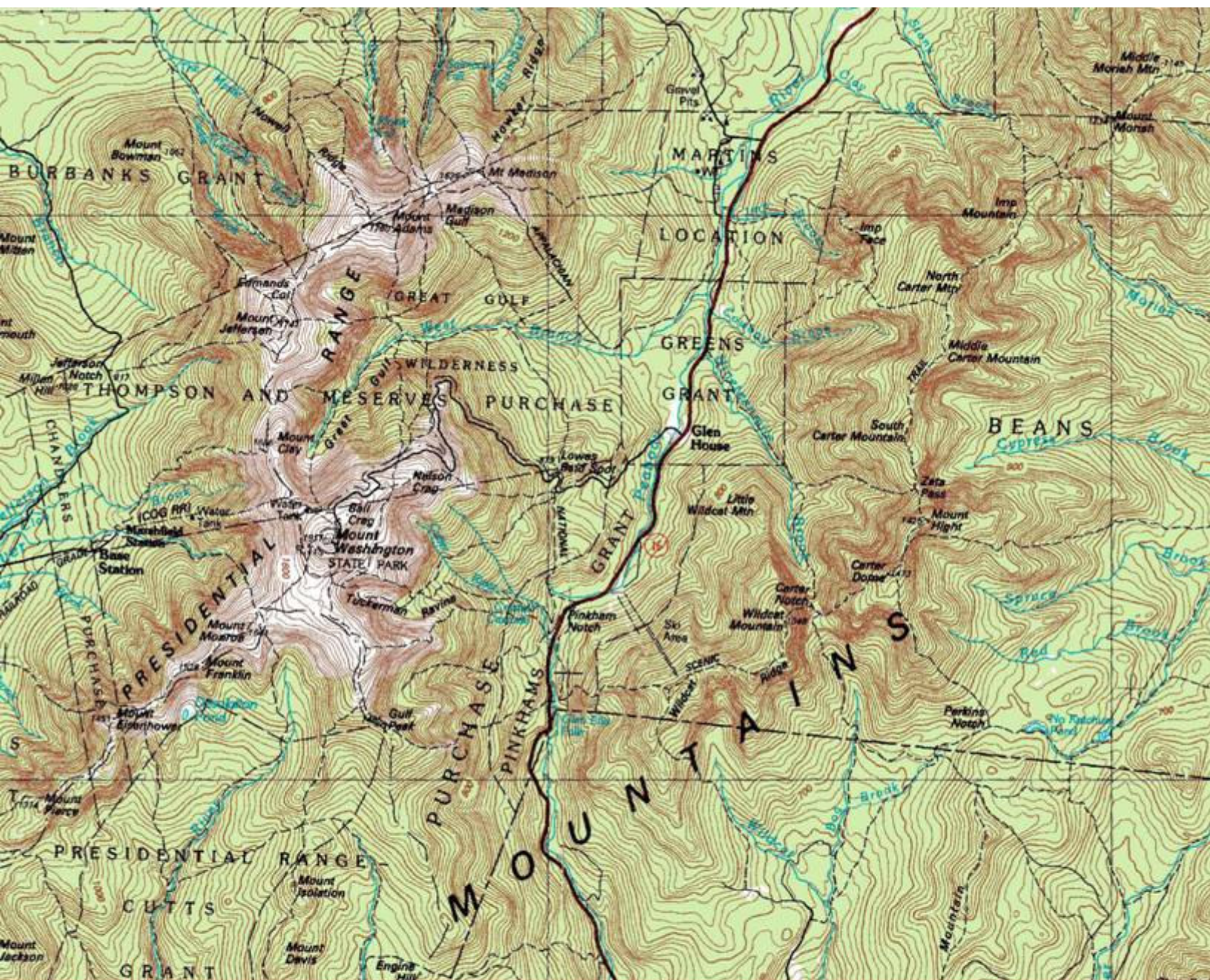


# Girls' *Angle* Bulletin

December 2009 • Volume 3 • Number 2

*To Foster and Nurture Girls' Interest in Mathematics*



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# From the Director

Happy Holidays and Happy New Year!

Our fifth session ended just 3 minutes late, when the girls managed to open both combination locks to the box that held their well-earned treasure. Read more about the treasure hunt in the Notes from the Club section.

In this issue, I'd like to draw particular attention to the interview with **Cathy Kessel**, the former president of the Association for Women in Mathematics. I've benefited a great deal from conversations I've had with her. She has examined gender issues in mathematics with great care and depth.

Also, **Amy Tai** starts us off with a highly graphical proof of the concurrency of the medians of a triangle. Please let us know if you like this style of proof.

Finally, I'm excited to announce that **Bjorn Poonen** has joined our Advisory Board. Bjorn is the Claude Shannon Professor of Mathematics at MIT. He can be seen in the recent documentary *Julia Robinson and Hilbert's Tenth Problem*.

All my best,  
Ken Fan  
Founder and Director

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## Girls' Angle Bulletin

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Girls' Angle welcomes submissions that pertain to mathematics.

Editor: C. Kenneth Fan

## Girls' Angle: A Math Club for Girls

*The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and empower them to tackle any field no matter the level of mathematical sophistication.*

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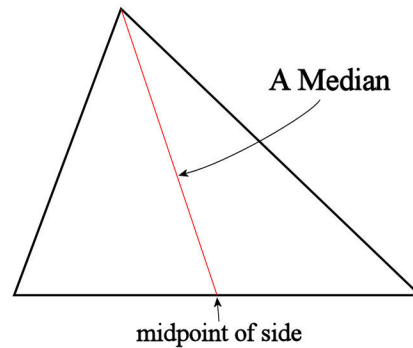
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On the cover: A topographical map of Mt. Washington, New Hampshire, courtesy of the U. S. Geological Society.

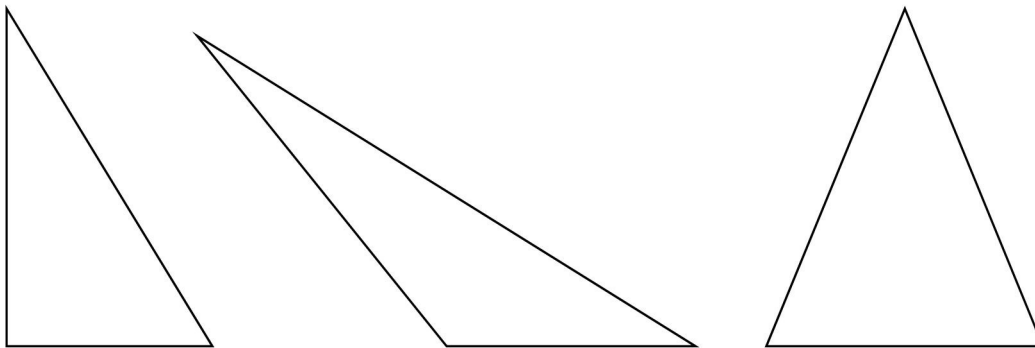
# Medians

by Amy Tai

A **median** of a triangle is a line segment that runs from a vertex of the triangle to the midpoint of the opposite side, as shown below.

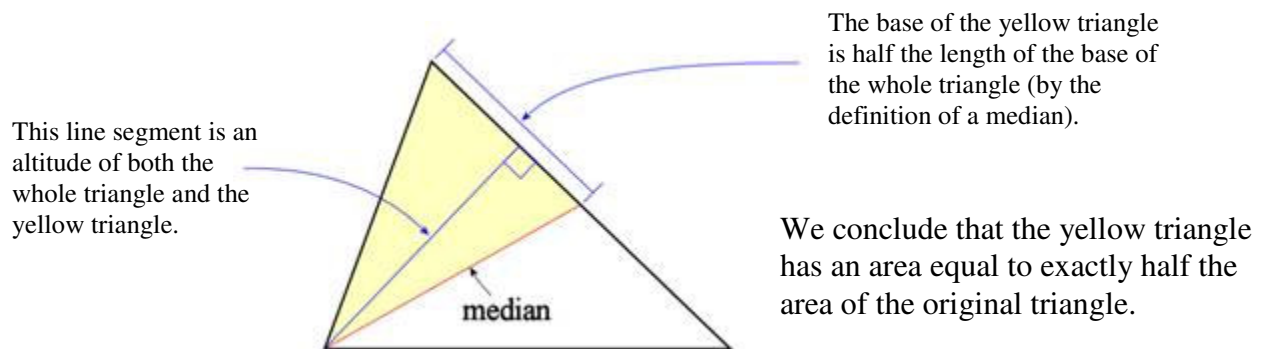


Because every triangle has 3 vertices, all triangles have 3 distinct medians. Draw all of the medians in the 3 triangles below.



