Girls' Angle Bulletin

June 2009 • Volume 2 • Number 5

To Foster and Nurture Girls' Interest in Mathematics

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Summer Fun

and More!

From the Director

Summer is here, and hence, also new Summer Fun problem sets! We've got a diverse group of writers to help you keep your mind active when the sun is shining. Don't miss these!

Also, we have a new policy with the Girls' Angle Bulletin: It is Free! That's right…anybody who wants it can be added to the subscriber list and you should feel free to pass the Bulletin along to anyone you please.

However, if you want to send in questions or answers and get a response, then you will need to purchase an "active" subscription or be a member of Girls' Angle. All members and active subscribers are welcome to submit questions and solutions about anything mathematical, including the Summer Fun problem sets. We'll put your solutions in the August issue!

In addition to the regular columns, Allison Henrich (co-author of "A Puzzling Problem for Penrose" in volume 1, number 5 of this Bulletin) returns to tell us about knot theory!

All my best,
Ken Fan
Founder and Director

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Girls' Angle Bulletin

The official magazine of
Girls' Angle: A Math Club for girls

girlsangle@gmail.com

This magazine is published about six times a year by Girls' Angle to communicate with its members and to share ideas and information about mathematics.

Girls' Angle welcomes submissions that pertain to mathematics. Subscriptions to the Girls' Angle Bulletin cost $20 per year and support club activities.

Editor: C. Kenneth Fan

Girls' Angle:
A Math Club for Girls

The mission of Girls' Angle is to foster and nurture girls' interest in mathematics and to empower girls to be able to tackle any field no matter the level of mathematical sophistication required.

On the cover: One final message to wrap up Session 4, encoded of course! Send the decoded message to girlsangle@gmail.com!

Why Knot?

by Allison Henrich

Knots are objects that people start thinking about at an early age, especially in cultures where shoes with laces are common. Maybe you first thought about knots when deciding whether to tie your shoelaces the "standard" way versus the "bunny ears" way.

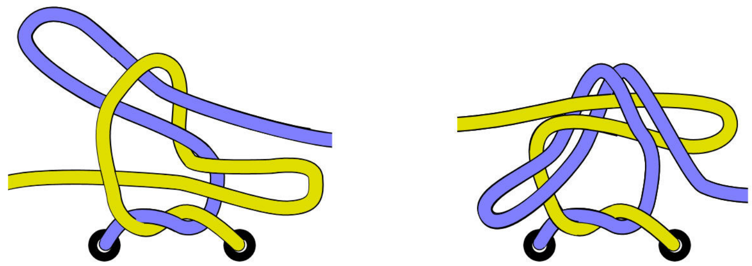


Figure 1: Two ways to tie your shoes. How do you tie your shoes?

Figure 2: A Celtic knot
en.wikipedia.org/wiki/File:Lindisfarne_StJohn_Knot2_3.svg

Maybe you've seen images of Celtic knots. You may be interested in sailing (where several interesting knots have a starring role) or making woven bracelets or even knitting. Even though knots may already be popping up in your life and imagination, I bet you've never thought of knots *mathematically*! It is a fact that many people find surprising that there are mathematicians all over the world who study knots for their research.

Without further ado, let us delve into the mathematical world of knots to answer a few questions: (1) What is the mathematical definition of a knot? (2) What are the main questions that mathematicians try to answer about knots? (3) What kinds of tools do they use to find out things about knots? We will aim to answer the first and second questions in some detail while we simply begin to scratch the surface of the answers to the third question. In future installments, we will explore answers to question (3) in greater detail.

Now, to get math into the picture, we have to decide exactly what we mean when we talk about knots. Actually, that's often one of the hardest things to do in math: making the definitions. How would *you* define a knot? I'll describe a definition that many mathematicians have found useful to study. It's the definition I'll use in these articles.

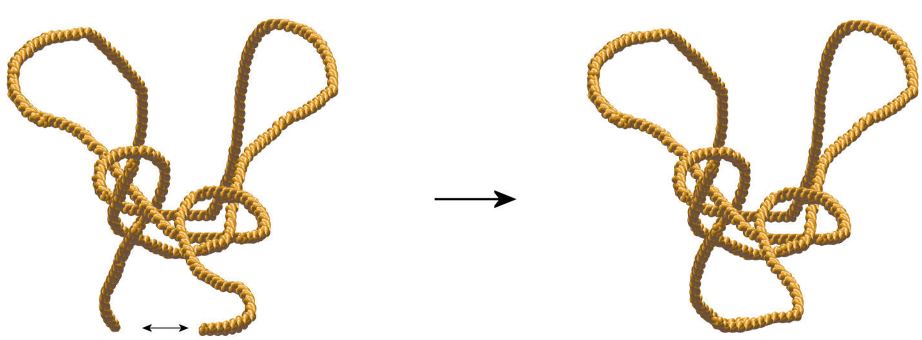


Figure 3: If the two ends are joined at left, you get the knotted circle at right.

First, picture a piece of rope with a knot tied in it.

Imagine gluing the two ends of the knot to each other, so you have a knotted circle. These knotted circles are what I'll mean by a "knot".

Often, when new objects are defined in mathematics, a helpful thing to try to do is to classify them or organize them into various categories. If you take a knot and just turn it over, it may look totally different, but I'd like to regard them as the same knot. In fact, let's consider a knot "the same" if you pull on it in various spots and tighten it in others, throw it in the air, ball it up in your pocket and take it to your friend's house. Let's say that the only thing you could do to change what kind of knot you have would involve cutting or breaking the rope, re-knotting it and gluing it back together in a different way. A mathematician who studies the current theory of knots would typically assume that anything you could do to the knot short of breaking it would preserve the knot's "sameness" or type. *Now try making a knot with a piece of rope or string and some tape! Rearrange your knot in different ways to see how different the same knot can look.* (Notice, for example, that a simple slip knot and an unknotted loop are considered the same in this scheme because you can get from one to the other without breaking the rope. Can you invent a theory of knots where they are regarded as distinct?)

If you taped the ends of your string together without tying a knot in the string, then you have what mathematicians call, the **unknot**. Otherwise, if you tied a single simple knot in your rope and attached the ends, you have the simplest knot that is not the unknot. This knot is called the **trefoil knot**. If you made the trefoil knot with your string and tape, show that the three diagrams of the trefoil below are actually picturing the same knot. Do this by arranging your knot in three different ways so that it looks like each of the pictures below.

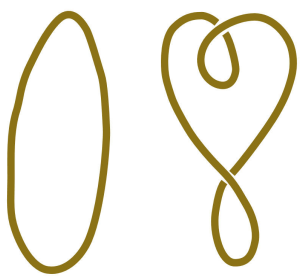


Figure 4: Two diagrams of the unknot

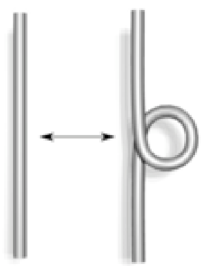
Figure 5: Three diagrams of the trefoil knot

As you can see, there are several different ways we can draw pictures, or **diagrams** of each knot. The idea that we can represent knots with diagrams (instead of rope) brings us to the core question in knot theory: *Given two diagrams of knots, how can you tell if they represent the same knot or two different knots?*

In the 1920s, Kurt Reidemeister thought about this question. Eventually, he taught himself enough about the problem that he was able to invent and prove an important theorem about when two knot diagrams represent the same knot.

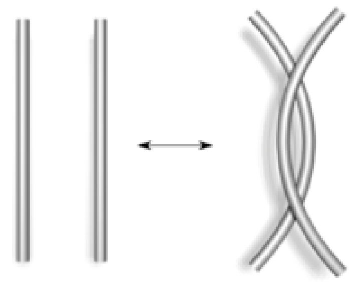
Reidemeister introduced three moves you could apply to a knot diagram without changing the knot that the diagram represents. These three moves, pictured on the next page, have come to be called Reidemeister moves I, II and III, respectively.

To see how the Reidemeister moves work, show that the two diagrams of the unknot from Figure 4 can be related to one another by a sequence of Reidemeister moves. Which Reidemeister move(s) did you need? Once you've related the two diagrams of the unknot, try a more challenging problem. Choose two of the diagrams of the trefoil from Figure 5 and find a sequence of Reidemeister moves relating them. How many moves did you need? Did you need to use all three moves, or could you do it with fewer?



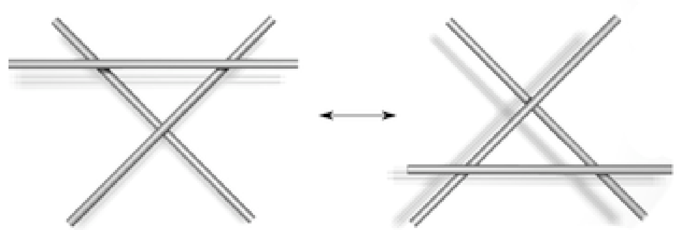
Reidemeister Move I

en.wikipedia.org/wiki/File:Reidemeister_move_1.png



Reidemeister Move II

en.wikipedia.org/wiki/File:Reidemeister_move_2.png



Reidemeister Move III

en.wikipedia.org/wiki/File:Reidemeister_move_3.png

Figure 6: Reidemeister moves I, II and III.

