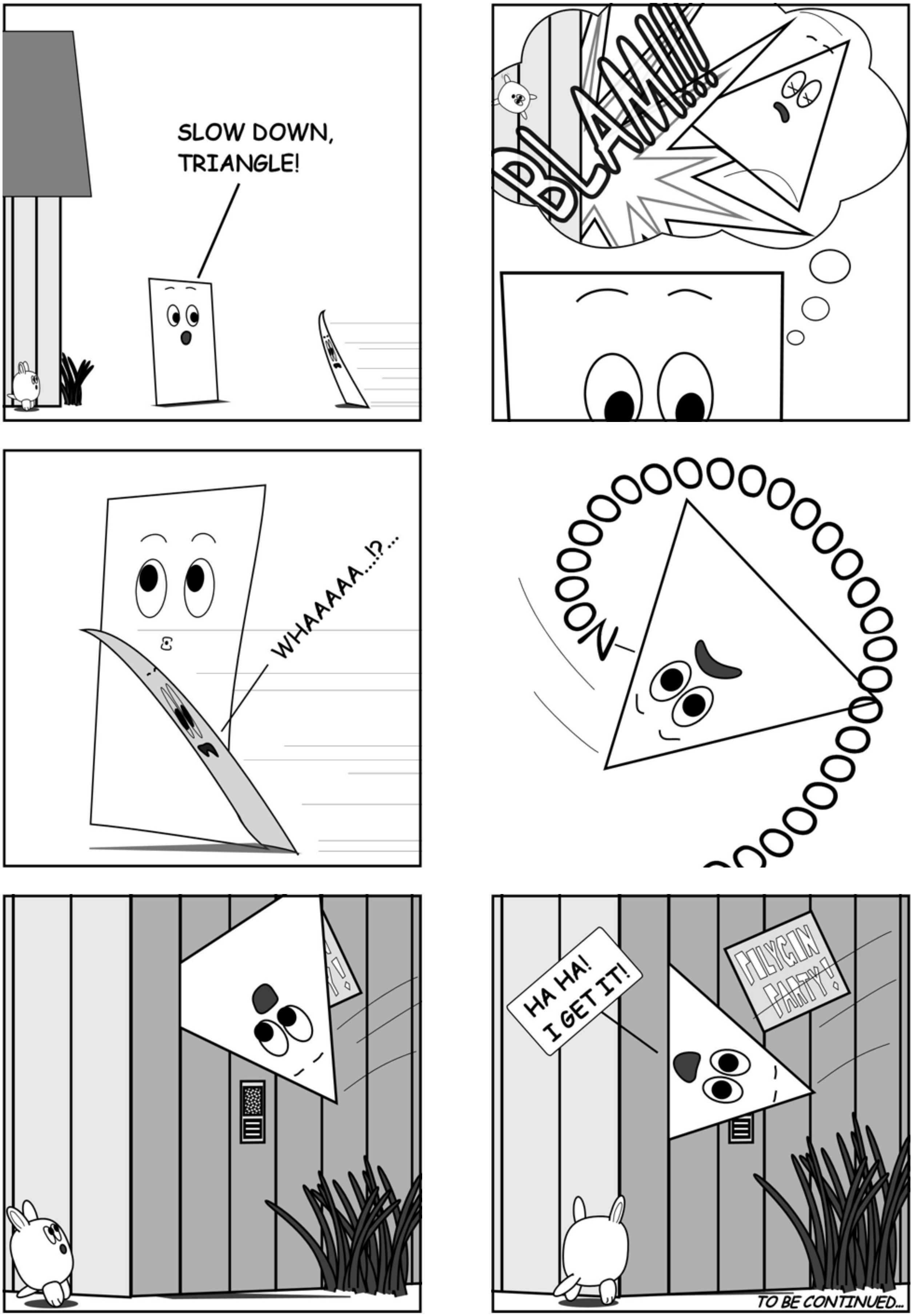
It Figures!

by CKFan




DO YOU HAVE HOLES?
HOLES! NO!
ARE YOU BOUNDED BY STRAIGHT EDGES?
SEEMS SO! YEP!
SOOO...ARE YOU A POLYGON?
UMMM...
YEP! I AM A POLYGON!
GREAT! ENTER BETWEEN THE FIRST AND SECOND SLAT!
WHAT ENTRANCE? I DON'T SEE AN ENTRANCE!
IT'S THERE! LOOK CAREFULLY!
I DON'T SEE IT!


BANG!
BANG!
BANG!
BANG!
POLYGIN PARTY!
OWWWWWW...
OHHHHHH...
HEY! STOP KNOCKIN'! JUST COME ON IN! YOU'RE MAKIN' A RACKET!
NEED TO TRY HARDER!
TRIANGLE? W-WHAT ARE YOU THINKING?
TRIANGLE? WHAT ARE YOU GOING TO DO?
POLYGONS! HERE... I... COME!!!!!


SLOW DOWN, TRIANGLE!
BLAM!!!
WHAAAAA...!?...
NOOOOOOOOOOOOOOOOOOOOOOOOOOOO
HA HA! I GET IT!
TO BE CONTINUED...