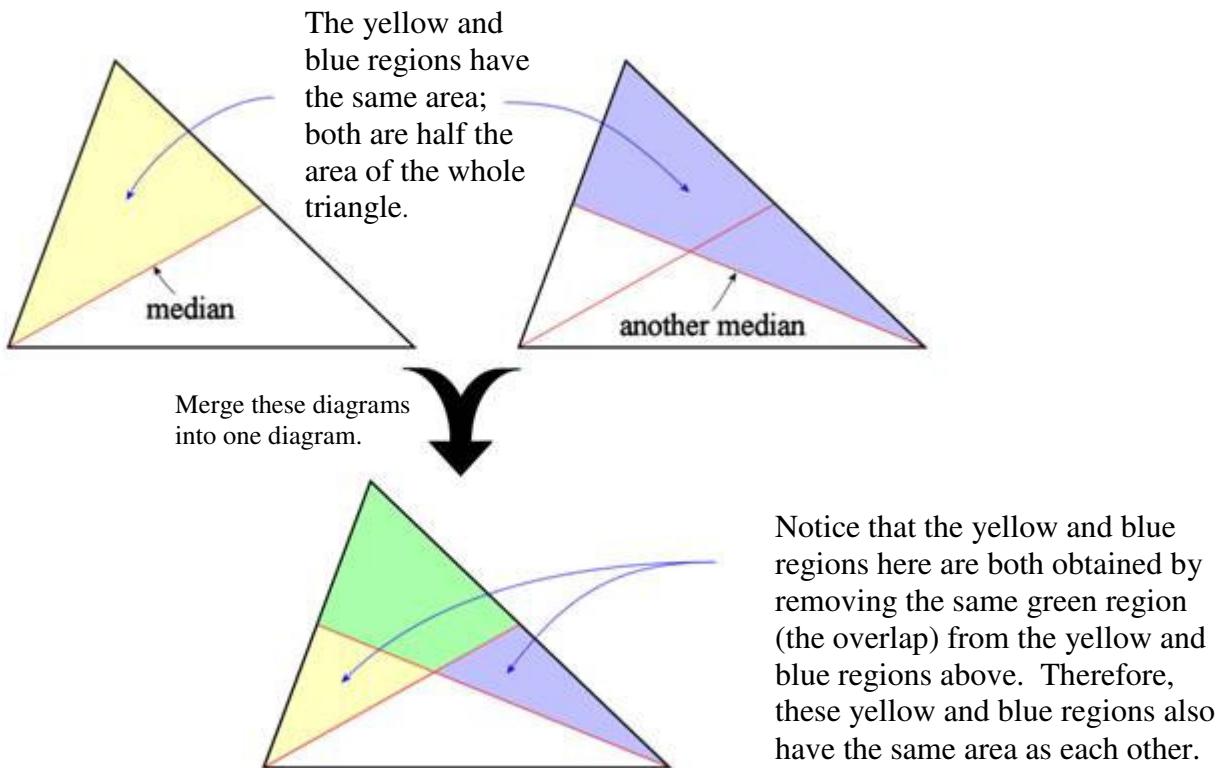
In each triangle, do your medians intersect in a point? If so, you've drawn them well! The 3 medians of any triangle always intersect in a single point<sup>1</sup>. Let's prove this!

Look at the median drawn below.

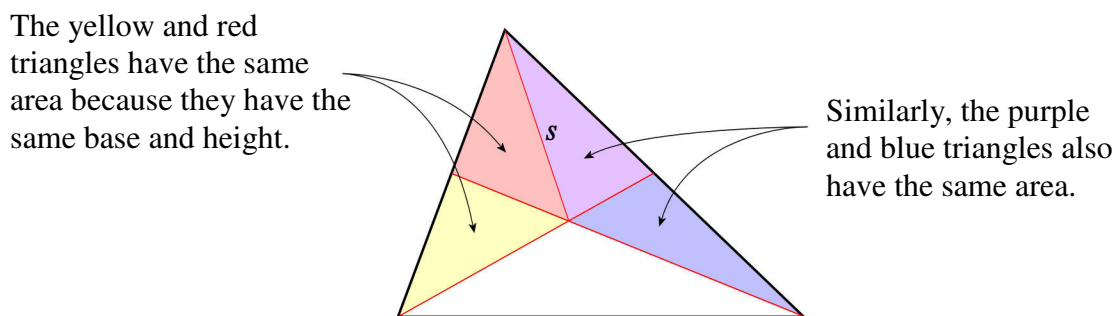


<sup>1</sup> The point where the medians intersect is called the **centroid**. If the triangle were made of a uniform material, the centroid is where the triangle would balance.

Draw a second median of the triangle. The blue region has an area equal to exactly half of the area of the original triangle, by the same reasoning as above. Therefore, the blue and yellow regions have the same area.



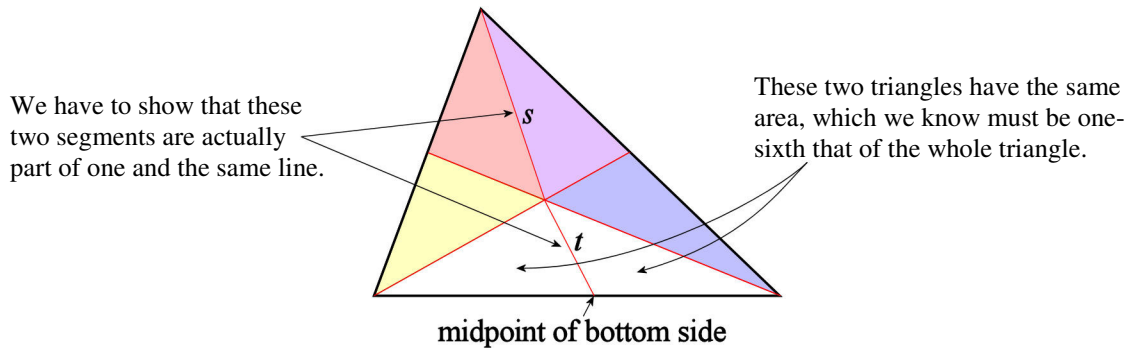
In this last figure, let's draw the line segment from the top vertex to the intersection of the two medians. I'll label this segment with the letter  $s$ .



We conclude that the four small triangles (yellow, red, purple, and blue) all have the same area. We've already seen that the triangle formed by the yellow, red, and purple triangles has area half that of the whole triangle. This implies that the yellow, red, purple, and blue triangles all have area one-sixth that of the whole triangle.

What we want to show is that if we extend  $s$  from the top vertex all the way to the opposite side, it will meet the opposite side exactly at its midpoint. Before going to the next page, try to complete the argument yourself.

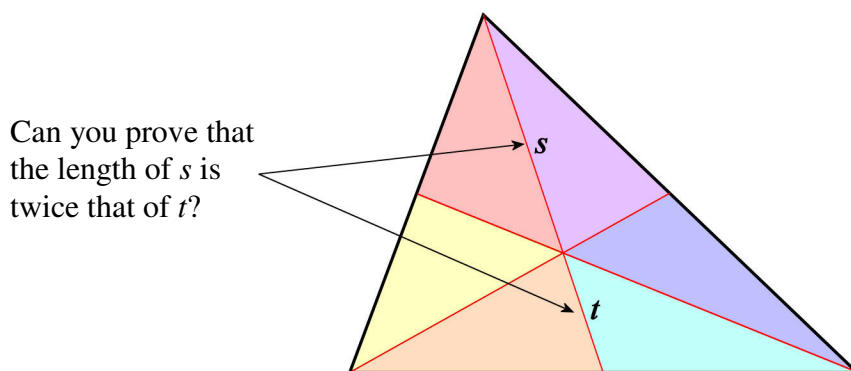
Let's add a line segment, call it  $t$ , that runs from the intersection of the two medians to the midpoint of the bottom side. (This segment is a median of the white triangle.) What we must show is that  $t$  actually extends segment  $s$ . If we drew everything very precisely, it would, in fact look like such an extension, because, in truth, it is! However, we have not yet *proven* this fact, and for all we know,  $t$  might *look* like an extension of  $s$  but actually be a bit off. So, in the diagram below, I'll deliberately draw in  $t$  so that it doesn't look like a linear extension of  $s$ .



Notice that the region consisting of the red triangle, the yellow triangle, and the left white triangle has area half of the whole triangle. Also, the region consisting of the purple triangle, blue triangle, and right white triangle has area half of the whole triangle. In other words, our original triangle is split into two equal area parts by the barrier consisting of segments  $s$  and  $t$ . But, the median from the top vertex, which also connects the top vertex with the midpoint of the bottom side, also splits our original triangle into two pieces of equal area. The only way this can happen is if the median and the barrier made of segments  $s$  and  $t$  are one and the same!

The proof is complete: The medians of a triangle are concurrent!

As a side benefit of this proof, we can also conclude that the three medians of a triangle partition the triangle into 6 triangles all with the same area.



The three medians of a triangle partition the triangle into six smaller triangles of equal area.

The medians of a triangle are examples of **Cevians**. We'll explore other Cevians in future issues of the Bulletin.

# An Interview with Cathy Kessel, Part 1

Cathy Kessel recently served as president of the Association for Women in Mathematics, a nonprofit organization created to encourage women and girls to study and to have active careers in the mathematical sciences, and promote equal opportunity and the equal treatment of women and girls in the mathematical sciences. I am fortunate to have had the opportunity to learn directly from her and I'm thrilled to present readers with this two-part interview.

**Ken:** You served as the president of the AWM. How did you become interested, first, in mathematics?

**Cathy:** My assumption is that everyone starts out interested in mathematics and science—as described by *The Scientist in the Crib*<sup>2</sup>. As the introduction says, “We are born with the ability to discover the secrets of the universe and of our own minds, and with the drive to explore and experiment until we do.” The way that I think about my interest in mathematics is that I was fortunate enough to have had my mathematical inclinations nourished as I grew up.

My mother told me that I insisted on playing with the pots and pans in the kitchen when I was very young. She put the pots and pans in the lowest shelves of the cupboards and let me play. She didn't say what I was doing with the pots and pans, but I imagine that I was figuring out spatial relationships, perhaps understanding containers.

One of my friends from New Zealand, who does research in mathematics education, told me that she interviewed parents of people who got undergraduate degrees in mathematics. The parents told similar stories about how their children insisted on playing in ways that involved spatial activities—creating patterns with toys, for example.

Another one of my mother's stories was about how I found cutting out cookies interesting and rolled out the dough many times—resulting in horrid cookies because the dough was toughened by such treatment!

Looking back on this as an adult, my guess is that it had to do with understanding area and volume. Anthropologists have found that Mexican children of potters, who learn to help their parents make pots when they are quite young, tended to be several years ahead of other Mexican children when they are tested on their understanding of volume. The technical term for this kind of understanding is “conservation of volume.” If you change a shape, say squash a ball of clay,

*At home, my parents didn't discourage me if I struggled with my homework.*

young children will often say that the mass of clay has changed its size—there is more or less of the clay than there used to be. Eventually, children learn that squashing a ball of clay or pouring a fixed amount of water

into a taller or shorter glass doesn't change the original volume of the clay or water. The Mexican potters' children learned this sort of thing before other Mexican children. My conjecture when they are quite young, people who later develop a talent for mathematics either seek out such experiences or happen to be in an environment that provides them. The Mexican potters' children are provided with opportunities to think about volume because they are asked to

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<sup>2</sup> *The Scientist in the Crib: What Early Learning Tells Us About the Mind*, by Gopnik, Meltzoff, and Kuhl.

help their parents—and there are similar examples from other studies. This suggests to me that the environments in which children grow up can provide opportunities for them to learn about shapes and space. Those opportunities might be at home, or in a pre-school or Montessori school. Or, children can create their opportunities—if their caretakers allow it.