Now that we've experimented with some diagrams and Reidemeister moves, let us look at Reidemeister's theorem. He showed that *two knot diagrams represent the same knot if and only if they can be related by a sequence of Reidemeister moves*. Another way of saying this is as follows. Not only do Reidemeister moves preserve the knot type of a diagram, but any two diagrams that represent the same knot *must* be related by a sequence of Reidemeister moves. Pretty astounding result! We will revisit this result and learn more about how you can tell knots apart in future installments. Until then, have fun playing with knots!

An Interview with Rebecca Goldin, Part I

Dr. Goldin is associate professor of mathematics at George Mason University and the director of research at STATS. She received her Ph.D. in mathematics from MIT. She is the first recipient of the Ruth I. Michler Memorial Prize given by the Association for Women in Mathematics.

"When I first arrived in college, there was only one major I was sure *I didn't want to pursue, and that was mathematics."*

Some mathematics that Dr. Goldin discusses in this interview is graduate level, so don't be discouraged if you don't recognize some terms! The main points of the interview can still be understood.

Ken: When did you realize that you wanted to be a mathematician? What got you interested in math?

Rebecca: I didn't realize I wanted to be a mathematician until I was about to take my qualifying exams[1] during graduate school. Until then, I always told myself and others that I didn't *really* care if I stayed in math or not– I think to protect myself in case I wasn't able to do it. When I was studying for my qualifying exams, I finally realized that I really did care, and that I wanted to be able to do research and stay in mathematics as a career.

I became interested in mathematics without actually admitting that I wanted to be a mathematician. When I first arrived in college, there was only one major I was *sure* I didn't want to pursue, and that was mathematics. But, just out of curiosity about upper level mathematics, I thought I would take a couple courses. The second course I took was Topology, taught by Andy Gleason[2]. I just fell in love with the material, and decided that I could major in mathematics while pursuing other interests such as philosophy and political science. I did pursue these other interests, but mathematics had a special pull.

Ken: Do you remember the first mathematical thought that you had that you found interesting?

Rebecca: Yes, it was the idea of a compact topological space. For some reason, I thought that the notion of compactness was incredibly beautiful, powerful, and cool. Compactness was such a simple idea[3] but it was so subtle to define and understand, and it had such big consequences.

Ken: Can you explain why mathematics has this "special pull" for you?

Rebecca: There are a lot of things I like about mathematics. It's incredibly intellectual and challenging, and there's a lot of beauty in the kind that attracts me most. Sometimes I think of it more like art than like science. I love the fact that when you know something you can really prove it, that is, no one will disagree with you. I also love the cleverness of even simple mathematical games or puzzles.

Ken: Can you explain to us one of your favorite results that you proved?

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[1] The qualifying exam in mathematics is given to graduate students to determine whether they are ready to begin the independent research that leads to the doctoral degree. It is often taken during the second year of graduate school.
[2] Andrew M. Gleason was Hollis Professor of Mathematicks and Natural Philosophy at Harvard.
[3] The notion of compactness is typically first encountered in a course on topology.

Rebecca: The work I'm most proud of is about something called an "orbifold". An orbifold is not a smooth object, like a sphere – but it's almost smooth. It can have some "singularities" that come from pinching points in a very careful way. So a lemon– with its two pinched points at the ends– could be an example of an orbifold. To be slightly more descriptive, it is locally a smooth space quotiented by a finite group.[4] I proved a result about how to tell orbifolds apart– the result involves computing an invariant[5] called orbifold cohomology. If the orbifold cohomology is different for two different orbifolds, then this invariant has distinguished the objects. The theorem I proved shows an easy way to compute this invariant for a large class of orbifolds.

Ken: How did you manage to prove it?

Rebecca: I spent a month at Mathematical Sciences Research Institute (MSRI) a few years ago. There I spent time talking with Tara Holm and Allen Knutson (now both professors at Cornell University) about this question. Eventually we had some insight and decided to publish these results together.

Ken: You're also the Director of Research at STATS. What do you see as your main mission in that capacity?

Rebecca: The main mission is educational. We hope to raise the standards of numerical proficiency in the general public and with journalists in general.

Ken: One of the goals we have at Girls' Angle is to increase our member's mathematical understanding so that they can think more clearly and make more rational decisions in their own lives. We hope to give them the tools that they can use to avoid being misled. Do you have any advice for our members about how to do that, especially in view of all the things that are published or aired in the media today?

Rebecca: My main advice with regard to not being misled by the media on scientific issues is this: if you really care, you should follow up with your own research and not blindly believe what you read. A lot of journalism is written to be sensational (and to sell newspapers) and may have exaggerated the effect of something. The media also tends to ignore scientific consensus while touting one new study or one analysis of some health issue. If you truly care about something and you think it's important to know what's going on, read some of the scientific literature yourself, try to get a feeling for the scientific consensus, and evaluate the science yourself. A media discussion of the topic is a good place to start, but not a good place to end.

Ken: You're also a mother of four handsome boys. I think that there is a feeling "out there" that, especially for women, it is hard to have a family and a career in academia. You are having both. Did you ever feel that your family life and your career were at odds? Is it difficult to be a mathematician and raise a family?

To be continued…

[4] Don't worry if you do not understand this now. Many mathematicians don't encounter the concept of "orbifold" until they are as old as twenty-something!

[5] Imagine that you have a set of things and that you declare some notion of when two things are equivalent. (See "Equivalence Relations" in Volume 1, Number 3 of this Bulletin.) An "invariant" is something that you can compute for each thing and has the additional property that if two things are equivalent, then you'll compute the same thing for both of them.

Prueba del 9: The Trick Explained!

By Hana Kitasei

In the fall, **littleMeme** brought a math trick to Girls' Angle. We've been building up to an explanation of this trick in a series of articles and we're finally ready to explain it!

See the box at right to review the trick and the notation we will use in this article.

The Trick

The math trick involved a "reduction" procedure: Start with any positive integer, a. If a has multiple digits, add its digits together to get a new integer. Repeat this process until you are left with a single digit integer, A. (See volume 2, number 1 of this Bulletin.)

The trick goes like this: Start with any two positive integers, a and b. Perform the reduction on a to get A and perform the reduction on b to get B. Compute the product, ab, and perform the reduction on ab to get a single digit number, X. Compute the product AB, and reduce it to get the number Y.

Amazingly, $X = Y$ every time!

Over the last several issues, we have explored modular arithmetic, especially modulo 9. For example, we proved that a and its reduction, A, have the same remainder when divided by 9. (I.e., a and A are equivalent modulo 9.)

Last issue, we learned how multiplication affects remainders. We learned that the product ab has the same remainder when divided by 9 as the product of the remainders of a and b. More generally, we showed that if m and n are equivalent modulo 9 and s and t are equivalent modulo 9, then mn is equivalent to st modulo 9.

$$
\begin{tikzcd}
a \text{ and } b \arrow[r, "\text{Multiply}"] \arrow[d, "\text{Reduce}"] & ab \arrow[r, "\text{Reduce}"] & X \\
A \text{ and } B \arrow[r, "\text{Multiply}"] & AB \arrow[r, "\text{Reduce}"] & Y
\end{tikzcd}
$$

Let's write $m \equiv n$ if m and n leave the same remainder when divided by 9 (or, equivalently, if 9 divides $m - n$). Part 2 of Prueba del 9 explained how a number and its reduction will both leave the same remainder when divided by 9. So, we know that $a \equiv A$, $b \equiv B$, $ab \equiv X$ and $AB \equiv Y$. Part 4 of Prueba del 9 explained how multiplication works in modular 9 arithmetic. So, we know that because $a \equiv A$ and $b \equiv B$, we also have $ab \equiv AB$. Thus, $X \equiv ab \equiv AB \equiv Y$, and because X and Y are both nonzero single digit numbers, they must be equal!

You may have been wondering what this has to do with the trick. Consider this: The product ab has the same remainder when divided by 9 as its reduced form, X. But from what we've also proven, if you reduce first and *then* multiply, you will get a number that will also have the same remainder when divided by 9. In other words AB has the same remainder as ab. But also, AB has the same remainder as its reduction Y. All together, this means that X and Y must leave the same remainder when each is divided by 9.

But X and Y are both nonzero single digit numbers. The only way two nonzero single digit numbers can have the same remainder if you divide them by 9 is if they are *equal*! And that's the trick!