Girls' Angle: A Math Club for Girls

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

What is Girls' Angle? Girls' Angle is a math club for girls that aims to foster and nurture girls' interest and ability in mathematics. Instead of modeling after the traditional classroom experience, Girls' Angle is inspired by the lively activity in math department common rooms. Our philosophy is that mathematical ability is best developed through interaction with people who have both a deep understanding of mathematics and a genuine interest in helping others learn. Rather than 'teach math' at the club, we'll have helpers who work on motivation, motivation, motivation! The helpers, who will mostly be women, will introduce the girls to all kinds of activities, objects, and ideas that are math related trying to hook their interest. Once hooked, we will encourage them to explore, to think, and to ask and seek the answers to questions. We will show them all kinds of techniques that help one find answers, and we will show them how to craft questions that promote progress. The goal is to empower girls to be able to tackle any level of mathematics in the future so that no field, no matter how technical, will be off limits. We aim to overcome math anxiety and build solid foundations, so we will be welcoming all girls, not just those deemed gifted in mathematics.

Who can join? Ultimately, we hope to open membership to all women. Initially, we will be opening the doors primarily to girls in grades 5-8. We welcome *all girls* regardless of perceived mathematical ability.

In what ways can a girl participate? There are 3 ways: **membership**, **premium subscriber**, and **basic subscriber.** **Membership** is granted for per session or per meet and includes access to the club, the math question email service, and a subscription to the Girls' Angle Bulletin. **Premium subscriptions** last one session and include the math question email service and subscription to the Girls' Angle Bulletin. **Basic subscriptions** are one-year subscriptions to the Girls' Angle Bulletin. We operate in 10-12 meet sessions, but girls are welcome to join at any time. The program is individually focused so the concept of "catching up with the group" doesn't apply.

What is the Girls' Angle Bulletin? The Girls' Angle Bulletin is a bimonthly (6 issues per year) electronic publication that will feature articles and information of mathematical interest as well as a comic strip that teaches mathematics.

What is the math question email service? The math question email service allows a subscriber to email math questions that will be answered by staff or addressed during club meetings. Please note that we will not do math problems that *appear to us* to be for homework.

What do members get? Members get a one-year subscription to the Bulletin and one session of access to the club and the math question email service. The club will be a friendly place staffed mainly by women who have been selected for their deep understanding of mathematics and their desire to truly help others learn math. Helpers will take a personal interest in each member, assessing her mathematical abilities and working with her to motivate an interest in mathematics and mathematical topics by encouraging questions and explaining strategies and techniques for finding answers. Helpers will also organize fun activities that serve to introduce, explain, and clarify mathematical topics.

Where is Girls' Angle located? Girls' Angle is located about 10 minutes walk from Central Square on Magazine Street in Cambridge. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. Please inquire about the calendar. It is very important that you pick up your child promptly at 5:45.

Can you describe what the activities at the club will be like? Girls' Angle activities will be tailored to each girl's specific needs. We will assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes. If you believe in our approach and goals and want to help support us, we appreciate any contribution you can make. Currently, Science Club for Girls, a 501(c)3 corporation, is holding our treasury. Please make donations out to **Girls' Angle c/o Science Club for Girls** and send checks to Ken Fan, 27 Jefferson St., Cambridge, MA 02141.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was an assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study, and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science and worked in the mathematics educational publishing industry. Ken believes that mathematics education in this country can be improved significantly. Also, through the years, he has witnessed instances of gender bias in mathematics and in math education. The last two summers Ken volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were hung at Boston Children's Museum. The girls of Science Club for Girls showed a lot of creativity and ingenuity and were able to realize their ideas in the final project, something that may not have happened in a co-ed environment. These experiences have motivated him to create Girls' Angle.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, graduate student in mathematics, UC Berkeley
Julia Elisenda Grigsby, NSF postdoctoral fellow, Columbia University
Lauren McGough, advanced high school student who founded her school's math club
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, Senior Research Scientist, Harvard Medical School.
Kathy Paur, graduate student in mathematics, Harvard
Lauren Williams, Benjamin Pierce assistant professor of mathematics, Harvard

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematical competence required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle helpers can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: A Math Club for Girls

Membership Application

Applicant's Name: (last) ______________________________ (first) ______________________________

Applying For:
- ☐ Member (Access to club, Math question email service, Receive Bulletin)
- ☐ Premium Subscriber (Math question email service, Receive Bulletin)
- ☐ Basic Subscriber (Receive Bulletin)

Parents/Guardians: ____

Address: ____ Zip Code: __________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. They will have to sign her out. Names: ____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about? ____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: For now, girls who are roughly in the grade 5-8 range are welcome. Although we will work hard to include every girl no matter her needs and to communicate with you any issues that may arise, Girls' Angle has the discretion to dismiss any girl whose actions are disruptive to club activities.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: ________________________

(Parent/Guardian Signature)

Membership-Applicant Signature: ____

☐ Enclosed is a check for (indicate one) (prorate as necessary)

☐ $216 for a 12 session membership ☐ $100 for a 12 week premium subscription

☐ $20 for a one year basic subscription ☐ I am making a tax free charitable donation.

☐ I will pay on a per session basis at $20/session. (Note: You still must return this form.)

Please make check payable to: **Girls' Angle c/o Science Club for Girls**. Mail to: Ken Fan, 27 Jefferson St., Cambridge, MA 02141. Paying on a per session basis comes with a one year subscription to the Bulletin, but not the math question email service. Also, please sign and return the Liability Waiver.

Girls' Angle: A Math Club for Girls

Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____ Date: ________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____________________________________

Girls' Angle

A Math Club for Girls