When I got older, I became interested in origami (paper folding), kirigami (paper cutting), and string figures—again, things that involved two- or three-dimensional space. I suspect that my mother's interest in art also contributed to my interest in spatial things. From fourth grade on, I was lucky enough to attend a private school with many good teachers—in art and science as well as mathematics.

At home, my parents didn't discourage me if I struggled with my homework. Recent research in psychology suggests letting me persist rather than telling me that it was OK to give up might have been a very helpful thing to do. It may have been especially helpful for mathematics, which often (in the U.S., at least) seems to be stereotyped as a subject where you either "get it" instantly or you don't. If you don't seem to understand something instantly, then you may be told by a well-intentioned person, perhaps even one of your parents, "That's OK, I was never good at math either" and not encouraged to work at understanding. I was very fortunate not to have encountered that kind of reaction when I was young. Mathematics, and probably any subject worth knowing, takes time to learn. But, I have noticed that years after I've learned something, it can seem very simple and obvious—even if I can remember that it wasn't when I first encountered it.

In high school, I took calculus and was fascinated by that. Calculus might have been the first time that I was really gripped by a school subject. Some people talk about the first sleepless night spent thinking about a problem as being a mathematical rite of passage and I remember that happening for a calculus problem.

I was fortunate in being able to go to college at the University of Chicago. There, I was fascinated by abstract algebra, which I think of as being about structure. Model theory (a branch of mathematical logic) is in some sense even more about structure—it's more abstract—and I liked that too. Model theory is a part of mathematical logic, thus, a neighbor of philosophy of mathematics—which has grown in interest to me over the years.

**Ken:** What a fascinating history! I can see how you became interested in math... so, how did you become interested in women and mathematics? What led you to want to lead the AWM?

**Cathy:** When you are the only woman in your class or one of a very few, it's hard not to notice. (And, if for some reason, you don't notice, someone is likely to point it out—at least that seemed to be the case when I was a student.) Different people give different amounts of time and energy to understanding why. I think that my interest in the foundations of mathematics and how people come to know mathematics is closely related to my interest in mathematics and gender.

Gottlob Frege, a prominent nineteenth-century logician, was famously against considering psychology in the foundations of mathematics. Anti-psychologism seems to be a strong tradition in foundations, but may also have spilled into mathematics itself.

In contrast to Frege, I am willing to consider psychology—and history, sociology, and cognitive science—in helping me to understand how mathematics evolved, how people learn it, and how it

is practiced. But, there's still a part of me that's a Platonist—I separate my ideas about what mathematics actually is from ideas about how we come to know and do it.

Because I've been able to spend substantial amounts of time around the School of Education at the University of California at Berkeley, I think that I've had an exceptional opportunity to pursue my interests. One person who has helped me understand a lot of relevant research on gender is Marcia Linn, an educational psychologist who has been involved for a long time with science education, designing curriculum and ways of teaching. She's expert in analyzing human behavior—and she has a long-standing interest in gender and cognition.

Other people have helped me to understand different pieces of the puzzle—for example, the idea that how universities are organized can make a difference in retaining women. When I went to graduate school, I didn't understand that different universities—and their math departments—can have quite different cultures. It's taken me some time to understand that these differences can be explained somewhat by their history and the way they're organized.

I've also been able to learn more about research in mathematics education; to be involved in some research projects, and to work as a researcher, and later to be involved in producing various documents, for example, recommendations for the mathematical education of teachers.

I don't think that many mathematicians have had similar opportunities for thinking and learning about mathematics with respect to gender and education. Because of that, I felt that I had something different and important to offer AWM which I hope has enriched the organization. And, it's given me an exceptional opportunity to learn more.

*...how universities are organized can make a difference in retaining women.*

**Ken:** What are the major needs that need to be addressed by organizations such as the AWM?

**Cathy:** I think there are two kinds of needs, inside and outside of mathematics. Outside of mathematics (and maybe also a bit inside), we need to address stereotypes about women and mathematics. Simply providing accurate and relevant information is a first step. When I am interviewed, I try always to mention that women have been getting between 40% and 48% of the undergraduate degrees in mathematics for the past 20 years. That's something that people often find surprising. Mathematics is actually one of the most gender-balanced undergraduate majors. And the percentage of women getting PhDs in mathematics has been increasing, slowly but steadily, since the 1970s.

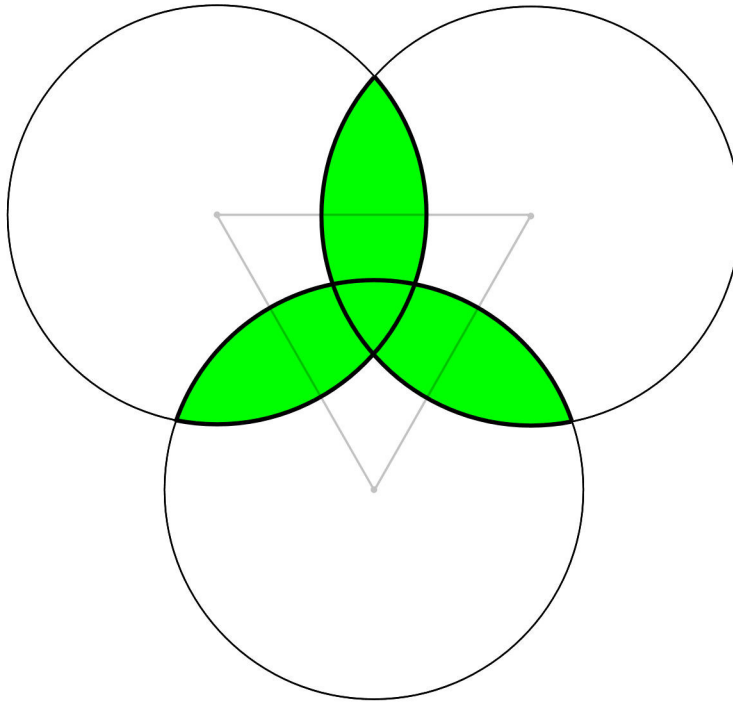
Within mathematics, we need to provide support and visibility for women, and accurate and useful information for the mathematical community. While I was president of AWM, I had first-hand experience of how many people are supportive of women in mathematics and how much they have to offer. Part of what AWM does is to help women and girls find what they are looking for—mentors, mathematicians with similar interests, and all kinds of information—about careers, getting through graduate school, or searching and applying for jobs.

**Ken:** In 2004, the AMS conducted a survey and found that at ten of the top math departments in the US, less than 6% of the tenured faculty were women. Do you think there is an explanation for this statistic? If so, what is your explanation?

*To be continued...*



# Zindler's Flower



The green region above is known as a **Zindler Flower**<sup>1</sup>. It is formed out of the intersection pattern of three circles of equal radii. The centers of the circles are located at the vertices of an equilateral triangle. The exact length of the radii isn't crucial, but, for the sake of definiteness, feel free to take these radii to be two-thirds of the length of the side of the equilateral triangle.

How can you split the Zindler Flower into two pieces of equal area using a single line segment?  
How many ways can you find to do this?

In fact, it's generally true for a shape that for a given direction, there is a unique line in that direction which will split the shape in such a way that the parts of the shape on one side of the line have the same area as the parts on the other side. Can you identify the line segment that cuts the Zindler Flower into equal area pieces for each direction?

Can you see that these area-halving line segments all have the same length?

Can you think of another shape whose area-halving line segments all have the same length?  
How many such shapes do you think there are?

How do these area-halving line segments split the perimeter of the Zindler Flower?

For more area halving problems, see page 15.

Send in your thoughts and observations to [girlsangle@gmail.com](mailto:girlsangle@gmail.com)!

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<sup>1</sup> Named after the German mathematician K. Zindler.

# Order Me!

Here's a problem from the Session 5 treasure hunt. Ten numbers are described in each box. Place these numbers in order from least to greatest. What position does the number 10 occupy? No calculators are allowed! Good luck!

The number of sheets of paper in the last issue of the Girls' Angle Bulletin.

$$\pi^2$$

$$\sqrt[3]{1111}$$

The reciprocal of  $0.00011001100110011001_2$ .

$$11$$

?

$$\sqrt{24} + \sqrt{26}$$

The distance between (0, 0) and (7, 7).

The circumference of a circle whose radius is the golden mean.

Area of an equilateral triangle of side length 5.

$$10$$

Feel free to send your answers to [girlsangle@gmail.com](mailto:girlsangle@gmail.com).  
The answers will appear in the next issue of the Bulletin.

# Anna's Math Journal

By Anna Boatwright

Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna Boatwright gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.

Here, Anna tackles a balance scale problem: There are 12 coins, all identical except for one that looks the same but is a different weight from the rest. Using a balance scale 3 times, can you determine which coin is the counterfeit and whether it is heavier or lighter than the rest?

What can I do with a balance scale?

ooo vs. ooo ?

Keep the number of coins on each side the same.

Assume that the counterfeit coin weighs more than the others.

On a balance, I place some coins on each side and the scale tells me which side is heavier. Does it ever make sense to put different numbers of coins on each side? If the coins are close in weight, I already can predict that the scale will tip to the side with more coins. So I won't learn anything new. Therefore, it only makes sense to put the same number of coins on each side.

To get a feel for this problem, I'll simplify it by assuming that the counterfeit is heavier than the others.