Table of Multiplication Modulo Nine

Here's the multiplication table from the last installment of *Prueba del 9* filled out. If you haven't tried to fill out the table yourself, please try to do that first before looking at this table. Figuring things out for yourself does take longer and is harder, but the benefits are well worth the effort.

Multiplication Modulo 9

m \ n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	0	1	2	3	4	5
2	0	2	4	6	8	1	3	5	7	0	2	4	6	8	1
3	0	3	6	0	3	6	0	3	6	0	3	6	0	3	6
4	0	4	8	3	7	2	6	1	5	0	4	8	3	7	2
5	0	5	1	6	2	7	3	8	4	0	5	1	6	2	7
6	0	6	3	0	6	3	0	6	3	0	6	3	0	6	3
7	0	7	5	3	1	8	6	4	2	0	7	5	3	1	8
8	0	8	7	6	5	4	3	2	1	0	8	7	6	5	4
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	1	2	3	4	5	6	7	8	0	1	2	3	4	5
11	0	2	4	6	8	1	3	5	7	0	2	4	6	8	1
12	0	3	6	0	3	6	0	3	6	0	3	6	0	3	6

What patterns do you see in this table?

Why does the second row (corresponding to $m = 1$) increase from 0 to 8 and then repeat? How does this compare with the behavior in the row corresponding to $m = 8$?

Why does the row corresponding to $m = 10$ look just like the row corresponding to $m = 1$?

Can you describe how this table would look if you extended the table to all positive integers? What happens of you include all integers, both positive, negative and zero?

Can you show that there is no integer k such that $k^2 - 5$ is divisible by 9? How can you relate this fact to the table?

What symmetries do you see in the table?

Anna's Math Journal

By Anna Boatwright

Mathematics is a journey of discovery. As mathematicians take this journey, they follow many wrong turns, believe many incorrect facts, and encounter many mysteries. Out of these twists and turns comes the reward of truth and understanding. However, if you look at math books, you might get the impression that mathematicians rarely err. In this column, Anna Boatwright gives us a peek into her mathematical process of discovery, bravely allowing us to watch even as she stumbles.

Dear Readers,

Recently, Anna transitioned from graduate school to the working world. She is now an actuarial analyst at Horizon Actuarial Services in Washington D. C.

Congratulations to Anna for her new job! We wish you success with your new career!

Because of these changes, we're skipping this issue's installment of Anna's Math Journal.

-Editor

More on Pick's Theorem

In the last issue, we stated Pick's theorem. It gives the area of a lattice polygon in terms of the number of lattices points on the polygon's boundary and the number in its interior.

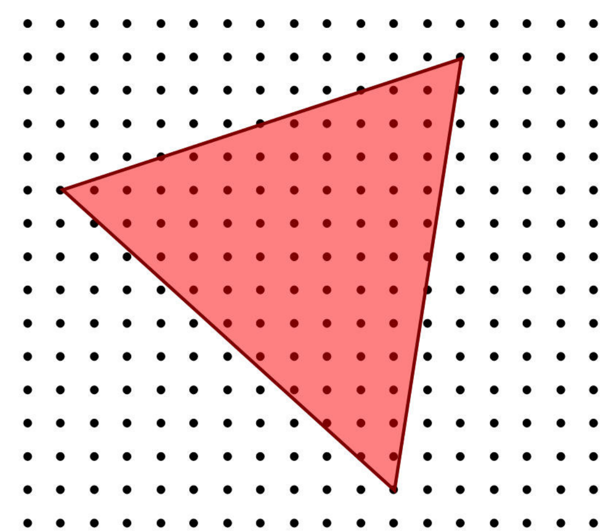
Here are some more questions for you to think about.

1. Based on Pick's theorem, what is the smallest possible area of a lattice polygon? If you have a lattice polygon of minimal area, what kind of polygon must it be?

2. Suppose a lattice polygon has vertices (0, 0), (a, b) and (x, y). (Because it's a lattice polygon, we know that a, b, x and y must all be integers.) What is the area of this lattice polygon?

3. If a lattice polygon is a perfect square, what could its area be? Can you find a *square* lattice polygon whose area is exactly equal to 1? 2? 3? 4? 5? How about 2009?

4. Can a lattice polygon ever be an equilateral triangle? (Girls' Angle mentor Annie Huang addressed this at the club once.)



Equilateral?

5. Suppose a lattice polygon has area A and contains no interior lattice points. How many lattice points are on its boundary (expressed in terms of A)?

6. How many lattice points are inside the triangle with vertices

$$(0, 0), (n + 2, 0) \text{ and } (0, n + 2),$$

where n is a positive integer? If you use Pick's theorem to solve this, you will have found another way to find the formula for the sum of the first n positive integers. Do you understand why?

7. This question was asked in the last issue: Suppose you have a polygon in the coordinate plane whose vertices all have *rational* horizontal and vertical coordinates. Can you prove that the area of such a polygon must be a rational number?

8. What is the minimum possible area for a lattice polygon with eight sides? Give an example of a lattice octagon of minimal area.

9. Can you *prove* Picks' theorem?

Send questions and answers to girlsangle@gmail.com!

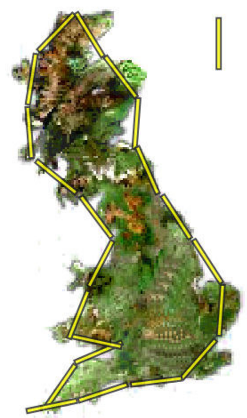
Fractals in Your World

By Katy Bold

MATH IN YOUR WORLD

Logo Design by Hana Kitasei

If you want to stump a friend with a trivia question, try this: How long is England's coastline?



Measuring the coast of England with two different sized rulers.

This is a trick question because the answer depends on the length of the ruler. Using a longer ruler, the bays, inlets, and peninsulas may be skipped over. With a shorter ruler, more of these features on the coastline are measured (see the figure at left).

There is something else interesting about the coast. The coast is very irregular, and if you look more closely at the coast, the same types of irregularity are seen again.

England's coast is an example of a **fractal**. A fractal is a special mathematical object that has the property of being **self-similar**: if you zoom in for a closer look, the close-up view looks a lot like the original view. Coasts are just one example of fractals found in nature; others include lightning, river systems, cauliflower, and veins of a leaf.

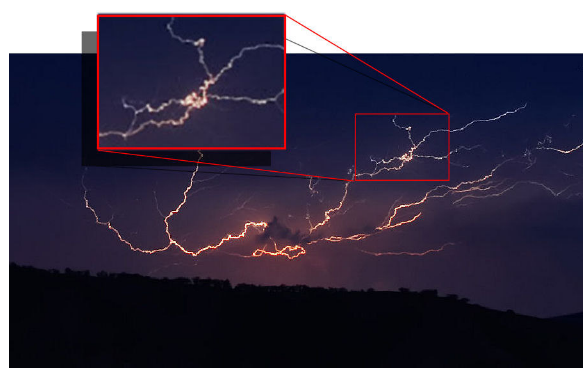


Photo modified from en.wikipedia.org/wiki/File:Cloud_to_cloud_lightning_strike.jpg

Figure 2. Lightning is self-similar. The close up view of lightning (inset) looks similar to the original view.

Let's look at two mathematical fractals: the Cantor set and the Koch snowflake.

The Cantor Set

To form the Cantor set, start with a line of unit length (this is level 0). Cut the line into thirds, and remove the middle third (this is level 1). How many pieces are left? How long is each piece? Each of the remaining pieces is just a smaller copy of the original line. Cut each of the two remaining pieces into thirds, and again remove the middle piece (this gives level 2).

How many pieces are left? How long is each piece? What is the total length of the pieces?

The same process can be repeated on each of the remaining 4 pieces.

What does the next level look like?

Try filling in the table below. What is the pattern for the number of pieces left at each level? The length of each piece? And the total length?

The Cantor Set				
Level		**Number of pieces**	**Length of each piece**	**Total length**
0	———	1	1	1
1	— —	2	$\frac{1}{3}$	$\frac{2}{3}$
2	- - - -	4	$\frac{1}{9}$	$\frac{4}{9}$
3				
4				
n		2^n		
∞ The Cantor Set				

The Cantor set is the collection of points that are “left over” at the end of the process. How many pieces are in the Cantor set? What is the length of each? And what is the total length? To answer these questions, look at your answers in the table: do the numbers tend to get bigger or smaller with successive levels?

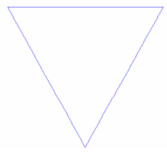
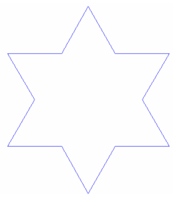
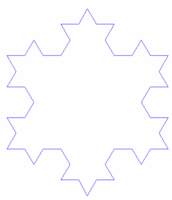
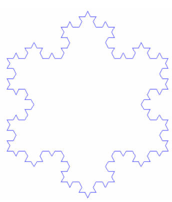
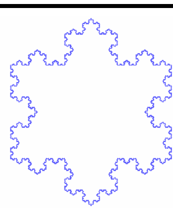
The Koch Snowflake

Another interesting, mathematical fractal is the Koch snowflake. It is made in a similar way as the Cantor Set. Instead of starting with a single line, we start with a triangle with each side of length 1/3 (this is level 0). Each side of the triangle is cut into thirds, and the middle third is replaced by two edges of a new, smaller triangle (see the figures in the table on the next page). Each of the three sides of the original triangle is replaced by 4 shorter line segments. The shorter line segments each have length 1/9. There are a total of 12 line segments in the level 1 curve.