The counterfeit has to be one of the blue coins.

Step I Use the balance scale as shown:

ooo vs. ooo  
ooo vs. ooo

- 1) Place 6 coins on each side
- 2) Determine which side is heavier, label all these blue

I'll just start by splitting the coins into two groups of six. If it doesn't work, I can always try something else later.

Step II

ooo vs. ooo  
ooo vs. ooo

- 1) Remove the white coins and divide the blue coins into 2 groups of 3
- 2) Determine which side is heavier, label these with a hat.

The counterfeit has to be one of the hatted coins.

Step III

ooo vs. ooo  
ooo vs. ooo

- 1) Pick any 2 of the blue coins with hat and place them on either side of the scale.

There are 2 possible outcomes to this final step:

Case 1: The 2 coins on the scale weigh the same.

This means that the other blue hat coin (not on the scale) is the counterfeit coin.

Case 2: The 2 coins on the scale do not weigh the same.

This means that the heavier coin is the counterfeit.

That's good! If the counterfeit coin is heavier, these steps give a way to find it.

Key:

Anna's thoughts

Anna's afterthoughts

Editor's comments



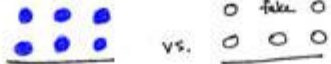
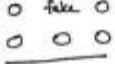
Assume that the counterfeit coin weighs less than the others.

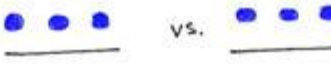

Repeat Steps I - III, but replace heavier with lighter each time. This process will find the counterfeit coin in just the same way.


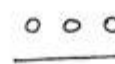
If we assume the counterfeit is lighter than the others, the same method will basically work. I just have to replace heavier with lighter throughout. I'll go ahead and make note of this for completeness sake.

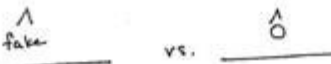
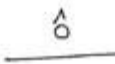
I want to be able to answer the question without needing prior knowledge about whether the counterfeit coin is heavier or lighter than the others. What happens if the assumption that the counterfeit coin is heavier is false but I use that method anyway?

I will pretend that I am assuming that the counterfeit coin is heavier but really the counterfeit coin is lighter than the rest (but I pretend I don't know that until the very end). Let's see what will happen...

**Step I**  vs.  ← I label the heavier coins blue.

**Step II**  vs.  ← This time the scale balances, so I know I've made a mistake and taken away the fake coin.

**Step III**  vs.  ← I weigh the other 6 coins that I took away after Step I. Since I now know that my assumption was wrong, I look for the lighter side this time and label with a hat.

**Step IV**  vs.  ← This is just like step III from Case 1. I find the fake as the coin that is lighter on the scale, or the one that is not on the scale if the scale is balanced.

But this has taken 4 steps. Can I do it in 3?

It's not so clear from my notes, but at this point, there is at least a way to solve the problem in 4 steps. One can proceed assuming that the counterfeit coin is heavier. At Step II, the validity of the assumption is revealed. If it is correct, Step II won't balance and I can proceed as on the first page. Otherwise, I continue as indicated here, knowing that the counterfeit coin is really lighter.

Key:

Anna's thoughts

Anna's afterthoughts

Editor's comments

Can you help Anna? Figure out if it is possible to find the counterfeit coin and determine if it is lighter or heavier than the others using the balance scale at most three times! Send your solutions to [girlsangle@gmail.com](mailto:girlsangle@gmail.com).

ABB 12.16.09

# Notation

by Ken Fan

Through time, some ways to notate mathematical ideas have become standard. Learning the notational conventions is important in order to be able to communicate. The best way to learn new notation is the same way we learned the alphabet: repetition. Practice makes perfect!

## Exponential Notation

It often happens that we have to multiply something by itself several times. For example:

What is the volume of a cube with side length  $s$ ? The answer is  $s$  times  $s$  times  $s$ .

At the recent treasure hunt at Girls' Angle, we used two combination locks whose combinations consisted of 3 digits. How many possible combinations are there? The answer is 10 times 10 times 10 for a single lock and 10 times 10 times 10 times 10 times 10 times 10 for both locks.

If you roll five six-sided dice (such as in the game Yahtzee), how many different ways can they come up? The answer is 6 times 6 times 6 times 6 times 6.

If you put money into a savings account that earns 2% interest every year by what factor would the original deposit grow after twenty years?

I'll write the answer to this last one shortly. But from these examples, it's easy to understand why people would invent notation to simplify the writing of such repeated multiplications. The notation in current use is called **exponential notation**. It looks like this:

The lower number is the number that is multiplied by itself. This number is also called the **base**.

→ 3<sup>8</sup>

The upper number tells how many times the lower number will be multiplied by itself. This number is called the **exponent**. It is written as a superscript: slightly smaller, elevated, and on the right.

This is read "three to the eighth power". Without exponential notation,  $3^8$  would have to be written something like this:  $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$ . Visually, isn't it difficult to see how many threes there are in that expression? With exponential notation, it's clear. Thus, the answers to the first 3 examples above can also be written:  $s^3$ ,  $10^3$  and  $10^6$ , and  $6^5$ .

If  $m$  and  $n$  are positive integers, notice that  $b^m b^n = b^{m+n}$ . Also,  $(b^m)^n = b^{mn}$ .

So far, our notation only makes sense for exponents that are positive integers. Can we extend the definition to include exponents that are zero or negative?

Well, since we are the creators of mathematical notation, we could extend our notation in any way we please! *However*, a neat idea people had is to try to extend the notation in such a way that the formula  $b^m b^n = b^{m+n}$  remains valid. If we try this, we find that for nonzero bases  $b$ , the

**Warning! Don't confuse  $s^3$  for  $3s$ . "3s" means 3 times  $s$  or  $s + s + s$ .**

definition can be extended in one and only one way. We are forced to define  $b^0 = 1$  and  $b^{-n} = \frac{1}{b^n}$

(can you see why?). For  $b = 0$  and positive exponents  $n$ , it is consistent with the original definition that  $0^n = 0$ . However, there is ambiguity when  $n = 0$ . So, people declare the value of  $0^0$  to suite their particular needs. In some cases, they just leave it undefined!

(By the way, computer programmers sometimes write “3^8” or “3\*8” for “3^8”.)

Here’s a worksheet for you to get used to exponential notation. Answers are below.

Exponentiation Worksheet				
Problem	Answer		Problem	Answer
$3^4 = ?$			If $3^3 3^2 3^5 = 3^n$ , what is $n$ ?	
Write $7 \times 7 \times 7$ using exponential notation.			Evaluate $2^2 + 2^3$ .	
Write $3 \times 3 \times 3 \times 3$ using exponential notation.			Evaluate $x^2 - 2x$ when $x = 1$ .	
Write 125 as an exponential with a base of 5.			True or False: $a^n a^m = a^{n+m}$ .	
Write 16 as an exponential in two different ways.			Simplify $1.234 \times 10^2$ .	
$10^6 = ?$			Simplify $1.234 \times 10^3$ .	
Evaluate $2^3 \times 5^3$ .			What is $b^1$ ?	
Evaluate $2x^3$ when $x = 3$ .			Suppose $(a^n)^m = a^p$ . What is $p$ in terms of $m$ and $n$ ?	
Evaluate $(2x)^3$ when $x = 3$ .			$\frac{4^6}{4^4} = 4^x$ . What is $x$ ?	
$0^{1000} = ?$			$\frac{7^{30}}{7^{18}} = 7^y$ . What is $y$ ?	
$1^{2009} = ?$			$\frac{b^m}{b^n} = b^z$ . What is $z$ ?	
If $2^4 \times 2^7 = 2^n$ , what is $n$ ?			If $\frac{1}{49} = 7^m$ , what is $m$ ?	

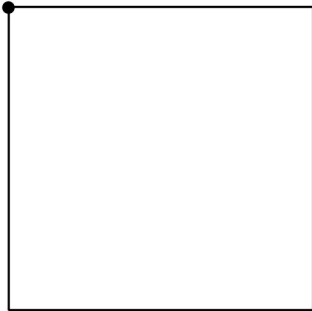
The answer to the savings account question is  $1.02^{20}$ .

The answers to the worksheet above are:  
 First column: 81, 7, 3, 5, 2, 1000000, 1000, 54, 216, 0, 1, 11.  
 Second column: 10, 12, -1, True, 1234, 4, 1234, 6,  $p = mn$ , 2, 12,  $z = m - n$ , -2.

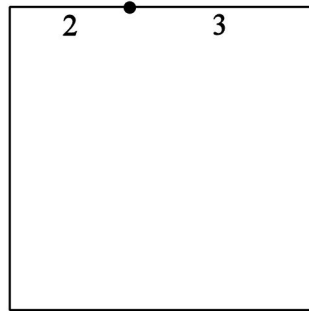


# Halving

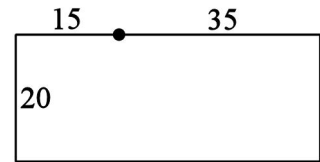
In each figure below, draw a line that passes through the black dot and cuts the figure into two pieces of equal area. These problems can all be solved precisely without extensive computation.



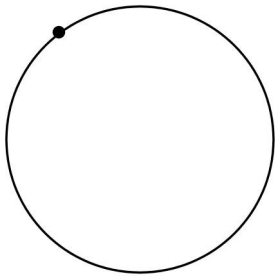
square



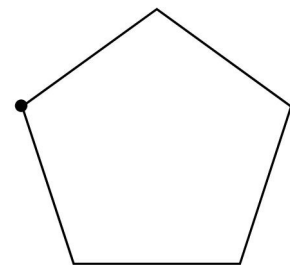
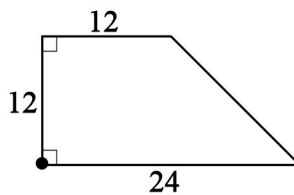
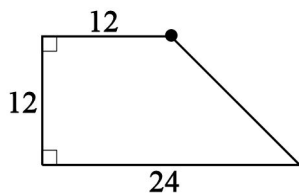
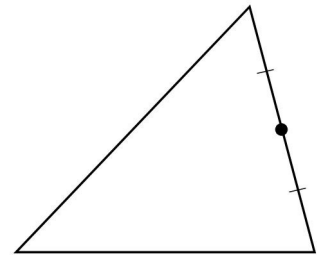
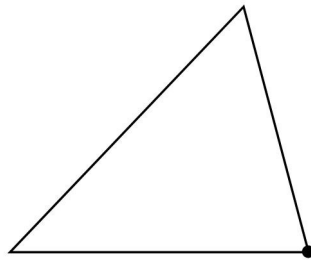
square



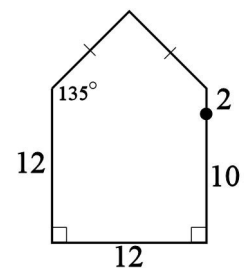
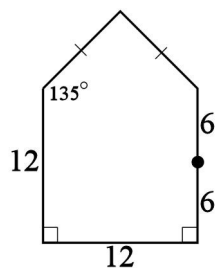
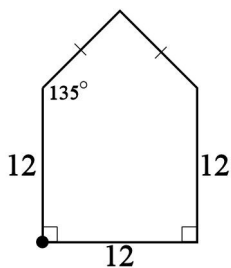
rectangle



circle



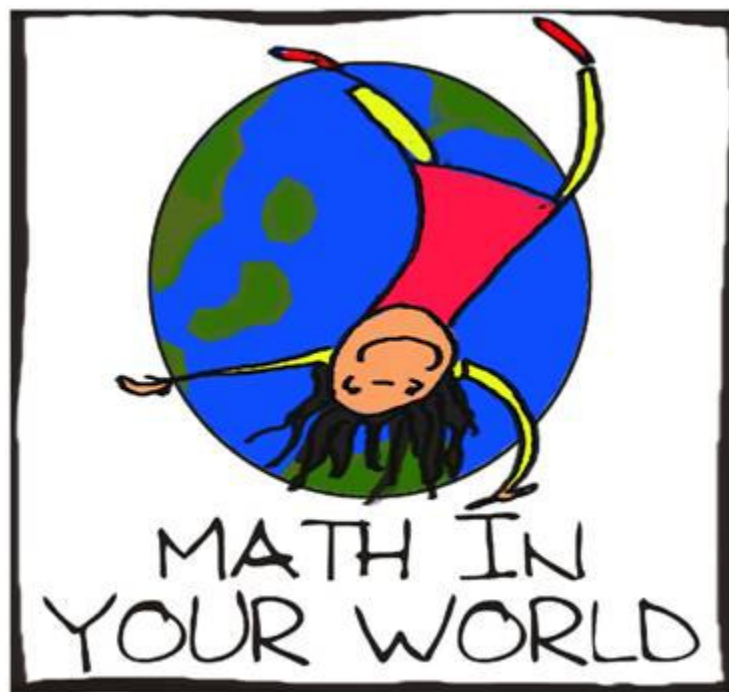
regular pentagon



# Climbing to the Top of the World

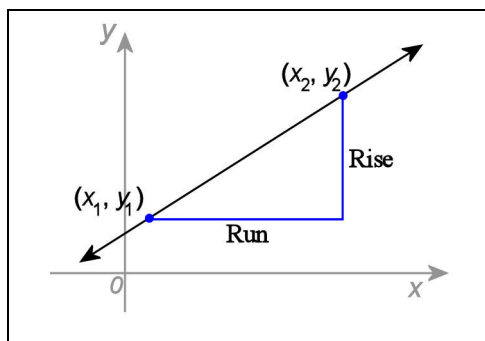
By Katy Bold

While walking around with a few friends one day, my friend Ed said, “I don’t want to go downhill because then we’ll just have to walk back uphill.” Ed is not usually lazy, but this was a hot day and we were on our way to meet the mayor at city hall. Since I was wearing heels, I agreed with Ed and we walked along a flat road. I started telling my friends that we were acting out a math problem— identifying the steepest direction of a surface. In this case, the surface in question was the surface of the earth, albeit covered with roads and buildings.



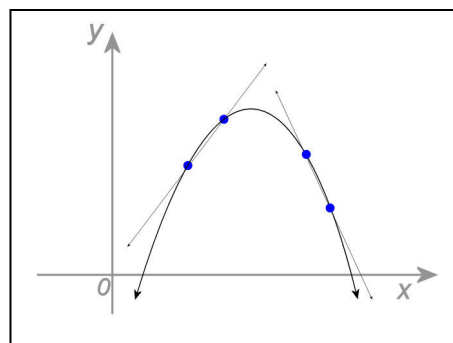
Let’s look at this as a math problem. The elevation of the earth’s surface<sup>1</sup> can be modeled as a function that depends on two variables: latitude and longitude. At any particular point on the earth’s surface, you could move around in different directions (changing your latitude, longitude, or both). Some of these directions may bring you to a higher or lower elevation. The direction that would increase your elevation fastest has a special name, the **gradient**<sup>2</sup>.

Think back to what you know about lines and slopes. See the figure at left. The slope of a line, which is a measure of steepness, can be calculated according to these formulas:  $\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$ . As the absolute value of the slope increases, so does the steepness of the line.



While a straight line has the same steepness at all points, steepness may vary over other types of curves. For example,

consider a parabola (at right). At different points along the curve, the steepness can often be approximated by finding the slope of a **secant** through nearby points. A secant is a line that passes through two points on a curve.



Where is the parabola the steepest? Where is it the least steep— or the most flat? The steepness is approximated by the secant line through two points. How could you make a more accurate estimate of the steepness of the parabola?

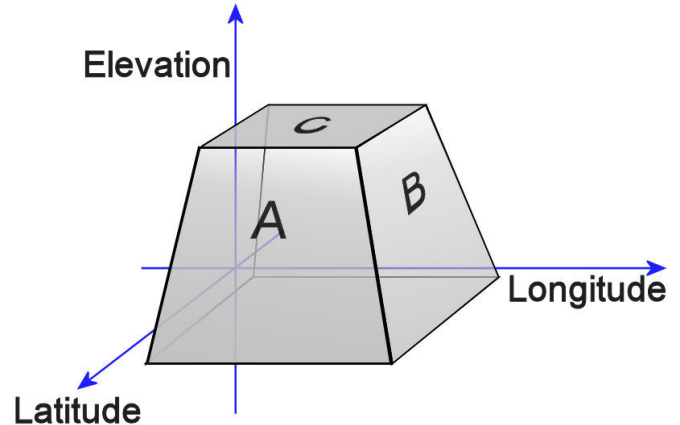
<sup>1</sup> This is a simplification because overhangs or other features could cause elevation not to be unique.

<sup>2</sup> If you know about vectors, the gradient is actually a vector. It encodes both the direction and the rate at which your elevation increases.



Going back to our surface example, we want to see how steep the earth's surface is as latitude and longitude change.

Imagine the mesa in New Mexico shown at right. On side A, the elevation changes as latitude changes, and there is no change in elevation as longitude changes. While on side B, we see the exact opposite—elevation changes with longitude but not latitude. Think about side C (the top of the mesa)—which direction is steepest? (This is a bit of a trick question!)



An idealized mesa in New Mexico.

The mesa shown is idealized, and in general, there would be much more variation in the elevation around the banks of the mesa. Unlike in the figure, elevation will tend to vary with both latitude and longitude.

Besides avoiding walking uphill on a hot afternoon, there are many practical reasons to determine the steepest direction. Engineers and scientists use algorithms that incorporate steepness in order to find the maximum or minimum of nonlinear functions in a range of applications. For example, these algorithms are a useful tool in analyzing a model of infectious diseases. Epidemiologists want to know the maximum number of people who may become infected or how fast the disease might spread.

### Take It To Your World

Imagine that you are about to set off on a very long walk. You will walk according to only one rule: With each step, walk uphill in the steepest direction.

- Where will you end up?
- Will you ever reach the ocean?
- Will you ever reach the peak of Mt. Everest?
- Where would you have to start in order to reach the peak of Mt. Everest?

If you start walking in the United States it is impossible to reach the peak of Mt. Everest because you will get stuck at a nearby hill, mountain, or mesa. Once you reach the top of a hill, mountain, or mesa, the elevation is flat or declining in all directions, so you have nowhere to go without violating the rule.

This is a problem that scientists and engineers face when using algorithms based on steepness. While we are usually interested in **global** solutions (i.e., the highest point in the whole world), these methods find **local** solutions (i.e., the hill by my house). That's why these types of methods work best when a good guess for the solution is already known. For example, if you knew the highest point in the world was in Nepal (and not the United States), you'd have a better chance of reaching the peak by starting in Nepal than if you started in Cambridge, Massachusetts. Better yet, if you knew to start near the Himalayas, you would have a better shot of reaching the peak!



# The Adventures of Emmy Newton

## Episode 2. *The Padlocked Doors*

by Maria Monks



*Last Time: Emmy and Melissa were sent to the principal's office to pick up a package for their math teacher. They accidentally went down one too many flights of stairs, and found a door leading to the basement. They opened the door and peeked around the corner...*

They were surprised to find themselves looking into a small, dimly lit square room, about the width of the hallways of the floor above. A single bare light bulb hung from the ceiling. Boxes filled with a seemingly random assortment of items were piled on top of each other haphazardly along the walls of the room.

"It looks like those two walls were put up pretty recently," said Emmy, pointing to two adjacent walls in the room, each of which was made of unpainted, unfurnished wood. "And each of those two doors on the new walls probably lead through to a hallway that looks like the ones on the first floor..."

She started sketching the school on her notebook. "The school is shaped like a square, and we're at a corner, right?"

"Yes," replied Melissa, "The hallways form a square around the courtyard in the middle, and the staircase is on the corner."