Just as for the Cantor set, the process is repeated on each of the shorter line segments. For each of the 12 line segments: the segment is cut into thirds, and the middle third is removed and replaced by two edges of a new, even smaller triangle.

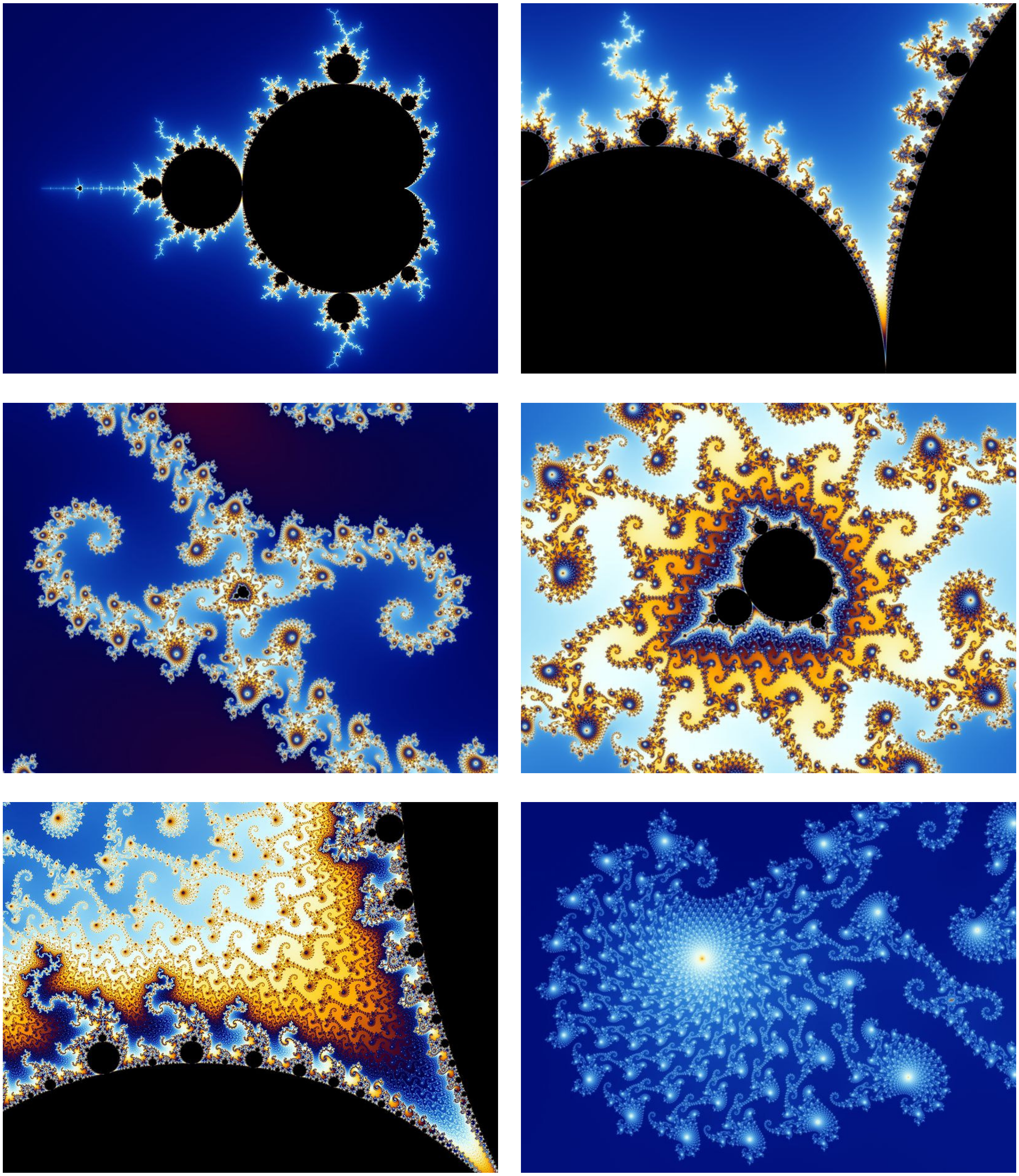
What happens to the total perimeter of the Koch snowflake?

Try filling in the table on the next page.

The Koch Snowflake				
Level		**Number of pieces**	**Length of each piece**	**Total length**
0		3	$\frac{1}{3}$	1
1		$3 \times 4 = 12$	$\frac{1}{9}$	$\frac{4}{9}$
2		$4 \times 12 = 48$		$\frac{16}{9} = \left(\frac{4}{3}\right)^2$
3				
n		3×4^n		
∞ The Koch Snowflake				

These two examples of fractals have some similarities and some differences. In each, there are more and more pieces at each successive level, and the pieces get shorter with each successive level. But, there is a big difference in the total length of all the pieces. For the Cantor set, the total length goes to 0, and it is infinite for the Koch snowflake. Which of these fractals is more like England's coast? Why?

Here we only looked at one feature of fractals, their length, but fractals have a lot of cool properties. In your math classes, you have probably learned about **dimensions** of objects: lines are 1d, squares are 2d, cubes are 3d. But those are not the only dimensions – fractals have **fractional dimension**. The dimension of the Cantor set is about 0.63, and the dimension of the Koch snowflake is about 1.26. Another nice property is that fractals can be really pretty!



Images courtesy of en.wikipedia.org/wiki/Mandelbrot_set

Images of the Mandelbrot set, one of the most famous fractals.

Take it to your world

Send an original picture of a fractal to girlsangle@gmail.com. Entries can be photographs from nature or a hand drawing of a fractal (you can make one up or look online for ideas). Entries may be featured in a future issue of the Girls' Angle Bulletin!

Summer Fun!

The best way to learn math is to do math!

We've made a bunch of fun problem sets for you to work on over the summer.

We invite Girls' Angle members and subscribers to the Bulletin to send any questions and solutions to girlsangle@gmail.com. We'll give you feedback and put your solutions in the Bulletin!

The goal may be the lake, but who knows what wonders you may discover on the way there?

In the August issue, we will give complete solutions. You could wait until the August issue to see the answers, but you will learn a lot more if you try to solve these problems before seeing solutions.

By the way, some of these problems are going to be very unlike those you will find at school. Usually, problems that you get at school are readily solvable. However, some of these problems were designed by the author to require time to solve and cannot be solved immediately.

If you are used to solving problems quickly, it can feel frustrating at first to work on problems that take years to solve. I've felt this frustration. But there can be things about the journey that are enjoyable. It's like hiking up a mountain or rock climbing. Getting to the top rewards one with a spectacular view, but during the journey, there's a lot to see and experience. So there's a meta-problem for those of you who feel frustrated at times doing these problems: see if you can dissolve that frustration and replace it with a relaxed, optimistic sense of adventure!

This is Summer Fun, not Summer Torture!

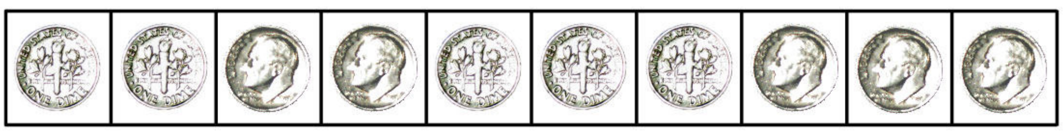
Summer Fun!

The Coin Flipping Game!

by Maria Monks

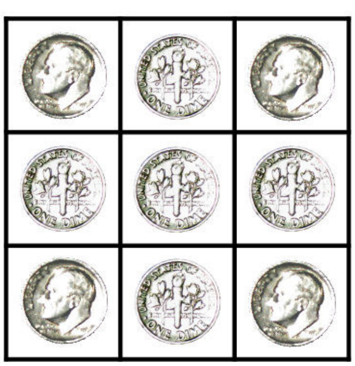
1. One coin is placed heads-up in each of the squares of a 1×10 grid as shown below.

You play a game where one move consists of first choosing one of the coins and then flipping over that coin along with any coin that is right next to it. For example, you can form the following pattern by choosing the first and sixth coins in the row of coins above, then flipping according to the rules:



You can make as many moves as you like. The goal is to eventually get the row to be all tails up. Can you do this? What if you are not allowed to choose the first or tenth coin, that is, you must always flip three coins? Can you do it if there were only 9 coins in a row?

2. Now coins are placed heads up in each of the triangles of a triangular grid with side length 3 units (each of the small triangles have side length 1 unit). A move consists of flipping a coin and all of its "neighbors," the coins that are in a triangle sharing an edge with the triangle containing the chosen coin. (In the grid shown at right, the coin in the bottom middle triangle was chosen and flipped together with its neighbors.) Can the grid be flipped to all tails in this manner? What if you start with a triangular grid that has side length 2? How about 4?



3. Now, suppose there is a coin in each of the squares of a 3×3 grid, starting heads up. As before, you are allowed to make moves consisting of choosing a coin and flipping over it and all its "neighbors," the coins in the squares sharing an edge with it. For instance, if you choose the coin in the middle to flip first, you will get the pattern shown. Can you make a sequence of moves that makes all the coins tails up? What if you start with a 4×4 grid? A 5×5?

Summer Fun!

The Pigeonhole Principle

by Lauren McGough

In this problem set, when I say "number", I mean an integer, that is, a counting number, the negative of a counting number, or zero.

1. Suppose you have a drawer that contains exactly six black socks, eight blue socks and nothing else. Imagine pulling socks out of the drawer with eyes closed. What is the minimum number of socks you need to pull from the drawer in order for you to be 100% sure that you have pulled out two socks of the same color? What if you start out with a drawer that contains four red socks as well as six black socks and eight blue socks – does your answer change? What if you start out with a drawer that contains socks of m different colors with four socks of each color?

2. This spring at Girls' Angle, we spent a lot of time discussing numbers written in different bases: we discovered that the *binary* number system, for example, is a way of representing numbers using only the digits 1 and 0, and that the *ternary* number system is a way of representing numbers using only the digits 0, 1 and 2. Can you show that, given any four ternary numbers, at least two of them must share the same last digit? What if you are given any nine binary numbers – what is the maximum number of them that is always guaranteed to share the same last digit? Suppose we write numbers in base N. Can you prove that, given $N + 1$ numbers written in base N, at least two of them must share the same last digit? Given $N^2 + 1$ numbers written in base N, can you prove that at least two of them must share the same last two digits? More generally, can you prove that, given $N^m + 1$ numbers written in base N, at least two of them must share the same last m digits?

3. Suppose that there are sixteen girls at the first meeting of Girls' Angle, and suppose that every girl shakes hands with some number of other girls (for example, Girl 1 might shake hands with five others and Girl 2 might shake hands with three others). Show that at least two of the girls shook hands with the same number of people.

4. You are given a collection of N integers. Show that there exists some pair of numbers in your collection whose difference is divisible by $N - 1$. Can you show that there is some (nonempty) subcollection whose sum is divisible by N? (A subcollection might contain only one integer, in which case, the sum of the integers in that subcollection is just the integer itself.)

5. The Fibonacci sequence is a famous sequence of numbers that is formed as follows: The first two numbers in the sequence are both 1, then each subsequent number is the sum of the two numbers that precede it (so the first few Fibonacci numbers are 1, 1, 2, 3, 5, ...). Prove that the last digit of the Fibonacci sequence is eventually periodic (it eventually repeats – for example, the sequence 1, 2, 3, 7, 9, 5, 8, 3, 9, 2, 9, 5, 2, 4, 9, 5, 2, 4, 9, 5, 2, 4, ... would be eventually periodic if it just continued repeating "9, 5, 2, 4" forever because even though it didn't start out periodic, it eventually became a repeating sequence). Can you also show that the last two digits of the Fibonacci sequence are eventually periodic? Can you extend this even further to show that the last n digits of the Fibonacci sequence are periodic?

Summer Fun!

“Distance” Between Numbers

by Elisenda Grigsby

We all know instinctively what we mean by numbers being “close” or “far.” For example, 4 is closer to 5 than it is to 6, and it’s *much* closer to 6 than it is to 1000. But how do we actually quantify what our instincts tell us? That’s easy! If we want to know the distance between two numbers, x and y, –let’s denote this distance by $d(x, y)$– we just take the absolute value of their difference: $d(x, y) = |x - y|$.

Recall that the absolute value of zero or a positive number is the number itself, and the absolute value of a negative number is -1 times that number. For example, $|5| = 5$, and $|-5| = 5$.

Using this notion of distance, we compute that $d(4, 5) = |4 - 5| = |-1| = 1$, $d(4, 6) = 2$, and $d(4, 1000) = 996$. In other words, the definition of “distance” I defined above matches our instincts. The larger $d(x, y)$ is, the “farther apart” x and y are.

I would like to tell you about a *completely different* (but, in a certain sense, just as valid!) notion of distance between numbers that is completely counter to our intuition about distance. Let’s define this new notion of distance as follows. Suppose x and y are whole numbers, and suppose a is the maximum number of times we can divide $|x - y|$ by 2 and still get a whole number. Let’s define the “2-adic” distance between x and y, denoted $\Delta_2(x, y)$, as follows: $\Delta_2(x, y) = \dfrac{1}{2^a}$. Since it is not clear what to do when $x - y = 0$, we also assert: $\Delta_2(x, x) = 0$.

So, for example, $\Delta_2(6, 14) = 1/2^3 = 1/8$, since $6 - 14 = -8$, and we can divide $|-8| = 8$ by 2 three times and still get a whole number.

1. Compute the following 2-adic distances: $\Delta_2(10, 6)$, $\Delta_2(90, 10)$ and $\Delta_2(194, 2)$.

2. Show that the 2-adic distance function satisfies the so-called “triangle inequality”: If x, y and z are numbers, then $\Delta_2(x, y) + \Delta_2(y, z)$ is greater than or equal to $\Delta_2(x, z)$. Show that the “normal” distance function I defined at the beginning also satisfies the triangle inequality.[1]

3. Make a list of whole numbers, x_1, x_2, x_3, x_4, etc. that satisfy the inequalities

$$\Delta_2(0, x_1) > \Delta_2(0, x_2) > \Delta_2(0, x_3) > \Delta_2(0, x_4) > \ldots$$

4. Suppose p is any prime number and let x and y again represent whole numbers. Define the “right” notion of the “p-adic” distance between x and y (in particular, you should be able to show that it satisfies the triangle inequality and the distance between two numbers should be 0 exactly when the two numbers are equal).

5. Suppose p is any prime number and x and y are fractions.[2] How should we define the p-adic distance between x and y?

[1] Every “self-respecting” distance function should satisfy the triangle inequality, which roughly says, “It is always shorter to go directly from x to z, without stopping at any other point, y, along the way.”

[2] A fraction is any number that can be written as a quotient of two whole numbers.

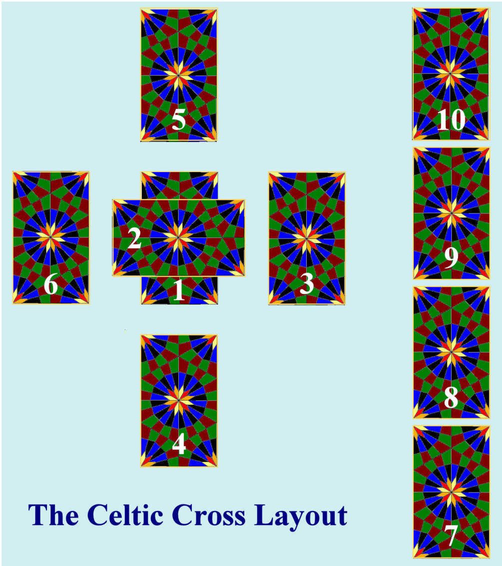
Summer Fun!

Math and Tarot Cards

by Gregg Musiker

A Tarot Deck consists of 78 cards. 56 of the cards are known as minor arcana and come in four suits (Wands, Cups, Swords, and Pentacles). Each suit consists of 14 cards (A, 2, 3, 4, 5, 6, 7, 8, 9, 10, Page, Knight, Queen and King). The remaining 22 cards are known as major arcana and include the Fool, Magician, High Priestess, Empress, Emperor, Hierophant, Lovers, Chariot, Strength, Hermit, Wheel of Fortune, Justice, Hanged Man, Death, Temperance, Devil, Tower, Star, Moon, Sun, Judgment, and World. Major arcana do not have suits.

Glenda is a Tarot card reader and recently has been seeing some eerie coincidences among the cards. She believes the cards may be telling her something. She is of the impression that she used to be strong-willed and until very recently was a star, but that she is presently being quite foolish. Foolish enough, she will be taken in by a cunning Emperor in the near future, with whom she will fall in love. However, it appears that this will lead to her downfall and she will be left alone after her fortunes turn. Her only hope is that a wise priestess or empress can allay her fear and help steer her towards a better final outcome.



Help set her mind at ease by computing probabilities! In all of these problems, ten cards are freshly dealt in the ten-card Celtic Cross layout (pictured at right).