Emmy looked up from her notebook and took a closer look at the room. As their eyes adjusted to the dim lighting, the girls soon noticed a cardboard sign hanging from the ceiling. Written in large green letters on the white background were the words:

## THIS WAY TO THE PRINCIPAL'S OFFICE!

An arrow pointing to one of the two doors underlined the message.

"See," giggled Emmy, pointing at the sign. "I *told* you we could get to the Principal's office this way."

Melissa just shook her head in disbelief. It was all very strange.

"Let's go," said Emmy. They entered the room cautiously, and the door slammed shut behind them. The girls jumped.

The cardboard sign was hanging from a string, and in the gust of wind caused by the slamming door it started to turn. The arrow now pointed to the second door. "Which door should we go through?" wondered Melissa.

"It probably doesn't matter," said Emmy, pointing to her sketch of the school. "We're here, on the southeast corner. This door leads west, and that door leads north. We're trying to go northwest, so either way should be fine."

Emmy stepped over a few metal bars and wooden planks strewn across the floor and reached the north door. She pulled the handle, but it was locked. She then noticed it was padlocked; there was a keypad attached to the metal handle, and she had no clue what the password was.

"I guess this one doesn't work. Let's try the west door."

Melissa stepped around some boxes and tried the second door. It was also padlocked. "No luck," she said. "Let's go back."

She pulled on the handle to the door leading back to the stairwell. It wouldn't budge. "Man, this door *is* heavy," said Melissa. "Emmy, can you help me out?"

The two girls pulled together as hard as they could, but the door was completely jammed.

"We're trapped!" exclaimed Melissa. "I *knew* we shouldn't go this way!" She stamped her foot and looked like she was ready to cry.

Emmy was scared as well, but she kind of liked the excitement of it. "Don't worry, Melissa, someone will find us soon," she said, trying to make sure her voice didn't waver. "But let's work on opening one of these doors," she continued with more resolve. "There is tons of stuff in here; maybe there's a tool we can use to open this jammed door, or maybe we can find the passcode to one of the padlocked doors."

She started rummaging around in the boxes. There were ping-pong balls and tennis rackets, vacuum cleaners and clothes hangers. There was an entire box filled with string and rope, and another box filled with white paper. Emmy lifted a light box containing pens, pencils, and markers to find a box containing shoes.

Melissa took a deep breath and decided to take a closer look at each of the doors. As she made her way to the north door, something crunched under her foot. A piece of paper was lying near the base of the door, as if it was posted next to it and had fallen off. She picked up the paper and smoothed it out. "Check this out!" she said.

Emmy dropped the chain of toothbrushes she had been observing and went over to read the paper with Melissa. It read:

*Dear Travelers on the Quest to the Principal's Office,*

*It has recently come to my attention that Mr. Wheel knows how to measure his own width. However, he would like to measure his girth. Unfortunately, he does not have a flexible measuring tape. Can you help him? He first needs to figure out the answers to these questions for greater accuracy:*

- 1. If a triangle with perimeter 12 has a circle inscribed in it with diameter 1, what is the area of the triangle?*
- 2. What is the units digit (the rightmost digit) in  $3^{1000}$ ?*
- 3. There are 24 ways to line up a certain group of students. How many students are in the group?*
- 4. There were 17 gnomes in a room. Some of them would always lie and some would always tell the truth. They played a game*

Below the third question, it appeared that there were more questions, but the bottom of the paper was too smudged and torn to read clearly.

"I bet the answers to these questions will somehow encode the password for this door!" Emmy said to herself. She hoped, anyway. It would be *so* cool...

"But what is all this about Mr. Wheel measuring his... girth? What's girth anyway?" said Melissa.

"I'm not sure what that has to do with anything, but girth is like circumference, or perimeter... it's how far it is around. In other words, Mr. Wheel wants to stretch a

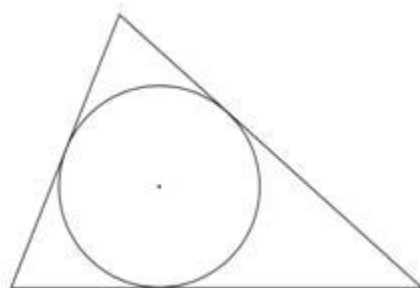


measuring tape around his stomach.” Melissa giggled, and Emmy continued, “But let’s solve these problems first.”

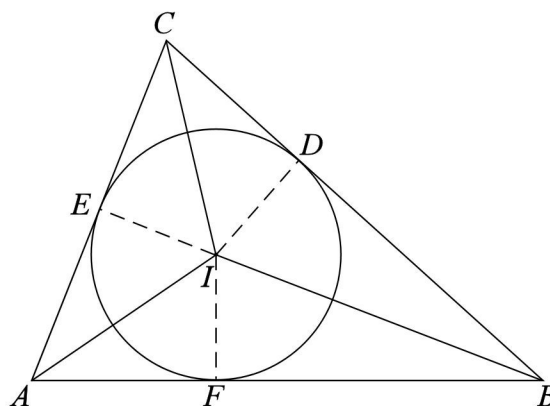
Emmy opened her notebook and drew a triangle with a circle inscribed inside.

“I think Mr. Wheel taught us a way to do this kind of problem the first day of class... remember?” said Emmy.

Melissa thought hard, and she vaguely remembered the formula Mr. Wheel wrote on the board. “Hmm, it was something like the area of a triangle is equal to half of the product of the radius of the inscribed circle and the perimeter.” She paused, and frowned. “But I don’t remember why.”



Emmy drew a few more lines on the diagram and labeled the points like this:



“Remember,” she said, “The area of triangle  $ABC$  is the sum of the areas of these three smaller triangles,  $ABI$ ,  $BCI$ , and  $CAI$ . Each of these triangles have area one-half of their base times their height. And we know their height is...”

“Oh, I know!” interrupted Melissa. “Each of those heights, which are the segments  $ID$ ,  $IE$ , and  $IF$ , are all radiuses of the circle! So they all have length one-half.”

“It’s *radii*, not radiuses,” sighed Emmy. Melissa used to say “radiuses” because she didn’t know it was wrong, but now she did it just to get Emmy riled up. “But right, the radius is half the diameter of a circle, so these lengths are all one-half. Since the lines are tangent to the circle at  $D$ ,  $E$ , and  $F$ , that means that the radii form right angles with these sides, so the heights of each of the smaller triangles is one-half.”

“Oh,” said Melissa. “So the area of one of the smaller triangles is one-quarter of the length of the base, because it’s one-half of the base times the height, and the height is one-half too. Since one-half times one-half is one-quarter, we get one-quarter of the length of the base.”

“Right,” said Emmy, “So the areas are  $\frac{AB}{4}$ ,  $\frac{BC}{4}$ , and  $\frac{CA}{4}$ , and their sum is then

$\frac{AB + BC + CA}{4}$ . But we know  $AB + BC + CA$ !”

“Yes, that’s the perimeter!” said Melissa. “So  $AB + BC + CA = 12$ , and so the answer is...”

“3,” said the girls in unison. They high-fived.

“And all because of the three radiuses,” said Melissa. Emmy sighed.

They looked at the next problem. “How in the world are we going to compute  $3^{1000}$ ?” asked Melissa. “Do you think there’s a calculator in here? Maybe there’s one hidden in one of these boxes.”





"I don't think we need a calculator," muttered Emmy, deep in thought. "It only asks for the units digit, not the whole number."

Emmy started writing out powers of 3, their values, and their units digit in a table:

$3^0$	$3^1$	$3^2$	$3^3$	$3^4$	$3^5$	$3^6$	$3^7$
1	3	9	27	81	243	729	2187
1	3	9	7	1	3	9	7

"See, the pattern 1, 3, 9, 7 repeats. I thought it would do something like that."

Melissa thought for a moment. "Oh, I see. If you multiply any number that ends in a 1 by 3, the new number ends in 3. Then you multiply by 3 again, and you get something that ends in 9, and so on. So once you're back to 1, it starts all over again."

"Right," said Emmy, "And since every fourth power of 3 starting from  $3^0$  ends in a 1, it must be that  $3^{1000}$  ends in a 1 too, because 1000 is divisible by 4."

Emmy wrote "1" down as the answer to the second problem.

"Oh, this next one is easy," said Emmy, looking at the third problem. "It's 4."

"How did you get that so fast?" asked Melissa, trying to imagine all the ways of lining up four people in a row.

"Remember, the number of ways to rearrange a list of  $n$  things is  $n$  factorial, that is, the product of the numbers from 1 to  $n$ . Say we have 4 people."

"Ok."

"There are 4 ways to choose who is first in line. After making this choice, there are 3 people left, so there are 3 ways to choose who goes in the second spot, for a total of  $4 \times 3$  ways of choosing the first two people."

"And then there are 2 ways to choose the next person, and 1 way to choose the last person, so it's  $4 \times 3 \times 2 \times 1$ ... oh, that is 24!" said Melissa. "But how did you figure it out so fast?" she demanded.

Emmy smiled. "I memorized that 4 factorial is 24."

Melissa smiled, impressed, and decided that she would memorize it too. "Ok, well we have that the three answers are 3, 1, and 4. But it looks like there are more digits to be discovered, and we can't read the remaining problems..."

"3, 1, 4..." Emmy got up and started pacing around the room. "Where have I heard that sequence before? Is there anything special about the number 314?"

"It's the first three digits of  $\pi$ ..." mused Melissa.

"Of course! It's  $\pi$ ! 3.14159265... And it all makes sense... Mr. Wheel wanted to measure his girth, which is like circumference, and he knows his width, which is like diameter. But  $\pi$  is the ratio of the circumference to the diameter of any circle, and it's a *wheel*, of course! Why didn't I think of that in the first place?"

Emmy walked to the padlock on the door where they found the problems and started pressing the digits of  $\pi$  in order. 3, 1, 4, 1, 5, 9, ...

The padlock clicked at 9 and Emmy turned the handle. The door opened smoothly and quietly, and the girls looked through with bated breath.