1. What is the probability that the Fool card is in Position 1 (the present) and the Emperor card is in Position 6 (the immediate future) in the same reading?

2. How probable is it that Positions 3 and 4 (the distant and recent past) include the Strength card and the Star Card, in either order, in the same reading?

3. What is the probability that the Judgment or Justice card is in Position 2 (the immediate challenge) and the Lovers or Chariot card is in Position 5 (the best outcome) in the same reading?

4. How likely is it that at least one of the cards dealt in Positions 7-10 is the Wheel of Fortune, Hermit, or Magician card?

5. What is the probability that the High Priestess and the Empress both appear in the ten card spread?

6. How likely is it that at least one of the cards dealt in Positions 1-6 is the Sun, Moon, or World?

7. Glenda does seven readings in a row, and, after finishing, notes that the High Priestess appeared three times (possibly more) in the seven readings. What is the probability of this event? Should Glenda be surprised? Explain.

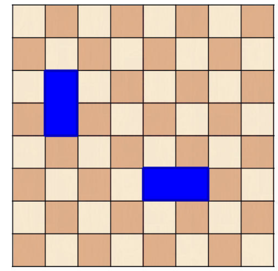
Same question, except now Glenda does fourteen readings? Twenty-one readings?

Summer Fun!

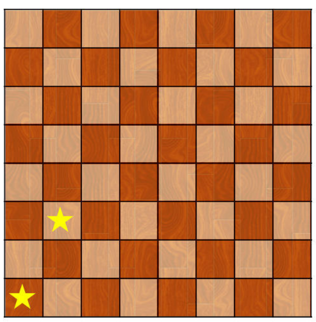
Chess Road Trip

by Grace Lyo

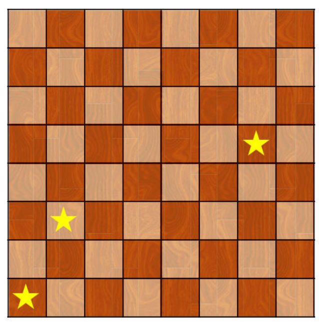
Taylor Walker and her older sister Casi are on a road trip to the Grand Canyon with their parents. They brought a chess set and dominos to pass the many hours they will be spending in the car. After playing dominos for a couple hours, they get bored and decide to play chess, only to discover that they forgot to bring the pieces! They decide to invent games and puzzles of their own instead.



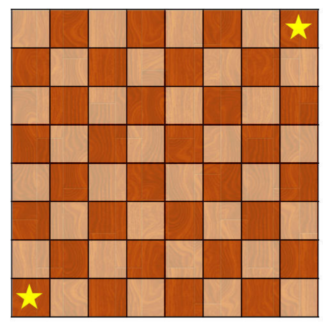
Dominos Casi comes up with a puzzle for Taylor first. Each domino is exactly the size of two squares on the chess board, so it can be placed on the board horizontally or vertically as in the diagram at right. Casi asks Taylor if in each of the scenarios below she can arrange the dominos so that every square except those marked with a yellow star is covered (see pictures (a) through (c)). Will Taylor be able to find domino tilings that work?



a.

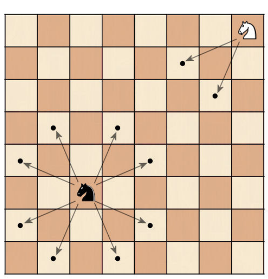


b.



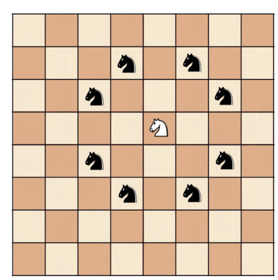
c.

(d) After solving (a), (b), and (c), Taylor asks Casi if there is any quick and easy way to determine whether a board minus a few squares can be tiled. Is there? If so, prove that your solution is correct.



Possible moves for the knights

Horses Taylor, having solved Casi's domino-tiling puzzle now gives Casi a puzzle of her own. A "knight" is a chess piece that looks like a horse. It moves in a very special way. In one turn, it can either move horizontally two squares and then vertically one square, or vertically two squares and horizontally one square.



Two chess knights are "attacking" each other if they can move to each other's squares in one turn. At right, the black knights are attacking the white knight and vice versa, but none of the black knights are attacking each other.

(a) What is the maximum number of knights that can be placed on the chessboard in such a way that no two knights are attacking one another?

(b) Challenge question: Prove that your answer is correct. Note that coming up with a configuration and then showing that no more knights can be added to that configuration is not a valid proof!

Cars and Goats

by Kay Kirkpatrick

You're a contestant on a game show, and you are presented with three doors.[1] Behind two of the doors there are goats; behind the third, a car. The game-show host knows what is behind each door and invites you to choose a door. Once you have chosen one of the doors (but not opened it), the host must open a different door to reveal a goat. Then you are invited to switch to the other unopened door if you wish. You get to take (or drive) home what is behind the door of your choice, and you want the car. Should you switch?

1. First, play this game with a partner and three cards, the ace of spades to represent the car and two red cards to represent the goats. One of you will be the host and will place the three cards face down, knowing what they are. The other will be the contestant and will select (but not look at) one card. Then, remembering the remaining two cards, the host will turn over one of them to reveal a red card. Then the contestant will decide whether to switch. After that, look to see whether you won the ace. Take turns being the contestant and the host, and play this game 10 or 15 times, recording whether you switched and whether you won. How often did you win when switching? When not switching? Do you see a pattern?

2. Next, imagine playing the game with 10 doors instead of three. In this variation, the host would invite you to choose one, and then would open 8 of the remaining doors, revealing 8 goats. Would switching increase your chances of winning the car? Why? (You can play this variation with a partner, the ace of spades, and 9 red cards.)

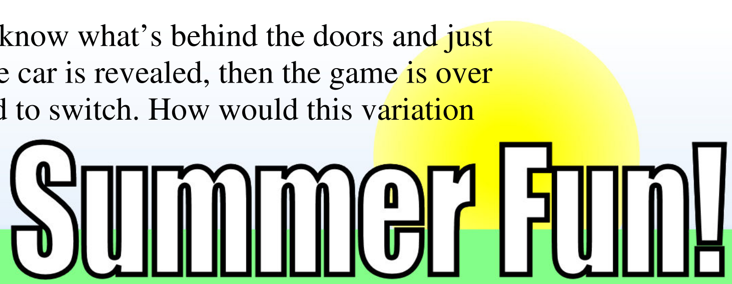
3. Let's analyze the 10-door variation of the game by cases. You will initially choose either the door with the car behind it or one of the others. You have one chance in 10 of initially choosing the door with the car behind it, or as we say, that happens with probability 1/10. Does switching result in winning in this case? On the other hand, with what probability do you initially choose a door with a goat behind it? Which doors will the host open then? And does switching result in winning in this case? Now look over your analysis and summarize it by answering this question: Is the strategy of always switching a good one?

4. We can analyze the original three-door problem similarly. With what probability do you initially choose the door with the car behind it? With what probability do you initially choose a door with a goat behind it? Which doors could or would the host open? Is the strategy of always switching a good one?

5. What if, in the 10-door variation, the host were to open only 7 doors, revealing 7 goats? Would switching increase your chances of winning? What about only 6? Only 1?

6. What if, in the three-door problem, the host doesn't know what's behind the doors and just opens one of the two remaining doors at random? If the car is revealed, then the game is over with no prize. If a goat is revealed, then you are invited to switch. How would this variation affect your strategy?

Goat image from en.wikipedia.org/wiki/File:Irish_Goat.jpg



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[1] The game described here is often referred to by the name of the host who popularized it. We'll reveal that host with the solutions.

A Potpourri of Problems

by Doris Dobi

If none of the other problems caught your fancy, perhaps one of these miscellaneous ones will!

1. Each natural number can be decomposed into a product of primes. For example, $24 = 2 \times 12$, $154 = 2 \times 7 \times 11$ and $23 = 23$. This is called decomposing a number into its prime factorization. Let us call a number "spunky" if its prime factorization consists of exactly three consecutive primes. The first spunky number is $30 = 2 \times 3 \times 5$. Find the fourth spunky number.

2. An analog clock reads 3:15. What is the angle between the minute hand and hour hand?

3. There are 3 black hats and 2 white hats in a box. Three men, which we'll call A, B, and C, each reach into the box and place one of the hats on his own head. They cannot see what color hat they have chosen. The men are situated in such a way that A can see the hats on B and C's heads, B can only see the hat on C's head and C cannot see any hats. When A is asked if he knows the color of the hat he is wearing, he says no. When B is asked if he knows the color of the hat he is wearing he says no. When C is asked if he knows the color of the hat he is wearing he says yes and he is correct. What color hat is C wearing and how can he know?

4. Imagine an analog clock set to 12 o'clock. Note that the hour and minute hands overlap. How many times each day do both the hour and minute hands overlap? How would you determine the exact times of the day that this occurs?

5. Alba, Ada and Antea are best friends. One of the girls always tells the truth, one always tells lies, and one answers yes or no randomly. The girls know each other very well so that each girl knows which girl is which. You may ask three yes or no questions to determine who is who. If you ask the same question to more than one person you must count each time you ask as one of the three questions asked. What three questions should you ask?

6. On a deserted island there live five people and a monkey. One day everybody gathers coconuts and puts them together in a community pile to be divided the next day. During the night one person decides to take his share himself. He divides the coconuts into five equal piles, with one coconut left over. He gives the extra coconut to the monkey, hides his pile, and puts the other four piles back into a single pile. The other four islanders then do the same thing, one at a time, each giving one coconut to the monkey to make the piles divide equally. What is the smallest possible number of coconuts in the original pile?

Summer Fun!

Palm tree modified from en.wikipedia.org/wiki/File:1859-Martinique.web.jpg

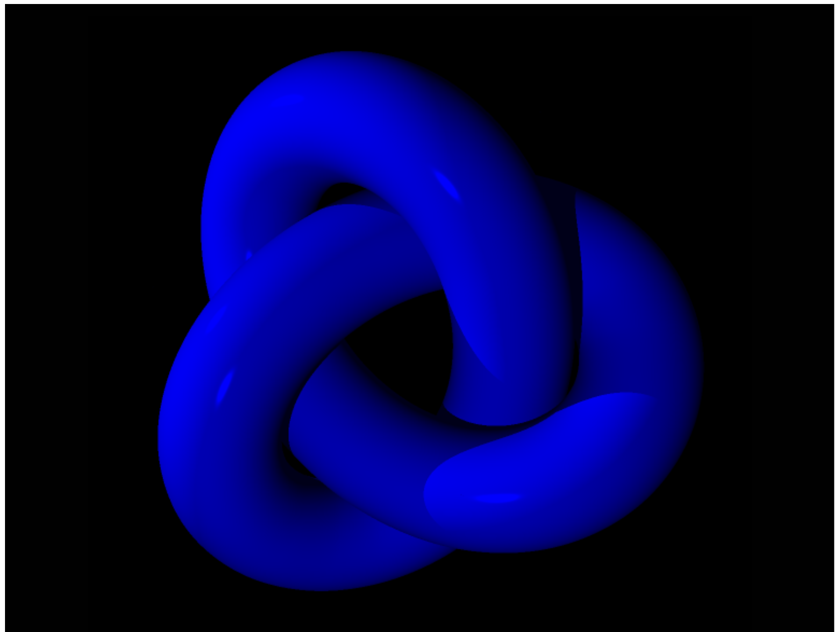
Notes from the Club

These notes cover some of what happened at Girls' Angle meets. They are not meant to be complete and, to nonmembers, they may not even be coherent!

Session 4 – Meet 10 – April 23, 2009

Mentors: Cammie Smith Barnes, Kay Kirkpatrick, Grace Lyo, Lauren McGough, Jennifer Melot, Mia Minnes, Maria Monks

Meet 10 featured the session's first Dream Time, led by Jennifer Melot. For this Dream Time, Jennifer described the everyday life of friends living on a strangely shaped world and the girls had to try to imagine the shape of this world in their mind's eye.



A trefoil knot.

Image modified from upload.wikimedia.org/wikipedia/commons/4/46/Trefoil_knot_arb.png

Kay and Mia worked with a group of girls to design geometric shapes for soap bars. **Hadassah** began thinking about how one might make a mold for a soap bar in the shape of a trefoil knot. It's not at all obvious and I hope she figures it out because it would be pretty cool to make soap bars of this shape! (See "Why Knot?" on page 3 for more on trefoil knots.)

Kay and Mia also talked about secret codes and error correcting codes. **Hadassah** had the idea of transmitting messages twice to detect errors and **Jo** extrapolated to three times and figured out the majority rule for decoding the final message.

Session 4 – Meet 11 – April 30, 2009

Mentors: Cammie Smith Barnes, Annie Huang, Kay Kirkpatrick, Grace Lyo, Lauren McGough, Jennifer Melot, Maria Monks

Special Visitor: Prof. Gigliola Staffilani, Math Department, MIT

The eleventh meet of Girls' Angle was an historic moment at Girls' Angle.

For the first time, we were visited by a tenured professor of mathematics! Dr. Staffilani is the Abby Rockefeller Mauzé Professor of Mathematics at the Massachusetts Institute of Technology.

But, before Prof. Staffilani's arrival, the mentors worked with the girls on a variety of ideas, activities, games and puzzles. For example, Maria's work led her to make a game out of finding the prime factorization of numbers. Cammie worked with **Ilana** on a number theoretic problem concerning squares with vertices on the integer lattice (see problem 3 on page 11). Jennifer worked with **Caitlin** and **Sylvie** converting binary representations of numbers to decimal. And Annie showed a group of girls how to make polyhedra using techniques of origami.

Dr. Staffilani started by giving a little background about herself. She was raised on a farm and during the summer, often would water the plants. While she watered plants, she would think about various things, and she discovered that one topic she loved to think about a lot was mathematics.

She then asked the girls, "Why are soap bubbles spherical?"

The discussion led to properties of soap films. Soap films minimize surface area. In other words, if you fashion a loop of wire into some shape and then dip this frame into a bucket of soapy water, a soap film will form. The boundary of the soap film will be the wire loop and the shape of the rest of the film will be a shape which minimizes surface area.

Dr. Staffilani showed the girls many soap films and she talked a little bit about applications of minimal surfaces. For example, in biology, cell membranes often behave like minimal surfaces. The girls then got to experiment with wire loops to see what the associated minimal surface would look like.

Kay made a particularly intriguing wire loop. She fashioned a piece of wire into the shape of a trefoil knot. Can you figure out what kind of minimal surface has a trefoil knot for a boundary? If you think you know, send in your answer to girlsangle@gmail.com!

<u>Session 4 – Meet 12 – May 7, 2009</u>

Mentors: Cammie Smith Barnes, Annie Huang, Kay Kirkpatrick, Lauren McGough
Jennifer Melot, Mia Minnes, Nike Sun

The last meet of the fourth session closed with a secret code treasure hunt!

Girls were given a number of secret codes. A number of math problems were provided whose solutions were hints to how to decipher the secret codes. One of the secret codes gave the location of a hidden treasure.

The solution was a true group effort. To all the girls who worked on the treasure hunt:

KYZGVMTSBMTEYZU GEVBU HYV USKKIIJEZG EZ KVMKCEZG MBB TFI KYJIU EZ TFI HEZMB TVIMUSVI FSZT MZJ HEZJEZG TFI FEJJIZ KFYKYBMTIU!

And, finally, a message to **Aba-ka-dabra**, **Anonymous**, **August**, **Caitlin**, **Cat**, **cat in the hat**, **Grace**, **Hadassah**, **Henriette**, **Honda**, **Ilana**, **Jo**, **littleMeme**, **Lucky**, **Mouse**, **Resday**, **Rowena**, **sports car**, **Sylvia, Sylvie**, **The Cat**, **Tree**, **Trisscar** and **Z:**

FMRI M GVIMT USAAIV!

We hope to see all of you on September 10!

Calendar

Session 4: (all dates in 2009)

January	29	Start of fourth session!
February	5	Sara Seager, Earth and Planetary Science, MIT
	12	
	19	Winter break - No meet
	26	Tanja Bosak, Earth and Planetery Sciences, MIT
March	5	Leia Stirling, Boston Children's Hospital
	12	
	19	Taylor Walker, DiMella Shaffer Architecture
	26	Spring recess - No meet
April	2	
	9	No meet - Rescheduled for April 23
	16	Eleanor Duckworth, Harvard Graduate School of Education
	23	
	30	Gigliola Staffilani, Mathematics, MIT
May	7	

Session 5: (all dates in 2009)

September	10	Start of fifth session!
	17	
	24	
October	1	
	8	
	15	
	22	
	29	No meet
November	5	
	12	
	19	
	26	Thanksgiving - No meet
December	3	
	10	

Girls' Angle: A Math Club for Girls

Gain confidence in math! Discover how interesting and exciting math can be! Make new friends!

What is Girls' Angle? Girls' Angle is a math club for girls and a supportive community for all girls and women engaged in the study, use and creation of mathematics. Our primary mission is to foster and nurture girls' interest and ability in mathematics and empower them to be able to tackle any field, no matter the level of mathematical sophistication required. We offer a comprehensive approach to math education and use a four component strategy to achieve our mission: Girls' Angle mentors, the Girls' Angle Support Network, the Girls' Angle Bulletin and Community Outreach.

Who are the Girls' Angle mentors? Our mentors possess a deep understanding of mathematics and enjoy explaining math to others. The mentors get to know each member as an individual and design custom tailored projects and activities designed to help the member improve at mathematics and develop her thinking abilities. Because we believe learning follows naturally when there is motivation, our mentors work hard to motivate. In order for members to see math as a living, creative subject, at least one mentor is present at every meet who has proven and published original theorems.