*TO BE CONTINUED...*



Editor: Grace Lyo

# Notes from the Club

These notes cover some of what happened at Girls' Angle meets. They are far from being complete. In these notes, we include some of the things that you can try or think about at home or with friends. We also include some highlights and some elaborations on meet material.

## Session 5 – Meet 8 – November 5, 2009

Mentors: Lauren Cipicchio, Wei Ho, Ariana Mann, Jennifer Melot,  
Maria Monks, Charmaine Sia, Rediet Tasfaye, Julia Yu

A special type of secret code puzzle has proven quite popular at the club. It is a type of secret code that is fairly easy to set up and can be used as a vehicle for many different types of math problem.

The idea is to start with a secret code, such as “I THINK THEREFORE I AM”. Write the letters of this code on separate index cards. For each index card, place a math problem. Construct these math problems in such a way that the answers to problems associated with letters that belong to the same word in the code are equal.

At meet 8, Julia prepared a secret code puzzle where the math problems were logic puzzles. Each logic puzzle had, as an answer, some color. The index cards with the letters in the word “THINK” had logic puzzles whose answers were all the same color. In meet 7, Lauren Cipicchio prepared a secret code puzzle where the puzzles were mathematical formulae that had to be simplified, and those cards that simplified to the same number corresponded to letters that belonged to the same word.

When the girls were given these index cards, it was natural for them to sort the index cards out according to the answers. After sorting the index cards into piles of equal answers, each pile would become an anagram puzzle. After the words were determined, they could then be put in order to produce the secret code.

In the case of a long secret code, math problems can be used whose answers belong to a structure with a natural order, and this natural order can be used to help sort the words.

### **Mr. Red, Blue, or Green?**

Here's one of the puzzles from Julia Yu's logic secret code. Can you solve it?

A notorious robber has been terrorizing the streets of Mathville. You are Head Inspector and have identified three suspects, Mr. Red, Mr. Blue and Mr. Green.

Your trusty sidekick tells you, “Of the three suspects, one of them makes two true statements; one makes one true and one false statement; and one suspect makes two false statements.”

However, before he left for vacation, he forgot to note which suspect is which. It is up to you to discover who is guilty:

**Mr. Red**

1. I am not the robber.
2. Mr. Green is the robber.

**Mr. Blue**

1. Mr. Green is innocent.
2. Mr. Red is the robber.

**Mr. Green**

1. I am not the robber.
2. Mr. Blue is innocent.

## Session 5 – Meet 9 – November 12, 2009

Mentors: Lauren McGough, Jennifer Melot, Mia Minnes, Charmaine Sia, Rediet Tasfaye

Special Visitor: JJ Gonson, Cuisine En Locale

JJ's presentation was quite different from most support network visits. Rather than discuss how she uses math in her work, she talked about her relationship to mathematics as she grew up. So many factors can influence our feelings about mathematics. We can be affected by the way we are initially exposed to the subject, by the kinds of teachers we encounter, and by how math relates to the things that we want to do. JJ was affected in all three ways and it was interesting to hear how different members perception of mathematics related to their life experiences.

After JJ's presentation we broke up into stations. One of the stations tackled a set of area halving problems (see page 15). For **Trisscar**'s clever solution to the last problem on that sheet, see page 25).

### Session 5 – Meet 10 – November 19, 2009

Mentors: Lauren Cipicchio, Ariana Mann, Lauren McGough, Jennifer Melot, Charmaine Sia, Rediet Tasfaye, Julia Yu

Special Visitor: Allie Anderson, MIT Aeronautics and Astronautics

Allie discussed the hazards of living in outer space. One of the hazards that astronauts face is muscle atrophy and bone loss. These hazards are partly caused by the lack of gravity. In orbit, one no longer has to push against the pull of the Earth because one is in a state of constant freefall. She then described her work with a group of MIT researchers to develop a space suit that simulates Earth's gravity when you wear it. The suit is specially designed with variable tension along its length to mimic the kinds of loads we experience when standing on Earth. Allie concluded by having the girls design custom spacesuits.

After Allie's visit, in one station, girls honed their communication skills in another Girls' Angle game: math pictionary. Can you get a friend to replicate the following images (up to scale) using oral communication alone? (Hand gestures are not allowed!)

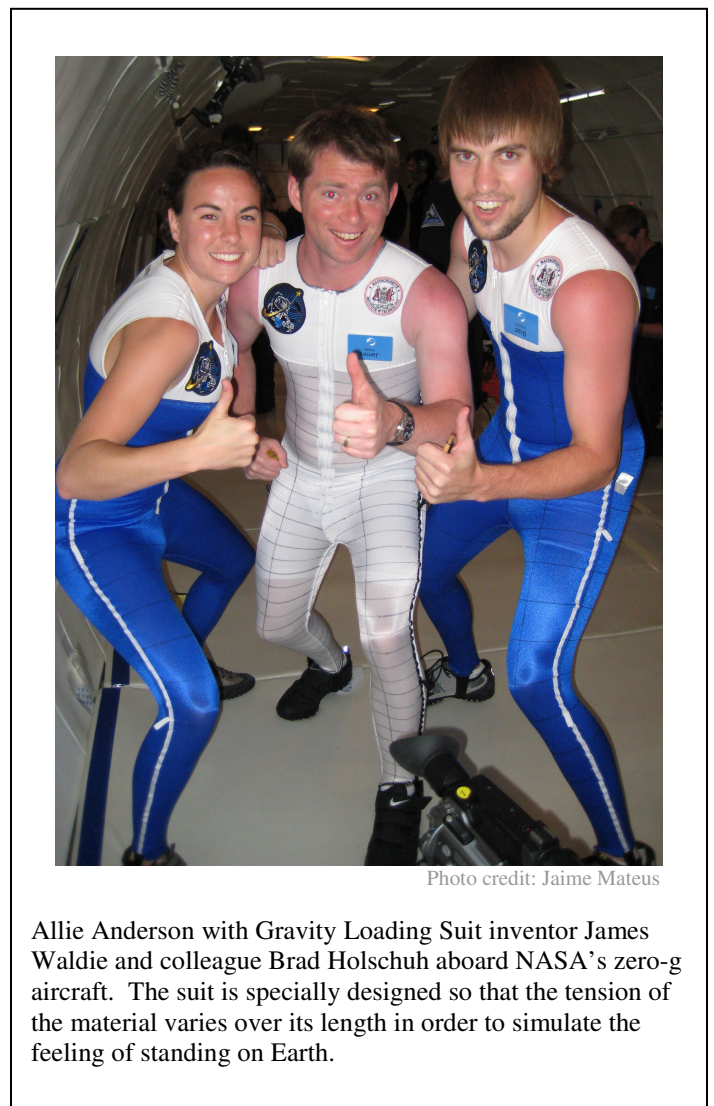
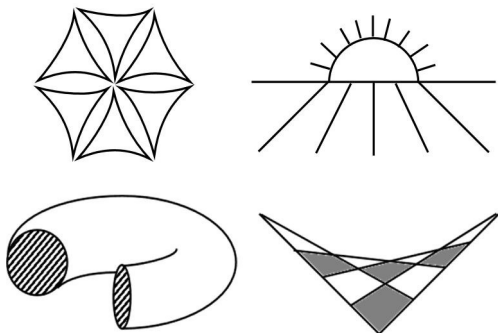


Photo credit: Jaime Mateus

Allie Anderson with Gravity Loading Suit inventor James Waldie and colleague Brad Holschuh aboard NASA's zero-g aircraft. The suit is specially designed so that the tension of the material varies over its length in order to simulate the feeling of standing on Earth.

### Session 5 – Meet 11 – December 3, 2009

Mentors: Lauren Cipicchio, Ariana Mann, Jennifer Melot,  
Maria Monks, Charmaine Sia, Rediet Tasfaye

Special Visitor: Meg Aycinena Lippow, MIT CSAIL

Meg discussed sorting algorithms. She had the girls play a game. In this game, letters were secretly associated with numbers. The girls had to identify which letter was associated with a given number. In the first exercise, the numbers were randomly assigned and the only way to find the letter was via a linear sort. In the second, the numbers increased in accordance with alphabetical ordering. In this case, the optimal sorting technique was a binary sort. In the third activity, the numbers were distributed into bins giving their remainders modulo 10. The optimal sorting technique in this last case used something called hashing.

In this way, Meg showed how algorithms can be more efficient if more information is known about a given situation. Using contextual information in this manner is one way AI researchers hope to make machines behave more intelligently.

### Session 5 – Meet 12 – December 10, 2009

Mentors: Lauren Cipicchio, Ariana Mann, Jennifer Melot, Maria Monks, Charmaine Sia

Treasure hunts have become something of an end-of-session tradition at Girls' Angle.

This time, treasures were wrapped in gold paper and securely tied with red ribbon and two combination locks. The girls were given a stack of clues and a secret code puzzle (made by Lauren). The girls worked on different parts in parallel, for there were far too many puzzles for any one person to solve on their own.

In the secret code puzzle, letters were sorted into piles according to the equality of mathematical formulae. These letters had to be anagrammed to form words. Gradually, words started to materialize: EIGHT, ANSWERS, NUMBER, SUM, SQUEAL...

Squeal?

Eventually, the girls realized that SQUEAL was wrong. What do you think they noticed? Hint: They didn't err on the math part!

See if you can figure out one of the numbers in the combination by solving the problem on page 10.

For **Cat in the hat's** solution to another Treasure Hunt problem, see page 25.

In a tremendous group effort, the girls narrowed down the number of possible combinations from one million to just a manageable handful. Just three minutes after the official end-of-class time, both locks were cracked open. Congratulations to all the members for all your efforts this session! We look forward to seeing you again at the start of session six!