What is the Girls' Angle Support Network? The Support Network consists of professional women who use math in their work and are eager to show the members how and for what they use math. Each member of the Support Network serves as a role model for the members. Together, they demonstrate that many women today use math to make interesting and important contributions to society. They write articles for the Bulletin, take part in interviews and visit the club.

What is the Girls' Angle Bulletin? The Girls' Angle Bulletin is a bimonthly (6 issues per year) electronic publication that features interviews, articles and information of mathematical interest as well as a comic strip that involves mathematics.

What is Community Outreach? Girls' Angle accepts commissions to solve math problems from members of the community. Our members solve them. We believe that when our members' efforts are actually used in real life, the motivation to learn math increases.

Who can join? Ultimately, we hope to open membership to all women. Currently, we are open primarily to girls in grades 5-10. We aim to overcome math anxiety and build solid foundations, so we welcome *all girls* regardless of perceived mathematical ability. There is no entrance test.

In what ways can a girl participate? There are 2 ways: **membership** and **active subscription** to the Girls' Angle Bulletin. **Membership** is granted per session and includes access to the club and extends the member's subscription to the Girls' Angle Bulletin to one year from the start of the current or upcoming session. You can also pay per session. If you pay per session and come for three meets, you will get a subscription to the Bulletin. **Active subscriptions** to the Girls' Angle Bulletin allow the subscriber to ask and receive answers to math questions through email. Please note that we will not answer email questions if we think that we are doing the asker's homework! We currently operate in 12 meet sessions, but girls are welcome to join at any time. The program is individually focused so the concept of "catching up with the group" doesn't apply. Note that you can receive the Girls' Angle Bulletin free of charge. Just send us email with your request.

Where is Girls' Angle located? Girls' Angle is located about 10 minutes walk from Central Square on Magazine Street in Cambridge, Massachusetts. For security reasons, only members and their parents/guardian will be given the exact location of the club and its phone number.

When are the club hours? Girls' Angle meets Thursdays from 3:45 to 5:45. For calendar details, please visit our website at www.girlsangle.org or send us email.

Can you describe what the activities at the club will be like? Girls' Angle activities are tailored to each girl's specific needs. We assess where each girl is mathematically and then design and fashion strategies that will help her develop her mathematical abilities. Everybody learns math differently and what works best for one individual may not work for another. At Girls' Angle, we are very sensitive to individual differences. If you would like to understand this process in more detail, please email us!

Are donations to Girls' Angle tax deductible? Yes. Currently, Science Club for Girls, a 501(c)(3) corporation, is holding our treasury. Please make donations out to **Girls' Angle c/o Science Club for Girls** and send checks to Ken Fan, P.O. Box 410038, Cambridge, MA 02141-0038.

Who is the Girls' Angle director? Ken Fan is the director and founder of Girls' Angle. He has a Ph.D. in mathematics from MIT and was an assistant professor of mathematics at Harvard, a member at the Institute for Advanced Study and a National Science Foundation postdoctoral fellow. In addition, he has designed and taught math enrichment classes at Boston's Museum of Science and worked in the mathematics educational publishing industry. Ken has volunteered for Science Club for Girls and worked with girls to build large modular origami projects that were displayed at Boston Children's Museum. These experiences and the enthusiasm of the girls of Science Club for Girls have motivated him to create Girls' Angle.

Who advises the director to ensure that Girls' Angle realizes its goal of helping girls develop their mathematical interests and abilities? Girls' Angle has a stellar Board of Advisors. They are:

Connie Chow, executive director of Science Club for Girls
Yaim Cooper, graduate student in mathematics, Princeton
Julia Elisenda Grigsby, NSF postdoctoral fellow, Columbia University
Grace Lyo, Moore Instructor, MIT
Lauren McGough, MIT '12
Mia Minnes, Moore Instructor, MIT
Beth O'Sullivan, co-founder of Science Club for Girls.
Elissa Ozanne, Senior Research Scientist, Harvard Medical School.
Kathy Paur, Ph.D., Harvard
Katrin Wehrheim, associate professor of mathematics, MIT
Lauren Williams, Benjamin Pierce assistant professor of mathematics, Harvard

At Girls' Angle, mentors will be selected for their depth of understanding of mathematics as well as their desire to help others learn math. But does it really matter that girls be instructed by people with such a high level understanding of mathematics? We believe YES, absolutely! One goal of Girls' Angle is to empower girls to be able to tackle *any* field regardless of the level of mathematics required, including fields that involve original research. Over the centuries, the mathematical universe has grown enormously. Without guidance from people who understand a lot of math, the risk is that a student will acquire a very shallow and limited view of mathematics and the importance of various topics will be improperly appreciated. Also, people who have proven original theorems understand what it is like to work on questions for which there is no known answer and for which there might not even be an answer. Much of school mathematics (all the way through college) revolves around math questions with known answers, and most teachers have structured their teaching, whether consciously or not, with the knowledge of the answer in mind. At Girls' Angle, girls will learn strategies and techniques that apply even when no answer is known.

Also, math should not be perceived as the stuff that is done in math class. Instead, math lives and thrives today and can be found all around us. Girls' Angle mentors can show girls how math is relevant to their daily lives and how this math can lead to abstract structures of enormous interest and beauty.

Girls' Angle: A Math Club for Girls

Membership Application

Applicant's Name: (last) ______________________________ (first) ______________________________

Applying For:
- ☐ Membership (Access to club, premium subscription)
- ☐ Subscription to Girls' Angle Bulletin
- ☐ Premium Subscription (interact with mentors through email)

Parents/Guardians: ____

Address: ____ Zip Code: __________

Home Phone: _________________ Cell Phone: _________________ Email: ______________________

Emergency contact name and number: ____

Pick Up Info: For safety reasons, only the following people will be allowed to pick up your daughter. They will have to sign her out. Names: ____

Medical Information: Are there any medical issues or conditions, such as allergies, that you'd like us to know about? ____

Photography Release: Occasionally, photos and videos are taken to document and publicize our program in all media forms. We will not print or use your daughter's name in any way. Do we have permission to use your daughter's image for these purposes? **Yes** **No**

Eligibility: For now, girls who are roughly in grades 5-10 are welcome. Although we will work hard to include every girl no matter her needs and to communicate with you any issues that may arise, Girls' Angle has the discretion to dismiss any girl whose actions are disruptive to club activities.

Permission: I give my daughter permission to participate in Girls' Angle. I have read and understand everything on this registration form and the attached information sheets.

____ Date: ________________________

(Parent/Guardian Signature)

Membership-Applicant Signature: ____

- ☐ Enclosed is a check for (indicate one) (prorate as necessary)
  - ☐ $216 for a 12 session membership
  - ☐ $50 for a one year active subscription
  - ☐ I am making a tax free charitable donation.
- ☐ I will pay on a per session basis at $20/session. (Note: You still must return this form.)

Please make check payable to: **Girls' Angle c/o Science Club for Girls**. Mail to: Ken Fan, P.O. Box 410038, Cambridge, MA 02141-0038. Please notify us of your application by sending email to girlsangle@gmail.com. Paying on a per session basis comes with a one year subscription to the Bulletin, but not the math question email service. Also, please sign and return the Liability Waiver.

Girls' Angle: A Math Club for Girls
Liability Waiver

I, the undersigned parent or guardian of the following minor(s)

____,

do hereby consent to my child(ren)'s participation in Girls' Angle and do forever and irrevocably release Girls' Angle and its directors, officers, employees, agents, and volunteers (collectively the "Releasees") from any and all liability, and waive any and all claims, for injury, loss or damage, including attorney's fees, in any way connected with or arising out of my child(ren)'s participation in Girls' Angle, whether or not caused by my child(ren)'s negligence or by any act or omission of Girls' Angle or any of the Releasees. I forever release, acquit, discharge and covenant to hold harmless the Releasees from any and all causes of action and claims on account of, or in any way growing out of, directly or indirectly, my minor child(ren)'s participation in Girls' Angle, including all foreseeable and unforeseeable personal injuries or property damage, further including all claims or rights of action for damages which my minor child(ren) may acquire, either before or after he or she has reached his or her majority, resulting from or connected with his or her participation in Girls' Angle. I agree to indemnify and to hold harmless the Releasees from all claims (in other words, to reimburse the Releasees and to be responsible) for liability, injury, loss, damage or expense, including attorneys' fees (including the cost of defending any claim my child might make, or that might be made on my child(ren)'s behalf, that is released or waived by this paragraph), in any way connected with or arising out of my child(ren)'s participation in the Program.

Signature of applicant/parent: ____Date: __________________

Print name of applicant/parent: ____

Print name(s) of child(ren) in program: ____

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