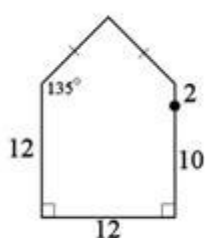
# Member's Thoughts

## Area Bisections and Trisections

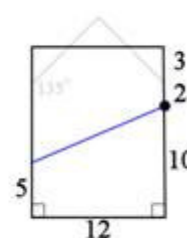
by Ken Fan

One of the ongoing themes in session 5 was that of area bisection: how can you cut a given region into two pieces of equal area? For examples of such problems, see page 15. Here, I'd like to describe two clever solutions to such problems, one by **Trisscar** and the other by **Cat in the hat**.

The last problem on page 15 asks for the line that passes through the indicated point on the perimeter of the figure below left which slices the figure in half. **Trisscar** made the following

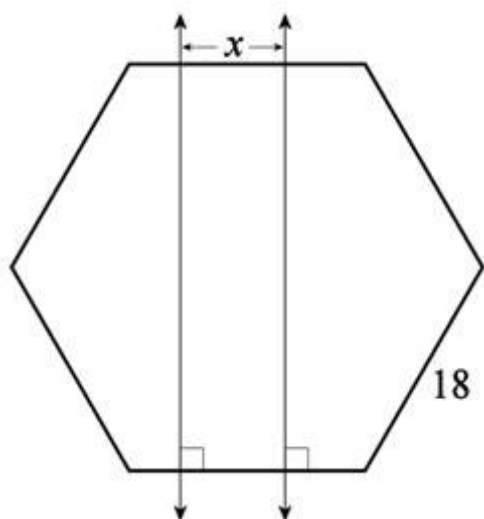


ingenious modification to the figure: replace the triangular top with a rectangle of the same area in such a way that the resulting figure is a large 12 by 15 rectangle. The length of the replacement rectangle would be 12 and its width would be half the height of the triangle, or 3 units. For a rectangle, the lines that halve the area are those which pass through the

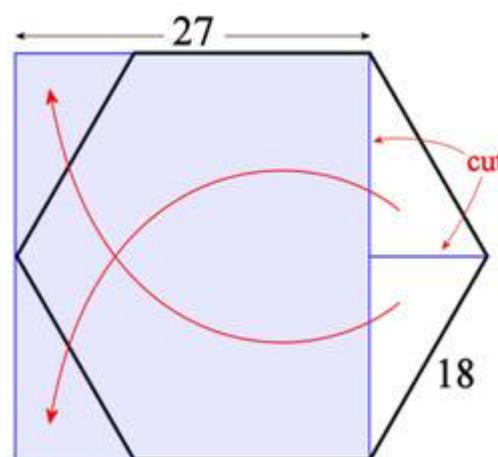


center of the rectangle, so the blue line segment whose left endpoint sits 5 units up the left side of the rectangle, by symmetry, must be such an area bisecting line. When one notes that the modification to the original takes place entirely on one side of the blue line segment, one realizes that this blue line segment is also the solution to the original problem!

For the last meet treasure hunt, girls were asked to find the value of  $x$  in the regular hexagon shown below when the two vertical lines trisect the area of the hexagon. For this problem, **Cat**



**in the hat** performed some simplifying surgery on the hexagon. She noted that the left side of the hexagon could be "squared off" with pieces obtained by chopping off the right side. The resulting rectangle (shown in blue at right) has top side length 27 units. To trisect the area of the blue rectangle with verticals is simple: trisect the horizontal sides. The separation between the two vertical



cuts would then be a third of 27 or 9 units. If these verticals are adjusted to trisect the area of the hexagon, they would have to be slid to the right by equal amounts (in order to maintain the area of the central piece), and, by symmetry, would have to intersect the hexagon within the top side. Therefore, the answer to the original question is  $x = 9$ .

In both cases, there's hardly any computing to do and it's easy to see that both are exactly right!

# Calendar

## Session 5: (all dates in 2009)

September	10	Start of fifth session!
	17	Tanya Khovanova, mathematician
	24	
October	1	
	8	Katherine Paur, Kiva Systems
	15	
	22	Jane Kostick, wood worker
	29	No meet
November	5	
	12	JJ Gonson, Cuisine En Locale
	19	Allie Anderson, MIT Aeronautics and Astronautics
	26	Thanksgiving - No meet
December	3	Meg Aycinena Lippow, MIT CSAIL
	10	

## Session 6: (all dates in 2010)

January	28	Start of sixth session!
February	4	
	11	Meike Akveld, ETH, Switzerland
	18	No meet
March	25	
	4	
	11	
	18	
	25	No meet
April	1	
	8	
	15	
	22	No meet
May	29	
	6	

# Girls' Angle: A Math Club for Girls

**Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!**

**What is Girls' Angle?** Girls' Angle is a math club for girls and a supportive community for all girls and women engaged in the study, use and creation of mathematics. Our primary mission is to foster and nurture girls' interest and ability in mathematics and empower them to be able to tackle any field, no matter the level of mathematical sophistication required. We offer a comprehensive approach to math education and use a four component strategy to achieve our mission: Girls' Angle mentors, the Girls' Angle Support Network, the Girls' Angle Bulletin and Community Outreach.

**Who are the Girls' Angle mentors?** Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

**What is the Girls' Angle Support Network?** The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society. They write articles for the Bulletin, take part in interviews and visit the club.

**What is the Girls' Angle Bulletin?** The Girls' Angle Bulletin is a bimonthly (6 issues per year) publication that features interviews, articles and information of mathematical interest. The electronic version is free. The printed version (beginning with volume 3, number 1) is free for members and can be purchased by others. Please contact us if you'd like to purchase printed issues.

**What is Community Outreach?** Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

**Who can join?** Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-11. We aim to overcome math anxiety and build solid foundations, so we welcome *all girls* regardless of perceived mathematical ability. There is no entrance test.

**In what ways can a girl participate?** There are 2 ways: **membership** and **active subscription** to the Girls' Angle Bulletin. **Membership** is granted per session and includes access to the club and extends the member's subscription to the Girls' Angle Bulletin to one year from the start of the current or upcoming session. You can also pay per session. If you pay per session and come for three meets, you will get a subscription to the Bulletin. **Active subscriptions** to the Girls' Angle Bulletin allow the subscriber to ask and receive answers to math questions through email. Please note that we will not answer email questions if we think that we are doing the asker's homework! We currently operate in 12 meet sessions, but girls are welcome to join at any time. The program is individually focused so the concept of "catching up with the group" doesn't apply. Note that you can receive the Girls' Angle Bulletin free of charge. Just send us email with your request.

**Where is Girls' Angle located?** Girls' Angle is located about 10 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

**When are the club hours?** Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at [www.girlsangle.org](http://www.girlsangle.org) or send us email.

**Can you describe what the activities at the club will be like?** Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

**Are donations to Girls' Angle tax deductible?** Yes, Girls' Angle is a 501(c)(3). As a nonprofit, we rely on public support. Join us in the effort to improve math education! Please make your donation out to **Girls' Angle** and send to Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038.

**Who is the Girls' Angle director?** Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was a Benjamin Pierce assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science and worked in the mathematics educational publishing industry. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum. These experiences have motivated him to create Girls' Angle.

**Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities?** Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls  
Yaim Cooper, graduate student in mathematics, Princeton  
Julia Elisenda Grigsby, assistant professor of mathematics, Boston College  
Kay Kirkpatrick, Courant Instructor/PIRE fellow, NYU  
Grace Lyo, Moore Instructor, MIT  
Lauren McGough, MIT '12  
Mia Minnes, Moore Instructor, MIT  
Beth O'Sullivan, co-founder of Science Club for Girls.  
Elissa Ozanne, Senior Research Scientist, Harvard Medical School.  
Kathy Paur, Kiva Systems  
Bjorn Poonen, professor of mathematics, MIT  
Gigliola Staffilani, professor of mathematics, MIT  
Katrin Wehrheim, associate professor of mathematics, MIT  
Lauren Williams, assistant professor of mathematics, UC Berkeley

**At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics?** We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.



# Girls' Angle: A Math Club for Girls

## Membership Application

Applicant's Name: (last) \_\_\_\_\_ (first) \_\_\_\_\_

Applying For: ☐ Membership  
☐ Active Subscription (interact with mentors through email)

Parents/Guardians: \_\_\_\_\_

Address: \_\_\_\_\_ Zip Code: \_\_\_\_\_

Home Phone: \_\_\_\_\_ Cell Phone: \_\_\_\_\_ Email: \_\_\_\_\_

**Emergency contact name and number:** \_\_\_\_\_

**Pick Up Info:** For safety reasons, only the following people will be allowed to pick up your daughter. They will have to sign her out. Names: \_\_\_\_\_

**Medical Information:** Are there any medical issues or conditions, such as allergies, that you'd like us to know about? \_\_\_\_\_

**Photography Release:** Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

**Eligibility:** For now, girls who are roughly in grades 5-11 are welcome. Although we will work hard to include every girl no matter her needs and to communicate with you any issues that may arise, Girls' Angle has the discretion to dismiss any girl whose actions are disruptive to club activities.

**Permission:** I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

\_\_\_\_\_  
(Parent/Guardian Signature) **Date:** \_\_\_\_\_

Membership-Applicant Signature: \_\_\_\_\_

- ☐ Enclosed is a check for (indicate one) (prorate as necessary)
- ☐ \$216 for a one session membership ☐ \$50 for a one year active subscription
- ☐ I am making a tax free charitable donation.
- ☐ I will pay on a per meet basis at \$20/meet. (Note: You still must return this form.)

Please make check payable to: **Girls' Angle**. Mail to: Girls' Angle, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to [girlsangle@gmail.com](mailto:girlsangle@gmail.com). Also, please sign and return the Liability Waiver or bring it with you to the first meet.

**Girls' Angle: A Math Club for Girls  
Liability Waiver**

I, the undersigned parent or guardian of the following minor(s)

\_\_\_\_\_,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: \_\_\_\_\_ Date: \_\_\_\_\_

Print name of applicant/parent: \_\_\_\_\_

Print name(s) of child(ren) in program: \_\_\_\_\_

